

IX H.W-5

1) Represent $\sqrt{5}$ on the number line.

2) Find the value of $\frac{3^{40} + 3^{39} + 3^{38}}{3^{41} + 3^{40} - 3^{39}}$

3) Simplify (i) $\sqrt{50} - \sqrt{98} + \sqrt{162}$

(ii) $4\sqrt{16} - 6\sqrt{343} + 18\sqrt{243} - \sqrt{196}$

4) Find the value of $(1^3 + 2^3 + 3^3)^{-\frac{3}{2}}$

5) Simplify $(\frac{1}{2})^{-2} + (\frac{2}{3})^{-2} + (\frac{3}{4})^{-2}$

6) Simplify $(\frac{x^a}{x^b})^{a+b} \times (\frac{x^b}{x^c})^{b+c} \times (\frac{x^c}{x^a})^{c+a}$

7) If $a = 2 + \sqrt{5}$ and $b = \frac{1}{a}$, find $a^2 + b^2$

8) If $x = 3 - 2\sqrt{2}$, find the value of $\sqrt{x} + \frac{1}{\sqrt{x}}$

9) If $\sqrt{2} = 1.414$, then find the value of $\frac{1}{\sqrt{2}+1}$

10) If $x = 4 - \sqrt{15}$, then find the value of $(x + \frac{1}{x})^2$

11) If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$; $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, find $x^2 + y^2$

12) Prove that $\frac{1}{3+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+1} = 1$

13) Find the values of a and b when $\frac{5+\sqrt{6}}{5-\sqrt{6}} = a + b\sqrt{6}$

14) If $f(x) = x^2 - 5x + 7$, evaluate $f(2) + f(\frac{1}{3}) - f(-1)$

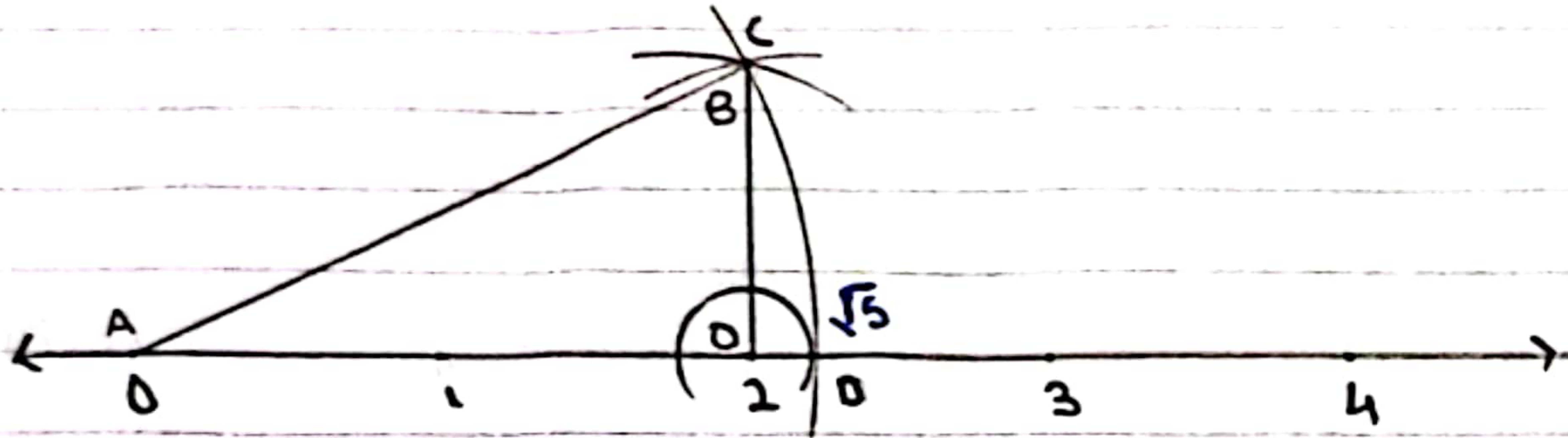
15) Verify if 1 and -3 are zeroes of the polynomial

$$3x^3 + 5x^2 - 11x + 3.$$

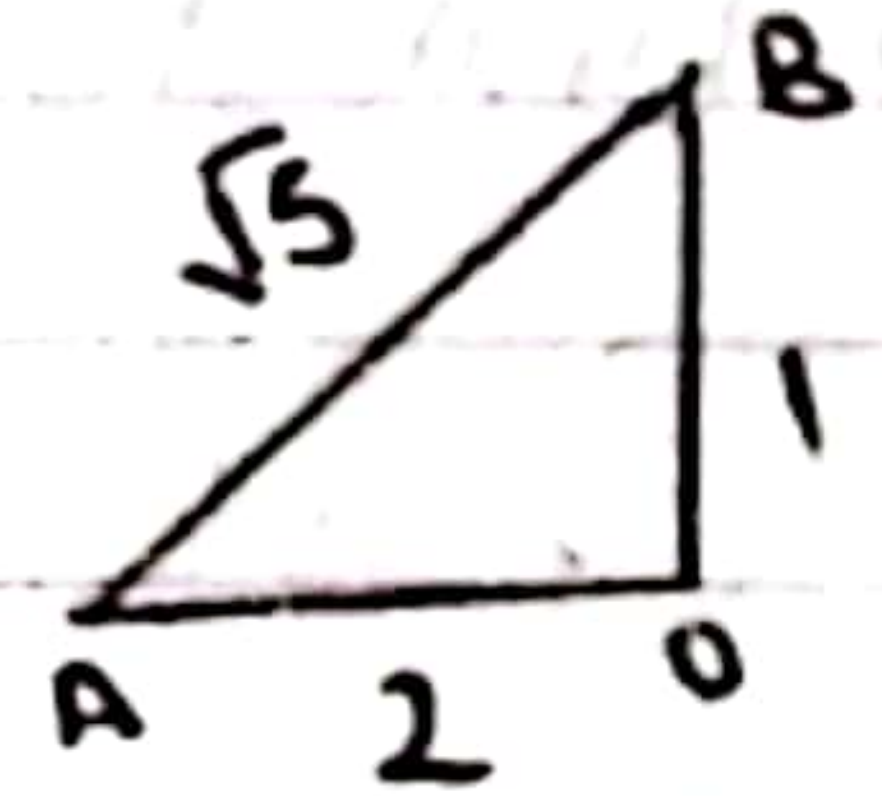
16) Find the value of k so that $x^3 + 3x^2 - kx - 3$ has -3 as a ~~factor~~ zero.

Homework V

1)



D represents $\sqrt{5}$ on the number line.



Acc. to Pythagoras
Theorem.

$$AO^2 + BO^2 = AB^2$$

$$2^2 + 1^2 = AB^2$$

$$4 + 1 = AB^2$$

$$\therefore AB = \sqrt{5}$$

H.W-5 answers

1) Construction

$$\frac{3^{40} + 3^{39} + 3^{38}}{3^{41} + 3^{40} - 3^{39}} = \frac{\cancel{3^{38}} (3^2 + 3^1 + 1)}{\cancel{3^{38}} (3^3 + 3^2 - 3)} = \frac{9+3+1}{27+9-3} = \frac{13}{33}$$

$$\begin{aligned} 3) (i) & \sqrt{50} - \sqrt{98} + \sqrt{162} \\ &= 5\sqrt{2} - 7\sqrt{2} + 9\sqrt{2} \\ &= \underline{\underline{7\sqrt{2}}} \end{aligned}$$

$$\begin{aligned} (ii) & \sqrt[4]{16} - 6\sqrt[3]{343} + 18\sqrt[5]{243} - \sqrt{196} \\ &= 2^{4 \times \frac{1}{4}} - 6 \times 7^{3 \times \frac{1}{3}} + 18 \times 3^{5 \times \frac{1}{5}} - 14 \\ &= 2 - 6 \times 7 + 18 \times 3 - 14 \\ &= 2 - 42 + 54 - 14 \\ &= 56 - 56 = \underline{\underline{0}} \end{aligned}$$

$$4) (1^3 + 2^3 + 3^3)^{-\frac{3}{2}} = (1+8+27)^{-\frac{3}{2}} = 36^{-\frac{3}{2}} = 6^{2 \times -\frac{3}{2}} = 6^{-3} = \frac{1}{6^3} = \underline{\underline{\frac{1}{216}}}$$

$$\begin{aligned} 5) \left(\frac{1}{2}\right)^{-2} + \left(\frac{2}{3}\right)^{-2} + \left(\frac{3}{4}\right)^{-2} &= 2^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{4}{3}\right)^2 \\ &= \frac{4 \times 36}{4 \times 9} + \frac{9 \times 9}{9 \times 4} + \frac{16 \times 4}{9 \times 4} \\ &= \frac{144+81+64}{36} = \underline{\underline{\frac{289}{36}}} \end{aligned}$$

$$\begin{aligned} 6) & \left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} \\ &= (x^{a-b})^{a+b} \times (x^{b-c})^{b+c} \times (x^{c-a})^{c+a} \\ &= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2} \\ &= x^{a^2-b^2+b^2-c^2+c^2-a^2} \\ &= x^0 = \underline{\underline{1}} \end{aligned}$$

$$7) a = 2 + \sqrt{5}$$

$$b = \frac{1}{a} = \frac{1}{2 + \sqrt{5}} = \frac{2 - \sqrt{5}}{2^2 - (\sqrt{5})^2} = \frac{2 - \sqrt{5}}{4 - 5} = \frac{2 - \sqrt{5}}{-1} = -(2 - \sqrt{5})$$

$$a^2 = (2 + \sqrt{5})^2 = 4 + 5 + 4\sqrt{5} = 9 + 4\sqrt{5}$$

$$b^2 = [-(2 - \sqrt{5})]^2 = (2 - \sqrt{5})^2 = 9 - 4\sqrt{5}$$

$$\therefore a^2 + b^2 = 9 + 4\sqrt{5} + 9 - 4\sqrt{5} = \underline{\underline{18}}$$

$$8) \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 = x + \frac{1}{x} + 2$$

$$x = 3 - 2\sqrt{2}$$

$$\frac{1}{x} = \frac{1}{3 - 2\sqrt{2}} = \frac{3 + 2\sqrt{2}}{9 - 8} = 3 + 2\sqrt{2}$$

$$\therefore \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 = 3 - 2\sqrt{2} + 3 + 2\sqrt{2} + 2$$

$$= 8$$

$$\therefore \sqrt{x} + \frac{1}{\sqrt{x}} = \underline{\underline{\sqrt{8} = 2\sqrt{2}}}$$

$$9) \frac{1}{\sqrt{2} + 1} = \frac{\sqrt{2} - 1}{2 - 1} = \sqrt{2} - 1 = 1.414 - 1 = \underline{\underline{0.414}}$$

$$10) x = 4 - \sqrt{15}$$

$$\frac{1}{x} = \frac{1}{4 - \sqrt{15}} = \frac{4 + \sqrt{15}}{16 - 15} = 4 + \sqrt{15}$$

$$\therefore \left(x + \frac{1}{x} \right)^2 = (4 - \sqrt{15} + 4 + \sqrt{15})^2 = 8^2 = 64$$

$$11) x = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \frac{(\sqrt{3} + \sqrt{2})^2}{3 - 2} = 3 + 2 + 2\sqrt{6} = 5 + 2\sqrt{6}$$

$$y = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} = \frac{(\sqrt{3} - \sqrt{2})^2}{3 - 2} = \frac{3 + 2 - 2\sqrt{6}}{1} = 5 - 2\sqrt{6}$$

$$\therefore x^2 + y^2 = (5 + 2\sqrt{6})^2 + (5 - 2\sqrt{6})^2$$

$$= 25 + 24 + 20\sqrt{6} + 25 + 24 - 20\sqrt{6}$$

$$= \underline{\underline{98}}$$

$$\begin{aligned}
 12) \text{ LHS, } & \frac{3-\sqrt{7}}{9-7} + \frac{\sqrt{7}-\sqrt{5}}{7-5} + \frac{\sqrt{5}-\sqrt{3}}{5-3} + \frac{\sqrt{3}-1}{3-1} \\
 &= \frac{3-\sqrt{7}}{2} + \frac{\sqrt{7}-\sqrt{5}}{2} + \frac{\sqrt{5}-\sqrt{3}}{2} + \frac{\sqrt{3}-1}{2} \\
 &= \frac{3-\cancel{\sqrt{7}}+\cancel{\sqrt{7}}-\cancel{\sqrt{5}}+\cancel{\sqrt{5}}-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}-1}{2} = \frac{2}{2} = \underline{\underline{1}}, \text{ RHS}
 \end{aligned}$$

$$13) \frac{(5+\sqrt{6})^2}{25-6} = \frac{25+6+10\sqrt{6}}{19} = \frac{31+10\sqrt{6}}{19} = \frac{31}{19} + \frac{10\sqrt{6}}{19}$$

$$\therefore a = \frac{31}{19}$$

$$b = \frac{10}{19}$$

$$14) f(x) = x^2 - 5x + 7$$

$$f(2) = 4 - 10 + 7 = 11 - 10 = 1$$

$$f\left(\frac{1}{3}\right) = \frac{1}{9} - \frac{5 \times \frac{1}{3}}{3 \times 3} + 7 = \frac{1 - 15 + 63}{9} = \frac{49}{9}$$

$$f(-1) = 1 + 5 + 7 = 13$$

$$\therefore f(2) + f\left(\frac{1}{3}\right) - f(-1) = 1 + \frac{49}{9} - 13 = \frac{49}{9} - 12$$

$$= \frac{49 - 108}{9} = -\frac{59}{9}$$

$$15) \text{ Let } p(x) = 3x^3 + 5x^2 - 11x + 3$$

$$p(1) = 3 + 5 - 11 + 3 = 11 - 11 = 0$$

$$\begin{aligned}
 p(-3) &= 3(-3)^3 + 5(-3)^2 - 11(-3) + 3 \\
 &= -81 + 45 + 33 + 3 \\
 &= -81 + 81 = 0
 \end{aligned}$$

$\therefore 1$ and -3 are zeroes of $p(x)$

$$16) \text{ Since } -3 \text{ is a zero of } p(x) = x^3 + 3x^2 - kx - 3, \text{ then}$$

$$p(-3) = 0$$

$$\Rightarrow (-3)^3 + 3(-3)^2 - k(-3) - 3 = 0$$

$$\Rightarrow -27 + 27 + 3k - 3 = 0$$

$$3k = 3$$

$$\underline{\underline{k=1}}$$