Test - 5

1) Find the zeroes of $p(x) = 7x^2 - \frac{11}{3}x - \frac{2}{3}$ and verify the relationship between the zeroes and their coefficients.

2) If m, n are the zeroes of $p(y) = 2y^2 + 5y + k$ satisfying the relation $m^2 + n^2 + mn = \frac{21}{4}$, then find the value of k.

3) If α and β are the zeroes of $f(x) = Ax^2 + Bx + C$, then find the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$.

Test - 5 Answers

1) Let $p(x) = 7x^2 - \frac{11}{3}x - \frac{2}{3} = 21x^2 - 11x - 2$ be of the form $ax^2 + bx + c$;

where $a = 7, b = \frac{-11}{3}, c = -\frac{2}{3}$

$p(x) = 21x^2 - 11x - 2 = 0$

S: -11 P: $-42 <$ -14, 3

$\Rightarrow 21x^2 - 14x + 3x - 2 = 0$

$\Rightarrow 7x(3x-2) + 1(3x-2) = 0$

$\Rightarrow (7x+1)(3x-2) = 0$

$\therefore x = \frac{-1}{7}, \frac{2}{3}$ are the zeroes of $p(x)$

Verification :- Let $\alpha = -\frac{1}{7}$ and $\beta = \frac{2}{3}$

$\alpha + \beta = -\frac{1}{7} + \frac{2}{3} = \frac{-3+14}{21} = \frac{11}{21} = -\left(\frac{-11}{21}\right) = \frac{-b}{a} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

$\alpha\beta = -\frac{1}{7} \times \frac{2}{3} = -\frac{2}{21} = \frac{c}{a} = \frac{\text{Constant term}}{\text{coefficient of } x^2}$

Hence verified

2) $a = 2, b = 5, c = k$

$m + n = -\frac{b}{a} = -\frac{5}{2}$

$mn = \frac{c}{a} = \frac{k}{2}$

$m^2 + n^2 + mn = \frac{21}{4}$

$\Rightarrow (m+n)^2 - 2mn + mn = \frac{21}{4}$

$\Rightarrow (m+n)^2 - mn = \frac{21}{4}$.

$\Rightarrow \frac{25}{4} - \frac{k}{2} = \frac{21}{4}$

$\Rightarrow -\frac{k}{2} = \frac{21}{4} - \frac{25}{4}$

$\Rightarrow \cancel{-}\frac{k}{2} = \cancel{-}\frac{4}{4}$

$\boxed{k = 2}$

3) $\alpha+\beta = -\frac{B}{A}$

$\alpha\beta = \frac{C}{A}$

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$= \frac{-\frac{B^3}{A^3} - 3\times\frac{C}{A}\times-\frac{B}{A}}{\frac{C}{A}}$$

$$= \frac{-\frac{B^3}{A^3} + \frac{3BC}{A^2}}{\frac{C}{A}}$$

$$= \frac{-B^3 + 3ABC}{A^{\cancel{3}2}} \times \frac{\cancel{A}^1}{C}$$

$$= \frac{3ABC - B^3}{A^2C}$$