X Homework-1

1) A quadratic polynomial, whose zeroes are -3 and 4 is

(a) x^2-x+12 (b) x^2+x+12 (c) $\frac{x^2}{2}-\frac{x}{2}-6$ (d) $2x^2+2x-24$

2) The zeroes of the quadratic polynomial x^2+kx+k; $k\neq 0$

(a) cannot be positive (c) are always equal

(b) cannot be negative (d) are always unequal

3) If the graph of a polynomial intersects the x-axis at exactly two points, then it

(a) cannot be a linear or a cubic polynomial

(b) can be a quadratic polynomial only

(c) can be a cubic or a quadratic polynomial

(d) can be a linear or a quadratic polynomial.

4) Find the zeroes of the polynomial $x^2+\frac{1}{6}x-2$ and verify the relation between the coefficients and zeroes of the polynomial.

5) If p and q are the zeroes of $f(x)=2x^2-7x+3$, find the value of p^2+q^2

6) Find the zeroes of the polynomial $\sqrt{3}x^2-11x+6\sqrt{3}$ and verify the relationship between the zeroes and the coefficients of the polynomials.

7) If sum of squares of the zeroes of $p(x)=x^2-8x+k$ is 40, find k.

8) If $\frac{2}{3}$ and -3 are the zeroes of the polynomial ax^2+7x+b, then find the values of a and b.

9) Which of the following is a quadratic polynomial having zeroes $-\frac{2}{3}$ and $\frac{2}{3}$?

(a) $4x^2-9$ (b) $\frac{4}{9}(9x^2+4)$ (c) $x^2+\frac{9}{4}$ (d) $5(9x^2-4)$

10) If α,β are zeroes of $p(x)=4x^2-3x-7$, then $\frac{1}{\alpha}+\frac{1}{\beta}=$ ___

(a) $\frac{7}{3}$ (b) $-\frac{7}{3}$ (c) $\frac{3}{7}$ (d) $\frac{-3}{7}$

X Homework - 1 (Answers)

1) Let $\alpha = -3$ and $\beta = 4$

$\alpha + \beta = -3 + 4 = 1$

$\alpha\beta = -3 \times 4 = -12$

$\therefore$ Required polynomial is $x^2 - (\alpha+\beta)x + \alpha\beta$

$= x^2 - 1x + (-12)$

$= x^2 - x - 12$

$= \frac{x^2}{2} - \frac{x}{2} - 6$ (c)

2) $a = 1, b = k, c = k$

Sum of zeroes $= -\frac{b}{a} = -k$, -ve

product of zeroes $= \frac{c}{a} = k$, +ve

Thus the zeroes cannot be positive (a)

3) can be a cubic or a quadratic polynomial (c)

4) Let $p(x) = x^2 + \frac{1}{6}x - 2 = \frac{1}{6}(6x^2 + x - 12)$

$a = 6, b = 1, c = -12$ and α, β be the zeroes.

put $p(x) = 0$

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1 $-72 < {-8 \atop 9}$

$\Rightarrow 6x^2 + x - 12 = 0$

$\Rightarrow 6x^2 + 9x - 8x - 12 = 0$

$\Rightarrow 3x(2x+3) - 4(2x+3) = 0$

$\Rightarrow (3x-4)(2x+3) = 0$

$\therefore x = \frac{4}{3}, -\frac{3}{2}$ are the zeroes of $p(x)$

Verification:- let $\alpha = \frac{4}{3}$ and $\beta = -\frac{3}{2}$

Sum of zeroes $= \alpha+\beta = \frac{4^{\times 2}}{3_{\times 2}} - \frac{3^{\times 3}}{2_{\times 3}} = \frac{8-9}{6} = \frac{-1}{6} = \frac{-b}{a} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of zeroes $= \alpha\beta = \frac{4}{3} \times \frac{-3}{2} = \frac{-12}{6} = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

Hence verified.

5) $a = 2,\ b = -7,\ c = 3$

$p+q = \frac{-b}{a} = \frac{7}{2}$

$pq = \frac{c}{a} = \frac{3}{2}$

$[a^2+b^2 = (a+b)^2 - 2ab]$

$p^2+q^2 = (p+q)^2 - 2pq$

$= \left(\frac{7}{2}\right)^2 - \cancel{2}^{1} \times \frac{3}{\cancel{2}_{1}}$

$= \frac{49}{4} - 3 = \frac{49-12}{4}$

$= \frac{37}{4}$

6) Let $p(x) = \sqrt{3}x^2 - 11x + 6\sqrt{3}$ be of the form ax^2+bx+c; where $a = \sqrt{3},\ b = -11,\ c = 6\sqrt{3}$ and α, β be the zeroes.

$p(x) = \sqrt{3}x^2 - 11x + 6\sqrt{3} = 0$

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-11 $18 < {-9 \atop -2}$

$\Rightarrow \sqrt{3}x^2 - 9x - 2x + 6\sqrt{3} = 0$

$\Rightarrow \sqrt{3}x(x - 3\sqrt{3}) - 2(x - 3\sqrt{3}) = 0$

$\Rightarrow (\sqrt{3}x - 2)(x - 3\sqrt{3}) = 0$

$\therefore x = \frac{2}{\sqrt{3}}, 3\sqrt{3}$ are the zeroes of $p(x)$

verification:- let $\alpha = \frac{2}{\sqrt{3}}$ and $\beta = 3\sqrt{3}$

Sum of zeroes $= \alpha+\beta = \frac{2}{\sqrt{3}} + 3\sqrt{3} = \frac{2+9}{\sqrt{3}} = \frac{11}{\sqrt{3}} = -\left(\frac{-11}{\sqrt{3}}\right) = \frac{-b}{a} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of zeroes $= \alpha\beta = \frac{2}{\sqrt{3}} \times 3\sqrt{3} = \frac{6\sqrt{3}}{\sqrt{3}} = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

Hence verified.

7) Let α and β be the zeroes of $p(x) = x^2 - 8x + k$

$a = 1, b = -8, c = k$

$\alpha + \beta = -\frac{b}{a} = 8$

$\alpha\beta = \frac{c}{a} = k$

ATQ, $\alpha^2 + \beta^2 = 40$

$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta = 40$

$\Rightarrow 64 - 2k = 40$

$\cancel{-}2k = \cancel{-}24$

$k = \frac{24}{2} = \underline{12}$

8) Let $p(x) = ax^2 + 7x + b$ be of the form $Ax^2 + Bx + C$; where

$A = a, B = 7, C = b$

Sum of zeroes $= \frac{2}{3} + (-3) = -\frac{B}{A}$

$\Rightarrow \frac{2-9}{3} = -\frac{7}{a}$

$\Rightarrow \frac{\cancel{-7}^{1}}{3} = \frac{\cancel{-7}^{1}}{a}$

$\therefore \boxed{a = 3}$

Product of zeroes $= \frac{2}{3} \times -3 = \frac{C}{A}$

$\Rightarrow -2 = \frac{b}{a}$

$\Rightarrow -2 = \frac{b}{3}$

$\therefore \boxed{b = -6}$

9) Sum of zeroes $= -\frac{2}{3} + \frac{2}{3} = 0$

Product of zeroes $= -\frac{2}{3} \times \frac{2}{3} = -\frac{4}{9}$

$\therefore$ the required polynomial $= x^2 - 0x - \frac{4}{9}$

$= x^2 - \frac{4}{9}$

$= 9x^2 - 4$

$= 5(9x^2 - 4)$ (d)

$a = 4, b = -3, c = -7$

10) $\alpha + \beta = -\frac{b}{a} = \frac{3}{4}$

$\alpha\beta = \frac{c}{a} = -\frac{7}{4}$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{3}{\cancel{4}}}{-\frac{7}{\cancel{4}}} = -\frac{3}{7}$ (d)