X Model Exam – Set 2

$$\begin{array}{r|l} 2 & 84 \\ 7 & 42 \\ 2 & 6 \\ & 3 \end{array}$$

1) $40 = 2^3 \times 5$

$84 = 2^2 \times 7 \times 3$

$HCF = 2^2 = 4$ cm (d)

2) $2x^2+5x-4$

$a=2, b=5, c=-4$

$p+q = \frac{-b}{a} = \frac{-5}{2}$

$pq = \frac{c}{a} = \frac{-4}{2} = -2$

$(1-p)(1-q) = 1-q-p+pq = 1-(p+q)+pq = 1+\frac{5}{2}-2$

$= \frac{5}{2}-1 = \frac{3}{2}$ (a)

3) $a=2, b=-10, c=k$

For equal roots, $b^2-4ac=0$

$100-8k=0$

$k = \frac{\cancel{100}^{25}}{\cancel{8}\,2} = \frac{25}{2}$ (a)

4) $a_1=-6, b_1=-2, c_1=-21$

$a_2=2, b_2=-3, c_2=7$

$\frac{a_1}{a_2} = \frac{-6}{2} = -3$

$\frac{b_1}{b_2} = \frac{-2}{-3} = \frac{2}{3}$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, intersecting lines (b)

5) $x-18 = y-x = -3-y$

$x+y = -3+18 = 15$ (c)

6) $3p+5-\cancel{3p}\cancel{+1} = 5p\cancel{+1}-\cancel{3p}-5$

$-2p = -10$

$p = 5$ (b)

7) When $x=0$, $2y=2$

$y=1$

$\therefore P(0,1)$ (c)

8) 7 units (d)

9) $2\times\left(\frac{1}{2}\right)^2 - 3\times\left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2 + 1$

$= 2\times\frac{1}{4} - 3\times\frac{1}{2} + 3 + 1$

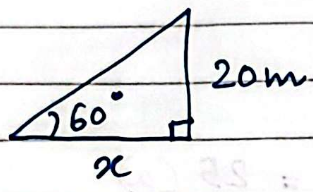
$= \frac{1}{2} - \frac{3}{2} + 4 = \frac{-2}{2} + 4 = -1+4 = 3$ (c)

10) $\cos A = 0$

$A = 90^\circ$

$\frac{1}{2}\sin\frac{90^\circ}{2} = \frac{1}{2}\sin 45^\circ = \frac{1}{2}\times\frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}}$ (d)

11)



$\tan 60^\circ = \frac{20}{x}$

$\sqrt{3} = \frac{20}{x}$

$x = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3}$ m (b)

12) Using Thales theorem, $\frac{x}{x+1} = \frac{x+3}{x+5}$

$\Rightarrow \cancel{x^2} + 5x = \cancel{x^2} + 4x + 3$

$x = 3$ (c)

13) $\angle POQ + \angle PTQ = 180^\circ$

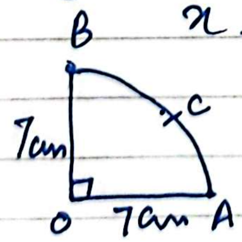
$\angle PTQ = 180^\circ - 90^\circ = 90^\circ$ (b)

14) $\angle OTP = 90^\circ$

Using exterior angle property in $\triangle OTP$,

$x = 90^\circ + 25^\circ = 115^\circ$ (d)

15)



Perimeter of the quadrant

$= OA + OB + \widehat{AB}$

$= 7 + 7 + \frac{\theta}{360^\circ}\times 2\pi r$

$= 14 + \frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 7 = 14 + 11 = 25\,cm$ (d)

16) $\pi r^2 = 154$

$\frac{22}{7} \times r^2 = 154$

$r^2 = \frac{154}{22} \times 7 = 49$

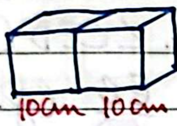
$r = 7m$

Perimeter $= 2\pi r = 2 \times \frac{22}{7} \times 7 = 44m$ (c)

17) lower limit = classmark $-$ $\frac{\text{classsize}}{2}$ $= 10 - \frac{6}{2} = 10 - 3 = 7$ (c)

18) favourable outcomes $= \{2, 3, 5\}$

P(a prime number) $= \frac{3}{6} = \frac{1}{2}$ (d)

19) 

$l = 20cm, b = 10cm, h = 10cm$

T.S.A $= 2(lb + bh + hl)$

$= 2(200 + 100 + 200) = 2 \times 500 = 1000\,cm^2$

(d) assertion is incorrect but Reason is correct.

20) P(doesnot hit a boundary) $= 1 - \frac{9}{45} = 1 - \frac{1}{5} = \frac{4}{5}$, True

Reason:- True

(a) both assertion and reason are correct and Reason is the correct explanation of assertion.

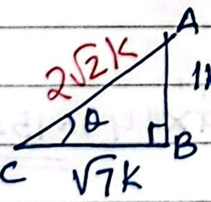
SECTION - B

21) HCF × LCM = product of no.s

$13 \times LCM = 65 \times 117$

$LCM = \frac{65 \times 117}{13} = \underline{\underline{585}}$

$117 \times 5 = 585$

22) 

Using Pythagoras theorem,

$AC^2 = k^2 + 7k^2 = 8k^2$

$AC = 2\sqrt{2}k$

$\operatorname{cosec}\theta = 2\sqrt{2}$

$\sec\theta = \frac{2\sqrt{2}}{\sqrt{7}}$

$$\therefore \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}} = \frac{\frac{48}{\cancel{7}}}{\frac{64}{\cancel{7}}} = \frac{\cancel{48}\,6}{\cancel{64}\,8} = \frac{3}{4}$$

OR

$\sin(A-B) = \frac{1}{2} \Rightarrow A - B = 30^\circ \rightarrow (1)$

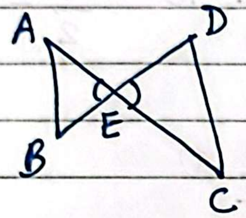
$\cos(A+B) = \frac{1}{2} \Rightarrow A + B = 60^\circ \rightarrow (2)$

$(1) + (2),\ 2A = 90^\circ$

$A = 45^\circ$ //

$B = 15^\circ$ //

23)



Given:- $\frac{EA}{EC} = \frac{EB}{ED}$

To prove :- $\Delta EAB \sim \Delta ECD$

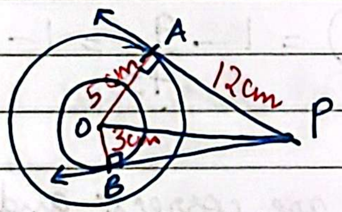
Proof:- In ΔAEB and ΔCED, $\angle AEB = \angle CED$ (VOA)

$\frac{EA}{EC} = \frac{EB}{ED}$ (given)

$\therefore \Delta AEB \sim \Delta CED$ (SAS Similarity)

Hence Proved

24)



$OA \perp PA$, $OB \perp BP$ } radius $\perp$ tangent through the point of contact.

In rt. ΔOAP, using Pythagoras theorem, $OP^2 = 25 + 144 = 169$

$OP = 13$ cm

Similarly, in rt. ΔOBP, $BP^2 = OP^2 - OB^2 = 169 - 9 = 160$

$BP = \sqrt{160} = 4\sqrt{10}$ cm

25) $r = 14$ cm

area of quadrant $= \frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154\ \text{cm}^2$

SECTION - C

26) Let us assume that $\sqrt{5}$ is rational.

Then, $\sqrt{5} = \frac{a}{b}$; where a and b are co-prime integers and $b \neq 0$

$\Rightarrow \sqrt{5}b = a$

$\Rightarrow 5b^2 = a^2 \longrightarrow (1)$

$\Rightarrow$ 5 divides a^2

$\Rightarrow$ 5 divides a

Put $a = 5c$, where c is any integer.

From eq: (1), $5b^2 = 25c^2$

$\Rightarrow b^2 = 5c^2$

$\Rightarrow$ 5 divides b^2

$\Rightarrow$ 5 divides b

Thus, 5 is a common factor for a and b. But this contradicts the fact that ~~$\sqrt{5}$~~ a and b are co-primes. (ie HCF = 1). This contradiction arises due to our wrong assumption that $\sqrt{5}$ is rational. Hence $\sqrt{5}$ is irrational.

27) Let the zeroes be α and β.

$\alpha + \beta = -1$

$\alpha\beta = -20$

$\therefore$ The required polynomial is $k[x^2 - (\alpha+\beta)x + \alpha\beta]$; where k is any non-zero real number.

$= k[x^2 + x - 20]$

$= x^2 + x - 20$; where $k = 1$

S	P
1	-20
	-4, 5

$(x-4)(x+5) = 0$

$x = 4, -5$

Hence, the zeroes are 4 and -5.

28) $a = 2, b = k, c = 3$

For equal roots, $b^2 - 4ac = 0$

$\Rightarrow k^2 - 24 = 0$

$k = \sqrt{24} = \underline{\underline{\pm 2\sqrt{6}}}$

$$\begin{array}{r|l} 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$$

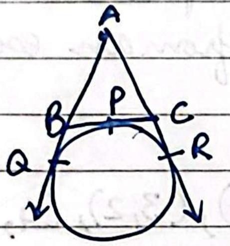
29) LHS, $\sec A\,(1 - \sin A)(\sec A + \tan A)$

$= \dfrac{1}{\cos A}(1 - \sin A)\left(\dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}\right)$

$= \dfrac{(1 - \sin A)(1 + \sin A)}{\cos^2 A} = \dfrac{1 - \sin^2 A}{\cos^2 A}$

$= \dfrac{\cos^2 A}{\cos^2 A} = 1$, RHS.

30)



Given:- AQ and AR are tangents from external point A.
Another tangent BC at P

To prove:- $AQ = \frac{1}{2}$ (perimeter of $\triangle ABC$)

Proof:- we know that the tangents drawn from an external point to a circle are equal in lengths,

$AQ = AR$ [$\because$ A is the external point] $\rightarrow (1)$

$BP = BQ$ [$\because$ B is the external point] $\rightarrow (2)$

$CP = CR$ [$\because$ C is the external point] $\rightarrow (3)$

$AQ = AB + BQ = AB + BP$ [from eq: (2)] $\rightarrow (4)$

$AR = AC + CR = AC + CP$ [from eq: (3)] $\rightarrow (5)$

(4)+(5), $AQ + AR = AB + (BP + CP) + AC$

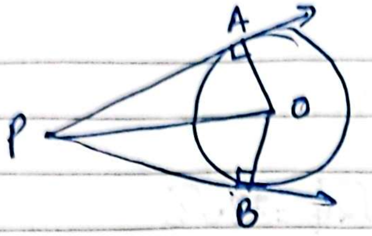
$\Rightarrow 2AQ = AB + BC + AC$ [from eq: (1)]

$\therefore AQ = \frac{1}{2}(AB + BC + AC)$

$= \frac{1}{2}$ (perimeter of $\triangle ABC$)

Hence Proved

OR



Given:- PA and PB are tangents drawn from external point P

To prove:- $PA = PB$

Construction:- Join OA, OB and OP

Proof:- $\angle OAP = \angle OBP = 90^\circ$ (radius $\perp$ tangent through the point of contact) $\rightarrow (1)$

In $\triangle OAP$ and $\triangle OBP$, $\angle OAP = \angle OBP$ (each 90°)

$OP = OP$ (common side)

$OA = OB$ (radii of the same circle)

$\therefore \triangle OAP \cong \triangle OBP$ (RHS congruency)

Thus, $PA = PB$ (by cpct)

Hence, the tangents drawn to ~~the~~ a circle from an external point are equal.

31) Total no. of outcomes = 36

(i) favourable outcomes = $\{(1,4), (2,3), (3,2), (4,1)\}$

$$P(E) = \frac{\text{no. of favourable outcomes}}{\text{Total no. of outcomes}}$$

$$P(\text{sum } 5) = \frac{4}{36} = \frac{1}{9}$$

(ii) favourable outcomes = $\{(4,6), (5,5), (6,4)\}$

$$P(\text{Sum } 10) = \frac{3}{36} = \frac{1}{12}$$

(iii) favourable outcomes = $\{(3,6), (6,3), (4,5), (5,4), (4,6), (5,5), (5,6), (6,4), (6,5), (6,6)\}$

$$P(\text{Sum atleast } 9) = \frac{10}{36} = \frac{5}{18}$$

SECTION-D

32) $x - y + 1 = 0$

$y = x + 1$

x	0	-1	1
y	1	0	2

$3x + 2y - 12 = 0$

$2y = 12 - 3x$

$y = \frac{12 - 3x}{2}$

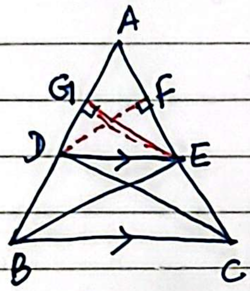
x	0	4	2
y	6	0	3

(graph)

From graph :- area $(\Delta) = \frac{1}{2} \times b \times h = \frac{1}{2} \times 5 \times 3$

$= \underline{\underline{7.5 \text{ sq units}}}$

33)



Given :- in ΔABC, $DE \parallel BC$

To prove :- $\frac{AD}{DB} = \frac{AE}{EC}$

Construction :- draw $DF \perp AC$ and $EG \perp AB$

Proof :-

$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta EDB)} = \frac{\cancel{\frac{1}{2}} \times AD \times \cancel{EG}}{\cancel{\frac{1}{2}} \times DB \times \cancel{EG}} = \frac{AD}{DB} \rightarrow (1)$

$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEC)} = \frac{\cancel{\frac{1}{2}} \times AE \times \cancel{DF}}{\cancel{\frac{1}{2}} \times EC \times \cancel{DF}} = \frac{AE}{EC} \rightarrow (2)$

we know that the Δ EDB and Δ DEC lie on the same base DE and between same parallels DE and BC,

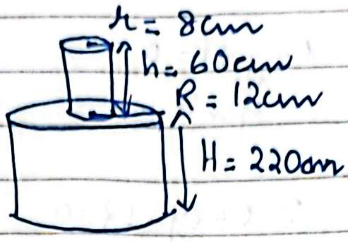
$\text{area}(\Delta EDB) = \text{area}(\Delta DEC) \rightarrow (3)$

From (1), (2) and (3), $\frac{AD}{DB} = \frac{AE}{EC}$

Hence proved.

34) mass = density × volume

density = $8 g/cm^3$



Volume of iron pole

= volume of ~~first~~ big cylinder + volume of small cylinder

$= \pi R^2 H + \pi r^2 h$

$= \pi (R^2 H + r^2 h) = 3.14(12 \times 12 \times 220 + 8 \times 8 \times 60)$

$= 3.14(31680 + 3840)$

$= 3.14 \times 35520$

$= 111532.8 cm^3$

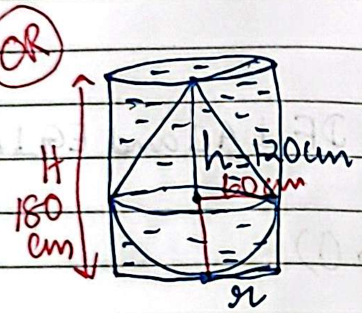
∴ mass of the pole = volume × density

$= 111532.8 \times 8$

$= 892262.4 g$

$= 892.26 Kg$

OR



h = 120cm

r = 60cm

H = 120 + 60 = 180cm

Volume of water left in the cylinder

= Volume of Cylinder − (volume of cone + volume of hemisphere)

$= \pi r^2 H - \left(\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right)$

$= \pi r^2 H - \pi r^2 \left(\frac{h}{3} + \frac{2}{3} r\right)$

$= \pi r^2 \left[H - \frac{h}{3} - \frac{2}{3} r\right] = \frac{22}{7} \times 60 \times 60 \left[180 - \frac{\cancel{120}^{40}}{\cancel{3}_1} - \frac{2 \times \cancel{60}^{20}}{\cancel{3}_1}\right]$

$= \frac{22}{7} \times 60 \times 60 \times 120$

$= \frac{7920000}{7} = 1131428.571 cm^3$

$= 1.13 m^3$

35)

C.I	f	c.f
65-85	4	4
85-105	x	$4+x$
105-125	13	$17+x$ c.f
$\boxed{125-145}$	20 f	$37+x$
145-165	14	$51+x$
165-185	y	$51+x+y$
185-205	4	$55+x+y$

median = 137

median class is 125-145

$f = 20$

$h = 20$

$c.f = 17+x$

$\frac{n}{2} = \frac{68}{2} = 34$

$n = 68 = 55+x+y \Rightarrow x+y = 13 \rightarrow (1)$

$$\text{median} = l + \frac{\frac{n}{2} - c.f}{f} \times h$$

$$137 = 125 + \frac{34-17-x}{\cancel{20}_1} \times \cancel{20}^1$$

$137 = 125 + 17 - x$

$137 = 142 - x$

$x = 142 - 137 = 5$ //

$y = 13 - 5 = 8$ //

SECTION-E

36) (i) 1, 3, 5, 7, 9, --- forms an AP with $a=1, d=2$

(ii) 2, 6, 10, 14, 18, ... forms an AP with $a=2, d=4$

(iii)(a) $S_{15} = \frac{n}{2}[2a+(n-1)d] = \frac{15}{2}[2+14\times2] = \frac{15}{2}\times 30 = 225$ squares

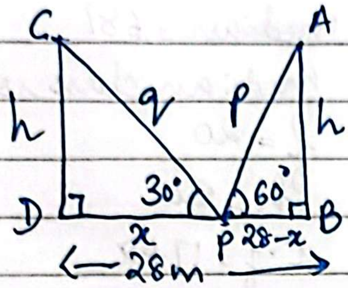
$\therefore$ area of shaded region $= 225 \times 2 \times 2 = 900\,cm^2$

OR (b) $S_n = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[4+(n-1)4]$

$= \frac{n}{\cancel{2}_1} \times \cancel{4}^2 [\cancel{1}+n-\cancel{1}] = \underline{\underline{2n^2}}$

$S_{10} = 2\times10\times10 = 200$ triangles //

37)
(i)



Let AB and CD be two poles of equal height.

(ii) (a) In rt. ΔCDP, $\tan 30° = \frac{h}{x}$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$x = h\sqrt{3} \rightarrow (1)$$

In rt. ΔABP, $\tan 60° = \frac{h}{28-x}$

$$\sqrt{3} = \frac{h}{28-x}$$

$28\sqrt{3} - x\sqrt{3} = h$

$28\sqrt{3} - 3h = h$ [from eq: (1)]

$4h = 28\sqrt{3}$

$h = 7\sqrt{3}m$

OR (b) In rt. ΔABP, $\sin 60° = \frac{\sqrt{3}}{2} = \frac{h}{p}$

$$h = \frac{\sqrt{3}p}{2} \rightarrow (2)$$

In rt. ΔCDP, $\sin 30° = \frac{1}{2} = \frac{h}{q}$

$$\Rightarrow h = \frac{q}{2} \rightarrow (3)$$

From (2) and (3) $\frac{\sqrt{3}p}{2} = \frac{q}{2}$

$$\therefore \boxed{q = \sqrt{3}p}$$

(iii) From eq: (1), $x = 7 \times 3 = 21m$
Hence the point of observation is at 21m and 7m away from the poles.

38) (i) distance covered by Niharika $= \frac{1}{4} \times 100 = 25m$

$\therefore$ coordinates of green flag $= (2, 25)$

(ii) distance covered by Preet $= \frac{1}{5} \times 100 = 20m$

(2, 25) —— (8, 20)

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(8-2)^2 + (20-25)^2}$$
$$= \sqrt{6^2 + (-5)^2} = \sqrt{36+25}$$
$$= \sqrt{61} \text{ m}$$

(iii) (a) $B(x, y) = B\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$= B\left(\frac{2+8}{2}, \frac{25+20}{2}\right) = B\left(5, \frac{45}{2}\right)$$
$$= B(5, 22.5)$$

OR

(b) $P(2, 25)$ —k— $R\left(4, \frac{45}{2}\right)$ —1— $Q(8, 20)$

$$R(x, y) = R\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$\left(4, \frac{45}{2}\right) = \left(\frac{8k+2}{k+1}, \frac{20k+25}{k+1}\right)$$

$$\therefore \frac{8k+2}{k+1} = 4$$
$$8k+2 = 4k+4$$
$$4k = 2$$
$$k = \frac{1}{2}$$

$\therefore$ The required ratio is 1 : 2