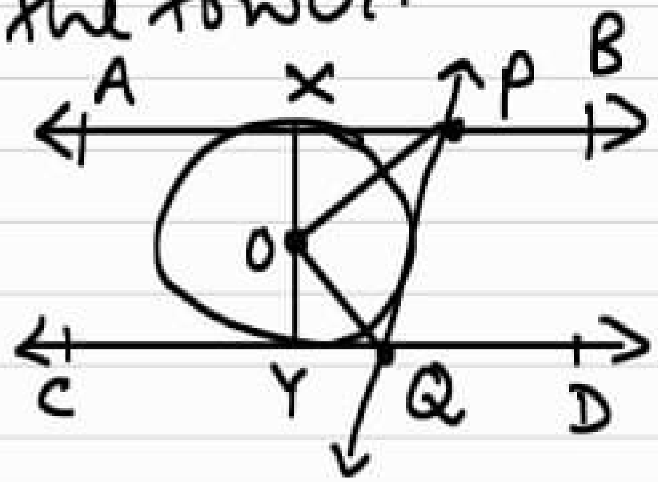
Test-22

1) P.T $\frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta} = \frac{1}{\sec\theta - \tan\theta}$

2) If $\sec\theta = \frac{a^2+b^2}{2ab}$, find other five T-ratios

3) A boy standing on the top of a building which is 10m above the ground, observes the angle of elevation of the top of a tower is 60° and angle of depression of its foot is 30°. Find the distance of the tower from the building and height of the tower.

4)



AB || CD

Prove that
$\angle POQ = 90^\circ$

5) If $u_i = \frac{x_i - 25}{10}$; $\Sigma f_i u_i = 20$ and $\Sigma f_i = 100$, find $\bar{x}$

6) If the difference of Median and Mean is 12, then find the difference of mode and median.

X Test - 22 (Answers)

1) LHS, $\dfrac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$

$= \dfrac{\sec\theta + \tan\theta - 1}{\tan\theta + 1 - \sec\theta}$

$= \dfrac{(\sec\theta + \tan\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta + 1 - \sec\theta}$ $[\sec^2\theta - \tan^2\theta = 1]$

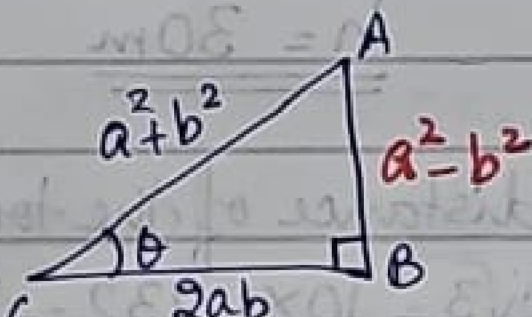
$= \dfrac{(\sec\theta + \tan\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta + 1 - \sec\theta}$

$= \dfrac{(\sec\theta + \tan\theta)\,[\cancel{1 - \sec\theta + \tan\theta}]^{1}}{\cancel{\tan\theta + 1 - \sec\theta}_{1}}$

$= \dfrac{\sec\theta + \tan\theta}{1}$

$= \dfrac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta} = \dfrac{\cancel{\sec\theta + \tan\theta}^{1}}{\cancel{(\sec\theta + \tan\theta)}_{1}(\sec\theta - \tan\theta)}$

$= \dfrac{1}{\sec\theta - \tan\theta}$, RHS



2) Using Pythagoras theorem in rt. $\triangle ABC$,

$AB^2 = AC^2 - BC^2$

$= (a^2+b^2)^2 - 4a^2b^2$

$= a^4 + b^4 + 2a^2b^2 - 4a^2b^2$

$= a^4 + b^4 - 2a^2b^2$

$= (a^2 - b^2)^2$

$\therefore AB = a^2 - b^2$

Other five Trigonometric ratios are

$\sin\theta = \dfrac{AB}{AC} = \dfrac{a^2-b^2}{a^2+b^2}$

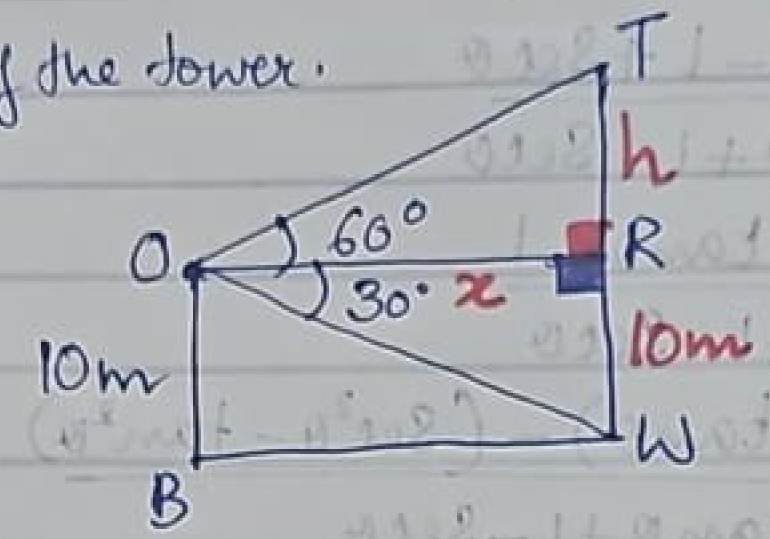
$\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{2ab}{a^2+b^2}$

$\tan\theta = \dfrac{AB}{BC} = \dfrac{a^2-b^2}{2ab}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{a^2+b^2}{a^2-b^2}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{2ab}{a^2-b^2}$

3) Let TW be the height of the tower.



In rt. ΔTRO,

$\tan 60° = \frac{TR}{OR}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow \boxed{x = \frac{h}{\sqrt{3}}} \to (1)$

In rt. ΔWRO, $\tan 30° = \frac{RW}{OR}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{x}$

$\Rightarrow \boxed{x = 10\sqrt{3}}$

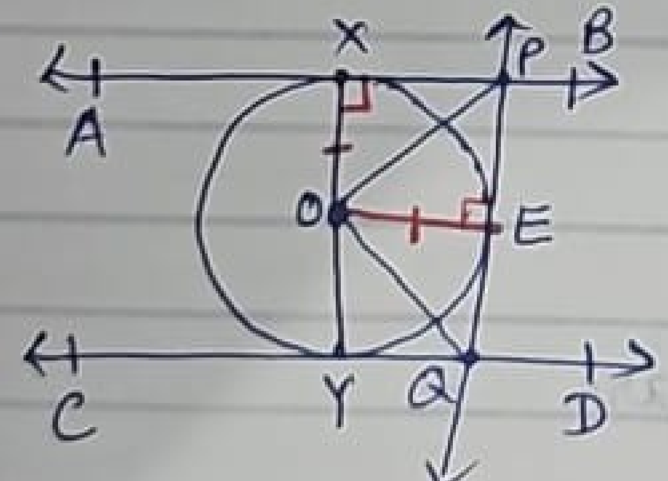
From eq: (1), $10\sqrt{3} = \frac{h}{\sqrt{3}}$

$\underline{\underline{h = 30m}}$

Hence, the distance of the tower from the building is
$= 10\sqrt{3} = 10 \times 1.732 = 17.32m$ //

Height of the tower $= h + 10 = 40m$ //

4)



Given:- AB, CD and PQ are tangents to the circle at centre O.

To prove:- $\angle POQ = 90°$

Construction:- Join OE

Proof:- In ΔOXP and ΔOEP, $\angle OXP = \angle OEP$ (radius ⊥ tangent through the point of contact)

OP = OP (common side)

OX = OE (radii of same circle)

$\therefore \Delta OXP \cong \Delta OEP$ (RHS congruency)

Thus, $\angle OPX = \angle OPE$ (by cpct) $\rightarrow (1)$

Similarly, we can prove $\triangle OEQ \cong \triangle OYQ$

and $\angle OQE = \angle OQY$ (by cpct) $\rightarrow (2)$

Since $AB \parallel CD$, $(\angle OPX + \angle OPE) + (\angle OQE + \angle OQY) = 180^\circ$

(co-interior angles)

$\Rightarrow 2\angle OPE + 2\angle OQE = 180^\circ$ (from eq's (1) and (2))

$\Rightarrow \angle OPE + \angle OQE = 90^\circ$

$\Rightarrow \angle OPQ + \angle OQP = 90^\circ \rightarrow (3)$

Using angle sum property in $\triangle OPQ$,

$\angle POQ = 180^\circ - (\angle OPQ + \angle OQP)$

$= 180^\circ - 90^\circ$

$\therefore \underline{\underline{\angle POQ = 90^\circ}}$ Hence Proved.

5) $u_i = \dfrac{x_i - a}{h} = \dfrac{x_i - 25}{10}$

$a = 25$

$h = 10$

$\Sigma f_i u_i = 20$

$\Sigma f_i = 100$

$\bar{x} = a + \dfrac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$= 25 + \dfrac{2\cancel{0}}{1\cancel{00}} \times 1\cancel{0}$

$= 25 + 2$

$= \underline{\underline{27}}$

6) Median − Mean = 12

$\Rightarrow$ Mean = Median − 12 $\rightarrow (1)$

Using Empirical formula,

Mode = 3Median − 2Mean

= 3Median − 2 (Median − 12)

= 3Median − 2Median + 24

= Median + 24

$\therefore$ Mode − Median = 24 //