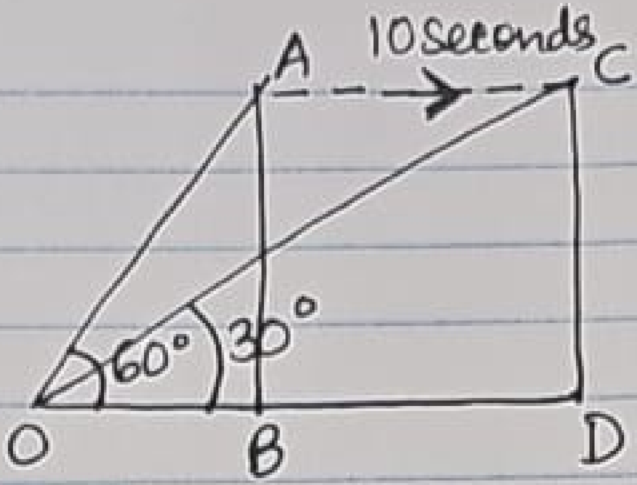
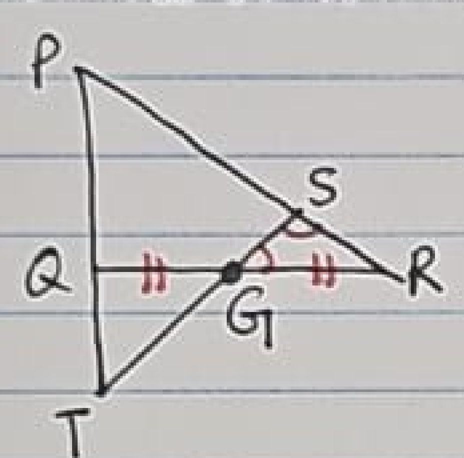
X Test-21

1) The angle of elevation of a hot air balloon at A from the ground is 60°. It reaches C in 10 seconds on the same height. The speed of hot air balloon is 720km/hr. Find the constant height at which the balloon is flying.



2)



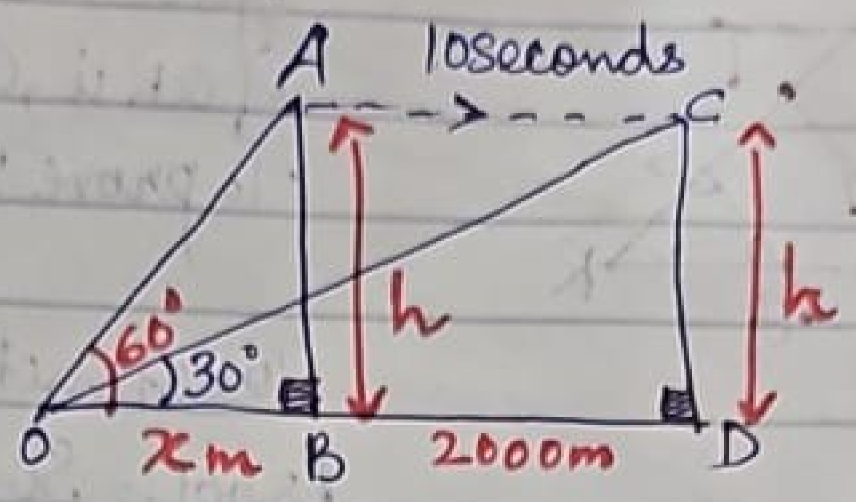
Given: $\angle RSG = \angle RGS$

G is the mid-point of QR

To prove :- $\frac{PT}{QT} = \frac{PS}{GR}$

X Test-21 (answers)

1) Let AC be the distance travelled by the balloon in 10 seconds.



Distance = speed × time

To find :- AB or CD

Speed = 720 km/hr = $\overset{40}{\cancel{720}} \times \frac{5}{\cancel{18}_{1}}$ = 200 m/s

AC = BD = 200 × 10 = 2000 m

In rt. ΔAOB, $\tan 60° = \frac{AB}{OB}$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \boxed{x = \frac{h}{\sqrt{3}}}$$

In rt. ΔCOD, $\tan 30° = \frac{CD}{OD}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+2000}$$

$$\Rightarrow x + 2000 = h\sqrt{3}$$

$$\Rightarrow \frac{h}{\sqrt{3}} + 2000 = h\sqrt{3}$$

$$\Rightarrow h + 2000\sqrt{3} = 3h$$

$$2h = 2000\sqrt{3}$$

$$h = 1000\sqrt{3}$$

$$= 1000 \times 1.732$$

$$= 1732 \text{ m}$$

$$= \underline{\underline{1.732 \text{ km}}}$$

Hence, the constant height at which the balloon is flying = 1.732 km (approx.)

2) If $\operatorname{cosec}\theta + \cot\theta = m$, then prove that $\frac{m^2-1}{m^2+1} = \cos\theta$

$$\operatorname{cosec}\theta + \cot\theta = m \rightarrow (1)$$

We know that, $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$\Rightarrow (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 1$$

$$\Rightarrow m(\operatorname{cosec}\theta - \cot\theta) = 1$$

$$\therefore \operatorname{cosec}\theta - \cot\theta = \frac{1}{m} \rightarrow (2)$$

$(1)-(2)$, $2\cot\theta = m - \frac{1}{m}$

$$\Rightarrow 2\cot\theta = \frac{m^2-1}{m}$$

$$\therefore \cot\theta = \frac{m^2-1}{2m} \rightarrow (3)$$

$(1)+(2)$, $2\operatorname{cosec}\theta = m + \frac{1}{m}$

$$\Rightarrow 2\operatorname{cosec}\theta = \frac{m^2+1}{m}$$

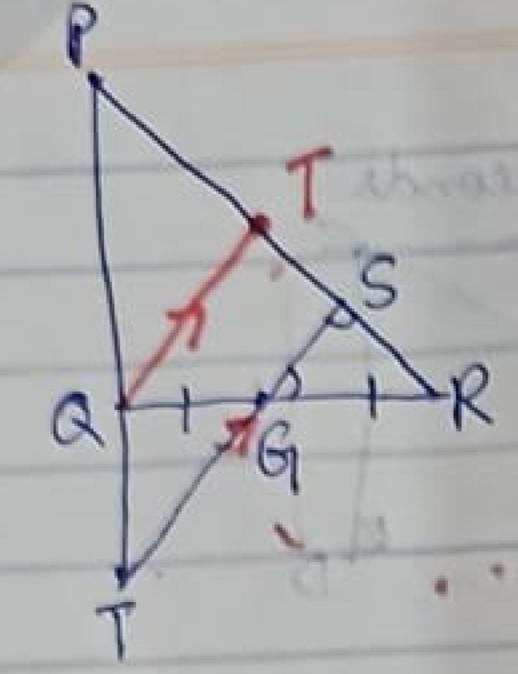
$$\therefore \operatorname{cosec}\theta = \frac{m^2+1}{2m} \rightarrow (4)$$

$$\frac{(3)}{(4)} \Rightarrow \frac{\frac{m^2-1}{\cancel{2m}}}{\frac{m^2+1}{\cancel{2m}}} = \frac{\cot\theta}{\operatorname{cosec}\theta}$$

$$\Rightarrow \frac{m^2-1}{m^2+1} = \frac{\cos\theta}{\cancel{\sin\theta}} \times \cancel{\sin\theta}$$

$$\therefore \frac{m^2-1}{m^2+1} = \cos\theta$$

2)



Given :- $\angle RSG = \angle RGS$. G is the mid-point of QR i.e., $QG = GR$

To prove :- $\frac{PT}{QT} = \frac{PS}{GR}$

Construction :- draw $QT \parallel TS$ to meet PR at T

Proof :- Since $\angle RGS = \angle RSG$,

$RG = RS$ (sides opposite to equal angles) $\rightarrow (1)$

Given, $RG = QG \rightarrow (2)$

Now, In $\triangle RQT$, Since $GS \parallel QT$ and G is the mid-point of RQ, using converse of mid-point theorem, S is also the mid-point of RT.

i.e, $RS = ST \rightarrow (3)$

From eq:s (1), (2) and (3), $RG = GQ = RS = ST \rightarrow (4)$

In $\triangle PTS$, since $QT \parallel TS$, using Thales theorem,

$$\frac{PQ}{QT} = \frac{PT}{TS}$$

$$\Rightarrow \frac{PQ}{QT} + 1 = \frac{PT}{TS} + 1$$

$$\Rightarrow \frac{PQ+QT}{QT} = \frac{PT+TS}{TS}$$

$$\Rightarrow \frac{PT}{QT} = \frac{PS}{TS}$$

$$\Rightarrow \frac{PT}{QT} = \frac{PS}{GR} \quad [\text{from eq:(4)}]$$

Hence Proved.