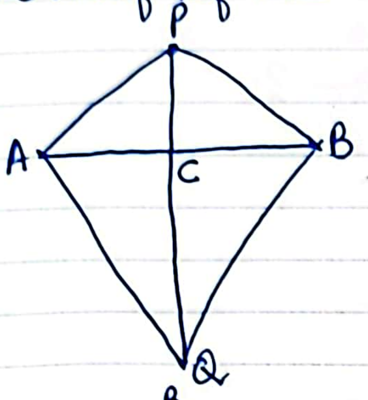
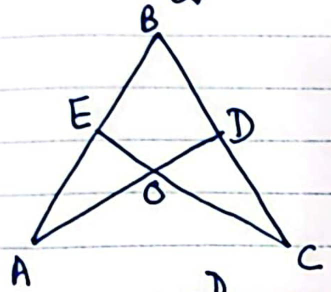
IX | H.W-14

1) Prove that the medians of an equilateral Δ are equal.

2) In a right triangle, prove that the line segment joining the mid-point of the hypotenuse to the opposite vertex is half of the hypotenuse.

3) 

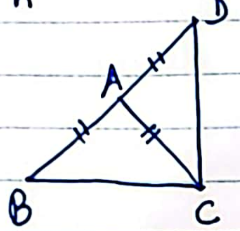
AB is a line segment P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B. Show that the line PQ is perpendicular bisector of AB.

4) 

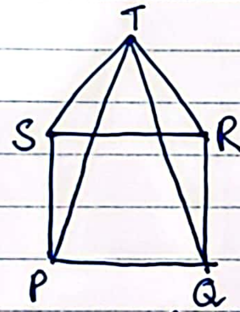
Given:- $\angle BAD = \angle BCE$

$AB = BC$

To prove:- $\Delta ABD \cong \Delta CBE$

5) 

ABC is an isosceles Δ in which $AB = AC$. Side BA is produced to D such that $AD = AB$. Show that BCD is a right angle.

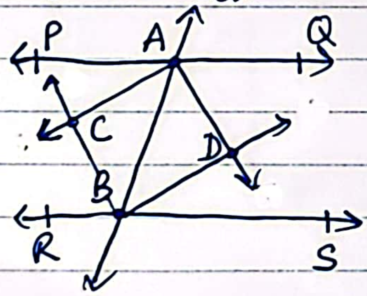
6) 

Given:- PQRS is a square

ΔSTR is an equilateral Δ

To prove:- (i) $PT = QT$

(ii) $\angle TQR = 15°$

7) 

Given:- $PQ \parallel RS$

AC bisects $\angle PAB$

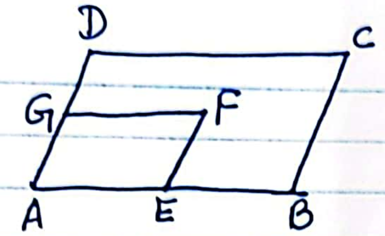
B bisects $\angle ABR$

B bisects $\angle ABS$

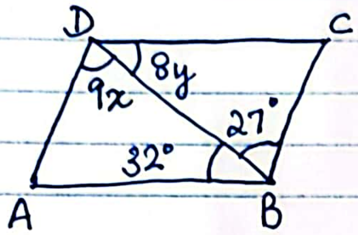
A bisects $\angle QAB$

To prove:- ACBD is a rectangle

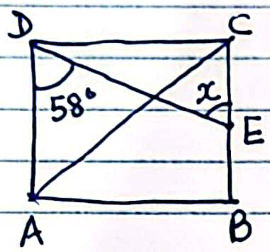
8) Prove that the bisectors of the angles of a parallelogram enclose a rectangle.

9) 

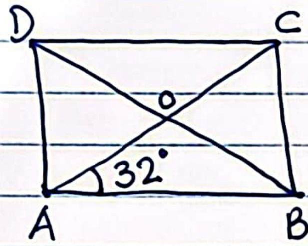
ABCD and AEFG are two parallelograms. $\angle C = 55°$. find $\angle F$.

10) 

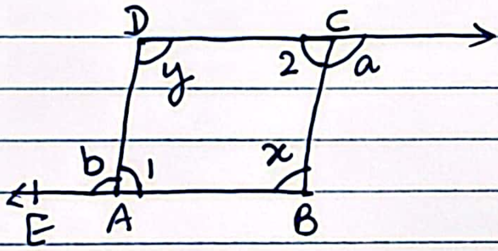
ABCD is a parallelogram. Find x and y.

11) 

ABCD is a square. find x

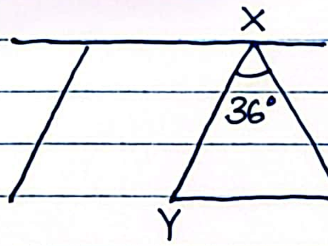
12) 

In rectangle ABCD, $\angle BAC = 32°$; find the measure of $\angle DBC$

13) 

ABCD is a quadrilateral. Prove that $x + y = a + b$

14) Case-Study



$XY = XZ = 110$ cm

$\angle YXZ = 36°$

(i) $\angle YXZ : \angle XZY =$

(a) 2:1 (b) 1:2 (c) 1:1 (d) 1:4

(ii) $\angle YXZ = 60°$; then length of side YZ is

(a) 120cm (b) 55cm (c) 110cm (d) cannot be predicted.

(iii) Which type of triangle is $\triangle XYZ$?

(a) acute and isosceles (b) acute and scalene

(c) obtuse and isosceles (d) right and isosceles

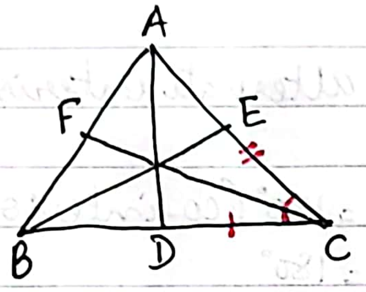
(iv) In two triangles, $\triangle ABC$ and $\triangle DEF$, $\angle A = \angle D$, $AB = DE$, $AC = DF$, then two $\triangle$s are congruent by (a) SSS (b) ASA (c) AAS (d) SAS

(v) Each angle of an equilateral $\triangle$ measures

(a) 50° (b) 80° (c) 60° (d) 20°

IX H.W-14 (Answers)

1)



Given:- in ΔABC, AD, BE and CF are the medians.

To prove:- $AD = BE = CF$

Proof:- In ΔADC and ΔBEC,

$AC = BC$ (sides of an equilateral Δ)

$\angle ACD = \angle BCE$ (common angle)

$\frac{1}{2}BC = \frac{1}{2}AC$

$\Rightarrow DC = EC$ ($\because$ D and E are the mid-points)

$\therefore \Delta ADC \cong \Delta BEC$ (SAS congruency)

Thus, $AD = BE$ (by cpct) $\rightarrow$ (1)

Similarly, in ΔADB and ΔCFB,

$AB = BC$ (sides of an equilateral Δ)

$\angle ABD = \angle CBF$ (common angle)

$\frac{1}{2}BC = \frac{1}{2}AB$

$\Rightarrow BD = BF$ ($\because$ D and F are the mid-points)

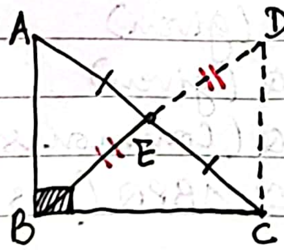
$\therefore \Delta ADB \cong \Delta CFB$ (SAS congruency)

Thus, $AD = CF$ (by cpct) $\rightarrow$ (2)

From eq:s (1) and (2), $AD = BE = CF$.

Hence, the medians of an equilateral Δ are equal.

2)



Given:- in rt. ΔABC, E is the mid-point of AC.

To prove:- $BE = \frac{1}{2}AC$

Construction:- produce BE to D such that $BE = ED$. Join DC

Proof:- In ΔAEB and ΔCED, $AE = EC$ ($\because$ E is the mid-point)

$\angle AEB = \angle DEC$ (VOA)

$BE = ED$ (by construction)

$\therefore \Delta AEB \cong \Delta CED$ (SAS congruency)

Thus, $AB = DC$ (by cpct) $\rightarrow (1)$

$\angle EAB = \angle ECD$ (by cpct)

But these angles form a pair of alternate interior angles only when $AB \parallel CD$.

Now, since $AB \parallel CD$, $\angle ABC + \angle DCB = 180°$ (co-interior angles)

$90° + \angle DCB = 180°$

$\angle DCB = 90°$ //

In $\triangle ABC$ and $\triangle DCB$, $AB = DC$ (proved above)

$\angle ABC = \angle DCB$ (each $90°$)

$BC = BC$ (common side)

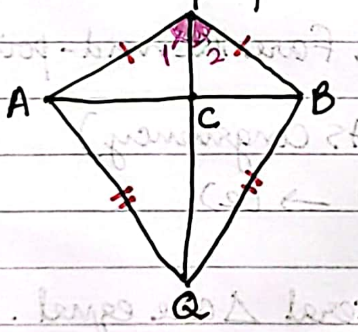
$\therefore \triangle ABC \cong \triangle DCB$ (SAS congruency)

Thus, $AC = BD$ (by cpct)

$\Rightarrow \frac{1}{2} AC = \frac{1}{2} BD$

$\therefore BE = \frac{1}{2} AC$ [$\because$ E is the mid-pt of BD]

Hence proved.

3) 

Given:- $AP = PB$

$AQ = BQ$

To prove:- $PQ \perp AB$

$AC = BC$

Proof:- In $\triangle APQ$ and $\triangle BPQ$, $AP = PB$ (given)

$AQ = BQ$ (given)

$PQ = PQ$ (Common side)

$\therefore \triangle APQ \cong \triangle BPQ$ (SSS congruency)

Thus $\angle APQ = \angle BPQ$ (by cpct)

$\Rightarrow \angle APC = \angle BPC \longrightarrow (1)$

In $\triangle APC$ and $\triangle BPC$, $AP = BP$ (given)

$\angle APC = \angle BPC$ (proved above)

$PC = PC$ (common side)

$\therefore \triangle APC \cong \triangle BPC$ (SAS Congruency)

Thus, $AC = BC$ (by cpct) $\rightarrow (2)$

Also, $\angle PCA = \angle PCB$ (by cpct).

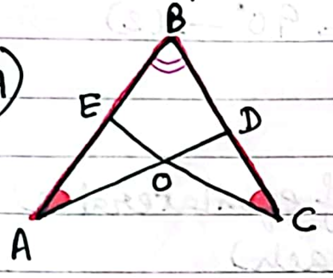
But, these angles form a linear pair and thus

$\angle PCA = \angle PCB = \frac{180^\circ}{2} = 90^\circ \rightarrow (3)$

From eq:s (2) and (3), PQ is a perpendicular bisector of AB.

Hence Proved.

4)



Given:- $\angle BAD = \angle BCE$

$AB = BC$

To prove:- $\triangle ABD \cong \triangle CBE$

Proof:- In $\triangle BAD$ and $\triangle BCE$,

$\angle BAD = \angle BCE$ (given)

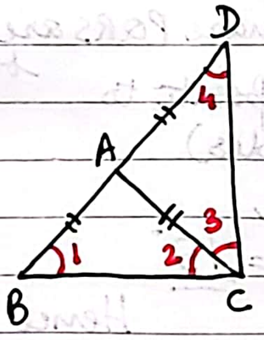
$AB = BC$ (given)

$\angle ABD = \angle CBE$ (common angle)

$\therefore \triangle ABD \cong \triangle CBE$ (ASA Congruency)

Hence Proved.

5)



Given:- in isosceles $\triangle ABC$, $AB = AC$

$AD = AB$

To prove:- $\angle BCD = 90^\circ$

Proof:- since $AB = AC$ in $\triangle ABC$, $\angle 1 = \angle 2$ (angles opposite to equal sides) $\rightarrow (1)$

Similarly, since $AD = AB \Rightarrow AD = AC$ ($\because AB = AC$)

$\Rightarrow \angle 3 = \angle 4 \rightarrow (2)$

Using angle sum property in $\triangle DCB$,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$

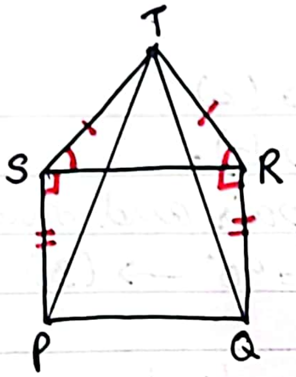
$\Rightarrow 2\angle 2 + 2\angle 3 = 180^\circ$

$\Rightarrow 2(\angle 2 + \angle 3) = 180^\circ$

$\therefore \angle 2 + \angle 3 = 90^\circ$

$\Rightarrow \angle BCD = 90^\circ$

Hence Proved.

6) 

Given :- PQRS is a square

ΔSTR is an equilateral Δ

To prove :- (i) $PT = QT$

(ii) $\angle TQR = 15°$

Proof :- Since ΔSTR is an equilateral Δ,

$\angle TSR = \angle TRS = 60° \longrightarrow (1)$

Since PQRS is a square, $\angle PSR = \angle QRS = 90° \longrightarrow (2)$

Thus, $\angle TSP = 60° + 90° = 150°$

and $\angle TRQ = 60° + 90° = 150°$

(i) In ΔTSP and ΔTRQ, $TS = TR$ (sides of equilateral Δ)

$\angle TSP = \angle TRQ$ (150° each)

$SP = RQ$ (sides of a square)

$\therefore \Delta TSP \cong \Delta TRQ$ (SAS congruency)

Thus, $PT = QT$ (by cpct)

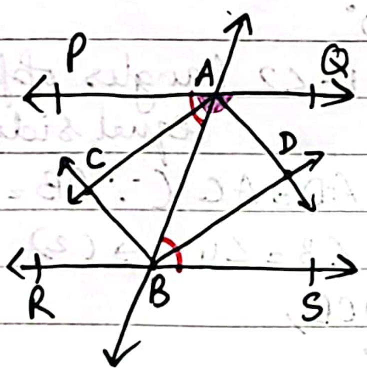
(ii) In ΔTRQ, $TR = RQ$ ($\because \Delta STR$ and square PQRS are on same base)

$\Rightarrow \angle RTQ = \angle RQT$ (angles opposite to equal sides)

Using angle sum property in ΔTRQ,

$\angle TQR = \dfrac{180° - 150°}{2} = \dfrac{30°}{2} = \underline{\underline{15°}}$

Hence Proved.

7) 

Given :- $PQ \parallel RS$

AC bisects $\angle PAB$

BC bisects $\angle ABR$

BD bisects $\angle ABS$

AD bisects $\angle QAB$

To prove :- ACBD is a rectangle.

Proof :- Since $PQ \parallel RS$, $\angle PAB = \angle ABS$ (alternate interior angles)

$\frac{1}{2}\angle PAB = \frac{1}{2}\angle ABS$

$\Rightarrow \angle CAB = \angle ABD$ ($\because$ AC and BD are angle bisectors)

These angles form a pair of alternate interior angles only when $AC \parallel BD$.

Similarly, we can prove $AD \parallel BC$ also.

Thus, ACBD is a parallelogram with both pairs of opposite sides parallel.

Now, $\angle PAB + \angle QAB = 180^\circ$ (linear pair)

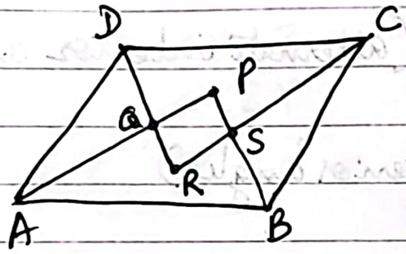
$\frac{1}{2}\angle PAB + \frac{1}{2}\angle QAB = 90^\circ$

$\Rightarrow \angle CAB + \angle DAB = 90^\circ$ [$\because$ AC and AD are angle bisectors]

$\therefore \angle CAD = 90^\circ$

Thus, ACBD is a rectangle with each angle measures 90°.

Hence Proved.

8) 

Given :- in parallelogram ABCD,
AP bisects $\angle A$,
BP bisects $\angle B$,
CR bisects $\angle C$
and DR bisects $\angle D$

To prove :- PQRS is a rectangle.

Proof :- $\angle A + \angle D = 180^\circ$ (adjacent angles of ||gm ABCD)

$\frac{1}{2}\angle A + \frac{1}{2}\angle D = 90^\circ$

$\Rightarrow \angle QAD + \angle QDA = 90^\circ$ [$\because$ AQ and DQ are angle bisectors]

Using angle sum property in $\triangle DQA$, $\angle DQA = 180^\circ - 90^\circ = 90^\circ$

Also, $\angle DQA = \angle PQR = 90^\circ$ (VOA)

Similarly, we can prove $\angle CSB = \angle PSR = 90^\circ$.

Now, $\angle A + \angle B = 180^\circ$ (adjacent angles of a ||gm)

$\frac{1}{2}\angle A + \frac{1}{2}\angle B = 90^\circ$

$\Rightarrow \angle PAB + \angle PBA = 90^\circ$ [$\because$ AP and BP are angle bisectors]

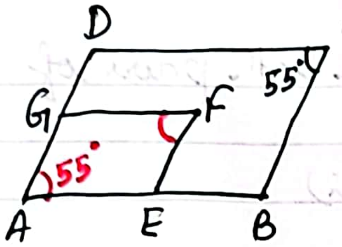
Using angle sum property in $\triangle APB$, $\angle APB = 180^\circ - 90^\circ = 90^\circ$

Similarly, we can prove in $\triangle DRC$, $\angle DRC = 90^\circ$

Thus, $\angle P = \angle Q = \angle R = \angle S = 90^\circ$ in quad. PQRS

Hence, PQRS is a rectangle with each angle measures 90°.

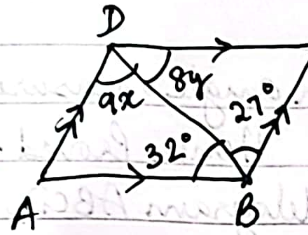
9)



Since ABCD is a parallelogram,
$\angle C = \angle A = 55°$ (opposite angles of a ||gm are equal)

Similarly, in parallelogram AEFG,
$\angle GAE = \angle GFE = 55°$ (opposite angles of ||gm AEFG)
$\therefore \angle F = 55°$

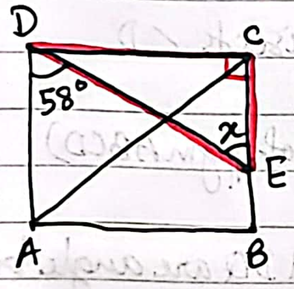
10)



Since AB || CD,
$8y = 32°$ (alternate interior angles)
$y = 4°$

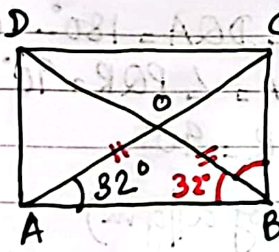
Since AD || BC, $9x = 27°$ (alternate interior angles)
$x = 3°$.

11)



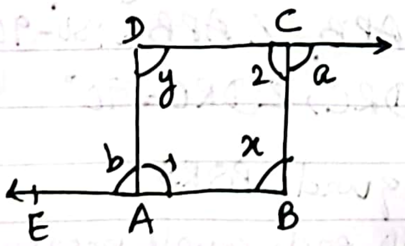
$\angle EDC = 90° - 58° = 32°$
In $\triangle DCE$, using angle sum property,
$x = 180° - (32° + 90°)$
$= 180° - 122°$
$= \underline{\underline{58°}}$

12)



Since ABCD is a rectangle,
diagonal AC = diagonal BD.
$\therefore$ OA = OB (diagonals bisect each other)
$\Rightarrow \angle OAB = \angle OBA$ (angles opposite to equal sides)
$= 32°$
$\therefore \angle DBC = 90° - 32° = \underline{\underline{58°}}$

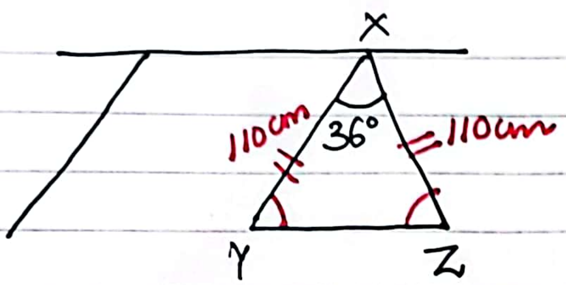
13)



$\angle 1 = 180° - b$ (linear pair)
$\angle 2 = 180° - a$ (linear pair)

In quad. ABCD, $180° - b + x + 180° - a + y = 360°$
$x + y + \cancel{360°} = \cancel{360°} + a + b$
$\therefore \underline{\underline{x + y = a + b}}$

14)



(i) In $\triangle XYZ$, $XY = XZ \Rightarrow \angle XYZ = \angle XZY$ (angles opposite to equal sides)

$= \frac{180° - 36°}{2} = \frac{144°}{2} = 72°$

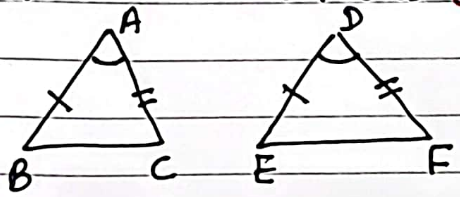
$\therefore \frac{\angle YXZ}{\angle XZY} = \frac{36°}{72°} = \frac{1}{2}$ (b)

(ii) $\angle YXZ = 60° \Rightarrow XYZ$ is an equilateral $\triangle$

$YZ = 110$ cm (c)

(iii) acute and isosceles (a)

(iv)



SAS (d)

(v) 60° (c)