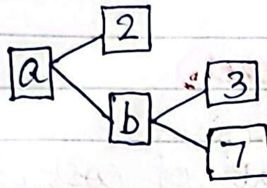
X HW-15

SECTION-A

1) 

The values of a and b in the factor tree are
(a) (2,3) (b) (2,5)
(c) (21,42) (d) (42,21)

2) If sum of two numbers is 1215 and their HCF is 81, then the possible no. of pairs of such numbers are
(a) 2 (b) 3 (c) 4 (d) 5

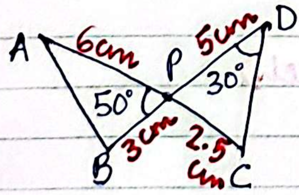
3) If one zero of the polynomial $6x^2+37x-(k-2)$ is reciprocal of the other, then what is the value of k?
(a) −4 (b) −6 (c) 6 (d) 4

4) Two linear equations in variables x and y are given below
$a_1x+b_1y+C=0$; $a_2x+b_2y+C=0$
Which of the following pieces of information is independently sufficient to determine if a solution exists or not for this pair of linear equation?
(i) $\frac{a_1}{b_1}=\frac{a_2}{b_2}=1$ (ii) $\frac{a_1}{a_2}=\frac{b_1}{b_2}$ (iii) $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq 1$ (iv) $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

5) The quadratic equation $x^2-4x+k=0$ has distinct real roots if
(a) $k=4$ (b) $k>4$ (c) $k=16$ (d) $k<4$

6) If the first three terms of an AP are $3p-1$, $3p+5$ and $5p+1$ respectively, then the value of p is
(a) 2 (b) −3 (c) 4 (d) 5

7) What is the ratio in which the line segment joining (2,−3) and (5,6) is divided by x-axis?
(a) 1:2 (b) 2:1 (c) 2:5 (d) 5:2

8) 

Line segments AC and BD intersect each other at P such that PA = 6cm, PB = 3cm, PC = 2.5cm, PD = 5cm, ∠APB = 50°, ∠CDP = 30°, ∠PBA = ____
(a) 50° (b) 30° (c) 60° (d) 100°

9) Which of these is equivalent to $\frac{2\tan x(\sec^2 x-1)}{\cos^3 x}$?

(a) $2\tan^3 x\,\mathrm{cosec}\,x$ (b) $2\cot^3 x\,\mathrm{cosec}^3 x$
(c) $2\tan^3 x\cdot\sec^3 x$ (d) $2\cot^3 x\sec^3 x$

10) The length of shadow of a pole on the play ground is $\frac{1}{\sqrt{3}}$ times the height of the pole, the angle of elevation of the Sun is

(a) 30° (b) 45° (c) 60° (d) 90°

11) The upper limit of the modal class of the given distribution is

Height (in cm)	Below 140	below 145	below 150	below 155	below 160	below 165
No. of girls	4	11	29	40	46	51

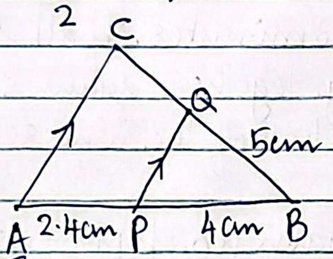
(a) 165 (b) 160 (c) 155 (d) 150

12) The length of a String between a kite and a point on the ground is 85m, if the string makes an angle θ with the ground level such that $\tan\theta = \frac{15}{8}$, then at what height is the kite from the ground?

(a) 75m (b) 79.41m (c) 80m (d) 72.5m

13) $4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \tan^2 45^\circ)$ is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

14) 

$PQ \parallel AC$. If $BP = 4cm$, $AP = 2.4cm$, $BQ = 5cm$, the length of BC = ___

(a) 8cm (b) 3cm (c) 0.3cm (d) $\frac{25}{3}$cm

15) The distance between the points $(0,0)$ and $(a-b, a+b)$ is

(a) $2\sqrt{ab}$ (b) $\sqrt{2a^2+ab}$ (c) $2\sqrt{a^2+b^2}$ (d) $\sqrt{2a^2+2b^2}$

16) The next term of the AP $\sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$ is

(a) $5\sqrt{2}$ (b) $5\sqrt{3}$ (c) $3\sqrt{3}$ (d) $5\sqrt{3}$

17) The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has

(a) two distinct real roots (b) two equal real roots
(c) no real roots (d) more than two real roots

18) If the list price of a toy is reduced by ₹2, a person can buy 2 toys more for ₹360. The original price of the toy is

(a) ₹18 (b) ₹20 (c) ₹19 (d) ₹21

19) The value of k, for which the pair of linear equation $kx + y = k^2$ and $x + ky = 1$ have infinitely many solution is

(a) ± 1 (b) 1 (c) -1 (d) 2

20) If α, β are the zeroes of the polynomial $p(x) = 4x^2 - 3x - 7$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to

(a) $\frac{7}{3}$ (b) $-\frac{7}{3}$ (c) $\frac{3}{7}$ (d) $-\frac{3}{7}$

SECTION-B

21) If α and β are the zeroes of the quadratic polynomial $f(x) = 3x^2 - 5x - 2$, then evaluate $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

22) The diagonals of a quadrilateral ABCD intersect each other at O, such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

23) If the roots of the equation $(a^2+b^2)x^2 - 2(ac+bd)x + (c^2+d^2) = 0$ are equal, P.T $\frac{a}{b} = \frac{c}{d}$

24) Find the 20th term from the last term of AP: 3, 8, 13, ... 253

25) If $\tan A = n \tan B$, $\sin A = m \sin B$, prove that $\cos^2 A = \frac{m^2-1}{n^2-1}$

SECTION-C

26) Three bells ring at intervals of 6, 12 and 18 minutes. If all three bells rang at 6am, when will they ring together again?

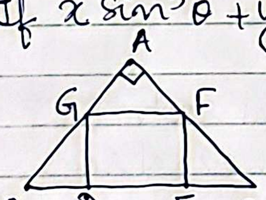
27) Find the coordinates of the points of trisection of the line segment joining the points (3, -1) and (6, 8).

28) ₹9000 were divided equally among a certain no. of persons. Had there been 20 more persons, each would have got ₹160 less. Find the original no. of persons.

29) Solve for x: $4x^2 - 4a^2x + (a^4 - b^4) = 0$

30) If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, P.T $x^2+y^2=1$

31)



DEFG is a square. $\angle BAC = 90°$.
Prove that:
(i) $\Delta AGF \sim \Delta DBG$ (ii) $\Delta AGF \sim \Delta EFC$
(iii) $\Delta DBG \sim \Delta EFC$ (iv) $DE^2 = BD \times EC$

SECTION-D

32) Solve for x: $9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$

33) Seven years ago Varun's age was five times the square of Swati's age. Three years hence, Swati's age will be two-fifth of Varun's age. Find their present ages.

34) If the ratio of sum of the first m and n terms of an AP is $m^2 : n^2$, show that the ratio of its mth and nth terms is $(2m-1) : (2n-1)$

35) AD and PM are median of triangles $\triangle ABC$ and $\triangle PQR$ respectively where $\triangle ABC \sim \triangle PQR$. Prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

SECTION-E

36) A rectangular floor area can be completely tiled with 200 square tiles. If the side length of each tile is increased by 1 unit, it would take only 128 tiles to cover the floor.

(i) Assuming the original length of each side of a tile be x units, make a quadratic equation from the above information.

(ii) Write the corresponding quadratic equation in standard form

(iii) Find the value of x, the length of side of a tile by factorisation.

(iv) Solve the quadratic equation for x, using quadratic formula.

37)

37. Push-ups are a fast and effective exercise for building strength. These are helpful in almost all sports including athletics. While the push-up primarily targets the muscles of the chest, arms, and shoulders, support required from other muscles helps in toning up the whole body.

Nitesh wants to participate in the push-up challenge. He can currently make 3000 push-ups in one hour. But he wants to achieve a target of 3900 push-ups in 1 hour for which he practices regularly. With each day of practice, he is able to make 5 more push-ups in one hour as compared to the previous day. If on first day of practice he makes 3000 push-ups and continues to practice regularly till his target is achieved.

Keeping the above situation in mind answer the following questions:

(*i*) **Form an A.P representing the number of push-ups per day and hence find the minimum number of days he needs to practice before the day his goal is accomplished?** **2**

(*ii*) **Find the total number of push-ups performed by Nitesh up to the day his goal is achieved.** **2**

Case study based questions are compulsory.

. The given figure alongside shows the path of a diver, when she takes a jump from the diving board. Clearly it is a parabola.

Annie was standing on a diving board, 48 feet above the water level. She took a dive into the pool. Her height (in feet) above the water level at any time 't' in seconds is given by the polynomial $h(t)$ such that $h(t) = -16t^2 + 8t + k$.

(*i*) What is the value of k? **1**

(*ii*) At what time will she touch the water in the pool? **1**

(*iii*) A polynomial $q(t)$ with sum of zeroes as 1 and the product as –6 is modelling Anu's height in feet above the water at any time t(in seconds). Then find the value of $q(t)$. **2**

(iv) **The zeroes of the polynomial $r(t) = -12t^2 + (k - 3)t + 48$ are negative of each other. Find value of k.**

37. The concept of the Cartesian coordinate system may seem simple, but it can be extended

X Homework - 15 (Answers)

1) $b = 3 \times 7 = 21$

$a = 2 \times 21 = 42$

$(42, 21)$ (d)

2) Let the no.s be $81x$ and $81y$

Then, $81x + 81y = 1215$

$$x + y = \frac{1215}{81} = 15$$

$\therefore$ Possible no. of pairs are $(1,14)$

$(2,13)$

$(4,11)$

$(7,8)$

4 pairs (c)

3) Let the zeroes be α and $\frac{1}{\alpha}$ and $a = 6$

$b = 37$

$c = -(k-2)$

Product of zeroes, $\alpha \times \frac{1}{\alpha} = \frac{c}{a}$

$$\Rightarrow 1 = \frac{-(k-2)}{6}$$

$\Rightarrow 6 = -k + 2$

$k = 2 - 6 = -4$ (a)

4) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (iv) and $\frac{a_1}{b_1} = \frac{a_2}{b_2} = 1$ (i)

5) $a = 1, b = -4, c = k$

For distinct and real roots, $b^2 - 4ac > 0$

$\Rightarrow 16 - 4k > 0$

$\Rightarrow -4k > 16$

$\Rightarrow 4k < 16$

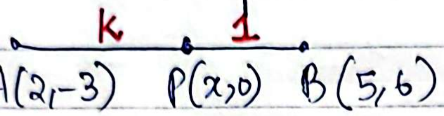
$k < 4$ (d)

6) $\cancel{3p}+5-\cancel{3p}+1 = 5p+1-3p-5$

$6 = 2p-4$

$2p = 10$

$p = 5$ (d)

7) 

$P(x,y) = P\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$(x,0) = \left(\frac{5k+2}{k+1}, \frac{6k-3}{k+1}\right)$

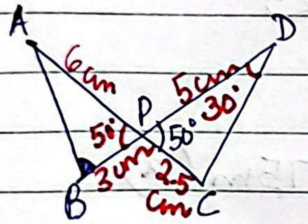
On equating y co-ordinates, $0 = \frac{6k-3}{k+1}$

$6k-3 = 0$

$6k = 3$

$k = \frac{1}{2}$

1:2 (a)

8) 

$\angle APB = \angle DPC = 50°$ (VOA)

$\frac{AP}{PD} = \frac{6}{5}$

$\frac{BP}{PC} = \frac{3}{2.5} = \frac{30}{25} = \frac{6}{5}$

$\angle APB = \angle DPC$ (VOA)

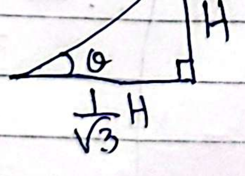
$\therefore \triangle APB \sim \triangle DPC$ (SAS similarity)

Thus, $\angle ABP = \angle PCD = 180° - (50° + 30°)$

$= 180° - 80° = 100°$ (d)

9) $\frac{2\tan x(\sec^2 x - 1)}{\cos^3 x} = \frac{2\tan x \cdot \tan^2 x}{\cos^3 x} = \frac{2\tan^3 x}{\cos^3 x}$

$= 2\tan^3 x \cdot \sec^3 x$ (c)

10) 

$\tan\theta = \frac{H}{\frac{1}{\sqrt{3}}H} = \sqrt{3}$

$\therefore \theta = 60°$ (c)

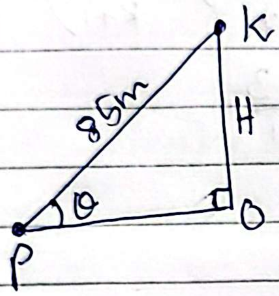
11)

C.I	C.f	f
135-140	4	4
140-145	11	7
$\boxed{145\text{-}150}$	29	18
150-155	40	11
155-160	46	6
160-165	51	5

Modal class = 145-150

Upper limit = 150 (d)

12)



$\tan\theta = \frac{KO}{OP} = \frac{15k}{8k}$

Using Pythagoras theorem,

$OP^2 + OK^2 = PK^2$

$64k^2 + 225k^2 = 7225$

$289k^2 = 7225$

$k^2 = 25$

$k = 5$

$\therefore OK = 15k = 15 \times 5 = 75m$ (a)

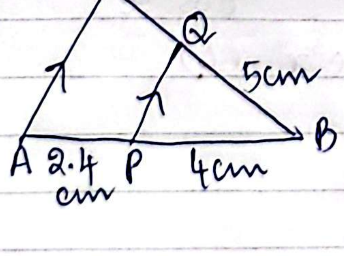
13) $4\left(\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right) - 3\left(\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right)$

$= 4\left(\frac{1}{16} + \frac{1}{16}\right) - 3\left(\frac{1}{2} - 1\right)$

$= \cancel{4}^{1} \times \frac{2}{\cancel{16}_{4}} - 3 \times -\frac{1}{2}$

$= \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$ (c)

14)



Using Thales theorem, since PQ||AC,

$\frac{BP}{PA} = \frac{BQ}{QC}$

$\Rightarrow \frac{4}{2.4} = \frac{5}{QC}$

$\therefore QC = \frac{5 \times 2.4}{4}$

$= 5 \times 0.6 = 3cm$

$\therefore BC = 5 + 3$

$= 8cm$ (a)

15) $d = \sqrt{x^2+y^2}$

$= \sqrt{(a-b)^2+(a+b)^2} = \sqrt{a^2+b^2-\cancel{2ab}+a^2+b^2+\cancel{2ab}}$

$= \sqrt{2a^2+2b^2}$ (d)

16) $\sqrt{8} = 2\sqrt{2}$

$\sqrt{18} = 3\sqrt{2}$

$\sqrt{32} = 4\sqrt{2}$

Next term is $5\sqrt{2}$ (a)

17) $a = 2, b = -\sqrt{5}, c = 1$

$b^2 - 4ac = 5 - 8 = -3 < 0$, no real roots (c)

18) Let the original price of the toy be x.

No. of toys bought $= \frac{360}{x}$

New list price $= x-2$

Then, the No. of toys bought $= \frac{360}{x-2}$

Difference in no. of toys = 2

ATQ, $\frac{360}{x-2} - \frac{360}{x} = 2$

$360\left[\frac{1}{x-2} - \frac{1}{x}\right] = 2$

$\overset{180}{\cancel{360}}\left[\frac{\cancel{x}-\cancel{x}+2}{x^2-2x}\right] = \cancel{2}^{1}$

$180 \times 2 = x^2 - 2x$

$x^2 - 2x - 360 = 0$

$(x+18)(x-20) = 0$

$x = -18, 20$

$\therefore$ original price = ₹20 (b)

$4 \,|\, 360$

$5 \,|\, 90$

18

S P

-2 -360

$18, -20$

19) $a_1 = k, b_1 = 1, c_1 = -k^2$

$a_2 = 1, b_2 = k, c_2 = -1$

For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\Rightarrow \underset{\text{I}}{\frac{k}{1}} = \underset{\text{II}}{\frac{1}{k}} = \underset{\text{III}}{\frac{-k^2}{-1}}$$

From I and II, $k^2 = 1$

$k = \pm 1$

From II and III, $k^3 = 1$

$k = 1$

$\therefore$ the required value of $k = 1$ (b)

20) $a = 4, b = -3, c = -7$

$$\alpha + \beta = \frac{-b}{a} = \frac{3}{4}$$

$$\alpha\beta = \frac{c}{a} = \frac{-7}{4}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{3}{4}}{\frac{-7}{4}} = \frac{-3}{7} \text{ (d)}$$

21) Let $f(x) = 3x^2 - 5x - 2$ be of the form $ax^2 + bx + c$; where $a = 3, b = -5, c = -2$

$$\alpha + \beta = \frac{-b}{a} = \frac{5}{3}$$

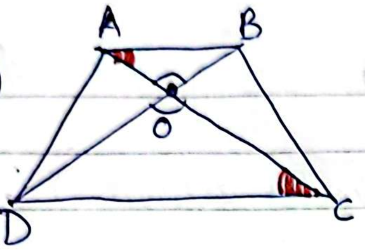
$$\alpha\beta = \frac{c}{a} = \frac{-2}{3}$$

$$\therefore \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} \qquad a^3 + b^3 = (a+b)^3 - 3ab(a+b)$$

$$= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$= \frac{\frac{125}{27} - \cancel{3}^{1} \times \frac{-2}{\cancel{3}_1} \times \frac{5}{3}}{\frac{-2}{3}} = \left(\frac{125}{27} + \frac{10 \times 9}{3 \times 9}\right) \times -\frac{3}{2}$$

$$= \frac{215}{\cancel{27}_9} \times -\frac{\cancel{3}^1}{2} = \frac{-215}{18}$$

22) 

Given :- in quadrilateral ABCD,
$\frac{AO}{BO} = \frac{CO}{DO}$

To prove :- ABCD is a trapezium.

Proof :- In ΔAOB and ΔCOD, $\frac{AO}{BO} = \frac{CO}{DO}$ (given)

$\Rightarrow \frac{AO}{CO} = \frac{BO}{DO}$

and $\angle AOB = \angle COD$ (VOA)

$\therefore \Delta AOB \sim \Delta COD$ (SAS similarity)

Thus, $\angle OAB = \angle OCD$ ($\because$ corresponding angles of similar Δs are equal)

These angles form a pair of ~~co~~ alternate interior angles only when $AB \parallel CD$.

Thus, quadrilateral ABCD is a trapezium with one pair of opposite sides parallel.

Hence Proved.

23) Let the given equation be of the form $Ax^2 + Bx + C = 0$;
Where $A = a^2+b^2$, $B = -2(ac+bd)$, $C = c^2+d^2$

For equal roots, $B^2 - 4AC = 0$

$\Rightarrow 4(ac+bd)^2 - 4(a^2+b^2)(c^2+d^2) = 0$

$\Rightarrow 4[\cancel{a^2c^2} + \cancel{b^2d^2} + 2abcd - \cancel{a^2c^2} - a^2d^2 - b^2c^2 - \cancel{b^2d^2}] = 0$

$\Rightarrow -a^2d^2 - b^2c^2 + 2abcd = 0$

$\Rightarrow a^2d^2 + b^2c^2 - 2abcd = 0$

$\Rightarrow (ad - bc)^2 = 0$

$\Rightarrow ad - bc = 0$

$\therefore ad = bc$

$\frac{a}{b} = \frac{c}{d}$

24) n^{th} term from the end $= l-(n-1)d$

$= 253 - 19 \times 5$

$= 253 - 95$

$= \underline{\underline{158}}$

25) $\tan B = \dfrac{\tan A}{n} \Rightarrow \cot B = \dfrac{n}{\tan A}$

$\sin B = \dfrac{\sin A}{m} \Rightarrow \operatorname{cosec} B = \dfrac{m}{\sin A}$

we know that, $\operatorname{cosec}^2 B - \cot^2 B = 1$

$\Rightarrow \dfrac{m^2}{\sin^2 A} - \dfrac{n^2}{\tan^2 A} = 1$

$\Rightarrow \dfrac{m^2}{\sin^2 A} - \dfrac{n^2\cos^2 A}{\sin^2 A} = 1$

$\Rightarrow \dfrac{m^2 - n^2\cos^2 A}{\sin^2 A} = 1$

$\Rightarrow m^2 - n^2\cos^2 A = 1 - \cos^2 A$

$\Rightarrow \cos^2 A - n^2\cos^2 A = 1 - m^2$

$\cos^2 A(1-n^2) = 1-m^2$

$\therefore \cos^2 A = \dfrac{1-m^2}{1-n^2} = \underline{\underline{\dfrac{m^2-1}{n^2-1}}}$

26) $6 = 2 \times 3$

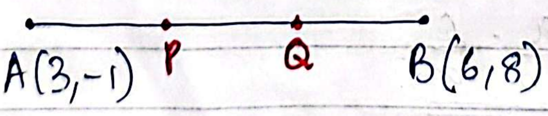
$12 = 2^2 \times 3$

$18 = 3^2 \times 2$

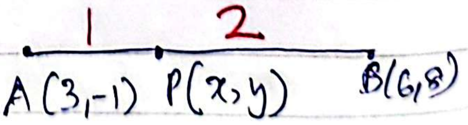
LCM $= 2^2 \times 3^2 = 4 \times 9 = 36$ minutes

$\therefore$ The bells ring together again at 6:36 am.

27)



P divides AB in the ratio 1:2



$P(x,y) = \left(\dfrac{m_1x_2 + m_2x_1}{m_1+m_2}, \dfrac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$$(x,y) = \left(\frac{6+6}{3}, \frac{8-2}{3}\right) = \left(\frac{12}{3}, \frac{6}{3}\right) = (4,2)$$

Q divides AB in the ratio 2:1

$A(3,-1) \overset{2}{\text{———}} Q(x,y) \overset{1}{\text{——}} B(6,8)$

$$Q(x,y) = Q\left(\frac{12+3}{3}, \frac{16+(-1)}{3}\right)$$

$$(x,y) = \left(\frac{15}{3}, \frac{15}{3}\right) = (5,5)$$

28) Let the original no. of persons be x.

Share of each person $= \frac{9000}{x}$

New no. of persons $= x+20$

Now, the share of each $= \frac{9000}{x+20}$

difference in amount $=$ ₹160

ATQ, $\frac{9000}{x} - \frac{9000}{x+20} = 160$

$\Rightarrow 9000\left[\frac{\cancel{x}+20-\cancel{x}}{x^2+20x}\right] = \cancel{160}^{8}$

$\Rightarrow \cancel{9000}^{1125} = \cancel{8}^{1}(x^2+20x)$

$\Rightarrow x^2+20x-1125=0$

$\Rightarrow (x-25)(x+45)=0$

$x = 25, -45$

S	P
20	-1125

$-25, 45$

$$\begin{array}{r|l} 5 & 1125 \\ \hline 5 & 225 \\ \hline & 45 \end{array}$$

x cannot be -ve, $\therefore$ required value of $x = 25$

Hence, the original no. of persons $= 25$

29) Let the given eq^n be of the form $Ax^2+Bx+C=0$; where $A = 4, B = -4a^2, C = a^4-b^4$

$B^2-4AC = 16a^4 - 4\times4(a^4-b^4) = \cancel{16a^4} - \cancel{16a^4} + 16b^4 = 16b^4$

$$x = \frac{-B \pm \sqrt{B^2-4AC}}{2A} = \frac{4a^2 \pm \sqrt{16b^4}}{8} = \frac{4a^2 \pm 4b^2}{8}$$

$x = \frac{4a^2+4b^2}{8} = \frac{4(a^2+b^2)}{8} = \frac{a^2+b^2}{2}$

$x = \frac{4a^2-4b^2}{8} = \frac{a^2-b^2}{2}$

$x \sin\theta = y \cos\theta \rightarrow (1)$

30) $x \sin^3\theta + y \cos^3\theta = \sin\theta \cos\theta$

$\Rightarrow \underline{x \sin\theta} \cdot \sin^2\theta + y \cos^3\theta = \sin\theta \cos\theta$

$\Rightarrow y \cos\theta \cdot \sin^2\theta + y \cos^3\theta = \sin\theta \cos\theta$

$\Rightarrow y \cos\theta \left[\sin^2\theta + \cos^2\theta \right] = \sin\theta \cos\theta$

$\therefore y \cancel{\cos\theta} = \sin\theta \cancel{\cos\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

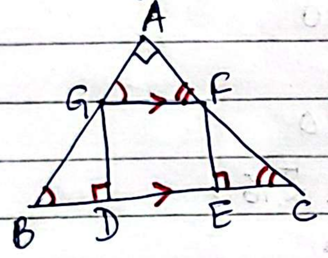
$y = \sin\theta \rightarrow (2)$

From eq: (1), $x \cancel{\sin\theta} = \cancel{\sin\theta} \cos\theta$

$x = \cos\theta \rightarrow (3)$

$\therefore x^2 + y^2 = \cos^2\theta + \sin^2\theta = \underline{\underline{1}}$

31)



Given:- in $\triangle ABC$, $\angle A = 90^\circ$

DEFG is a square.

To prove:- (i) $\triangle AGF \sim \triangle DBG$

(ii) $\triangle AGF \sim \triangle EFC$

(iii) $\triangle DBG \sim \triangle EFC$

(iv) $DE^2 = BD \times EC$

Proof:- (i) In $\triangle AGF$ and $\triangle DBG$, $\angle GAF = \angle BDG$ (each 90°)

$\angle AGF = \angle GBD$ (Corresponding angles)

$\therefore \triangle AGF \sim \triangle DBG$ (AA similarity) $\rightarrow (1)$

(ii) In $\triangle AGF$ and $\triangle EFC$, $\angle GAF = \angle FEC$ (each 90°)

$\angle AFG = \angle FCE$ (corresponding angles)

$\therefore \triangle AGF \sim \triangle EFC$ (AA similarity) $\rightarrow (2)$

(iii) From (1) and (2), ~~$\triangle GBD$~~ $\triangle DBG \sim \triangle EFC$

(iv) Thus, $\dfrac{BD}{EF} = \dfrac{DG}{EC}$

$\Rightarrow BD \times EC = DG \times EF$

$\Rightarrow BD \times EC = DE \times DE$ $(\because DG = EF = DE)$

$\therefore DE^2 = BD \times EC$.

Hence Proved.

32) Let the given equation be of the form $Ax^2+Bx+C=0$; where

$A=9$, $B=-9(a+b)$, $C=2a^2+5ab+2b^2$

$$B^2-4AC = 81(a+b)^2 - 4\times 9(2a^2+5ab+2b^2)$$
$$= 81(a+b)^2 - 36(2a^2+5ab+2b^2)$$
$$= 81(a^2+b^2+2ab) - 72a^2 - 180ab - 72b^2$$
$$= 81a^2+81b^2+162ab-72a^2-180ab-72b^2$$
$$= 9a^2+9b^2-18ab$$
$$= (3a-3b)^2$$

$$x = \frac{-B \pm \sqrt{B^2-4AC}}{2A} = \frac{9(a+b) \pm \sqrt{(3a-3b)^2}}{18}$$
$$= \frac{9(a+b) \pm (3a-3b)}{18}$$

$\therefore x = \frac{9a+9b+3a-3b}{18}$ | $x = \frac{9a+9b-3a+3b}{18}$

$= \frac{12a+6b}{18}$ | $= \frac{6a+12b}{18}$

$= \frac{2a+b}{3}$ | $= \frac{a+2b}{3}$

33) Let the age of Varun seven years ago be $5x^2$ and Swati's age be x.

Varun	Swati
$5x^2$	x

Present age of Varun $= 5x^2+7$

Swati's age $= x+7$

After 3 years, Varun's age $= 5x^2+10$

Swati's age $= x+10$

ATQ, $x+10 = \frac{2}{5}(5x^2+10)$

$\Rightarrow 5x+50 = 10x^2+20$

$\Rightarrow 10x^2-5x-30=0$

$\Rightarrow 2x^2-x-6=0$

$\Rightarrow 2x^2-4x+3x-6=0$

$\Rightarrow 2x(x-2)+3(x-2)=0$

S	P	
-1	-12	-4, 3

$\Rightarrow (2x+3)(x-2)=0$

$\therefore x = -\frac{3}{2}, 2$

x cannot be $-$ve,

$\therefore$ req. value of $x=2$

Present age of Varun $= 27$ years

Swati's $= 9$ years

34) Let the first term and common difference be a and d respectively.

$$\frac{S_m}{S_n} = \frac{\frac{\cancel{m}}{\cancel{2}}[2a+(m-1)d]}{\frac{\cancel{n}}{\cancel{2}}[2a+(n-1)d]} = \frac{m^{\cancel{2}}}{n^{\cancel{2}}}$$

$$\Rightarrow n[2a+(m-1)d] = m[2a+(n-1)d]$$

$$\Rightarrow 2an + n(m-1)d = 2am + m(n-1)d$$

$$\Rightarrow 2an - 2am = [m(n-1) - n(m-1)]d$$

$$\Rightarrow 2a(n-m) = [\cancel{mn} - m - \cancel{mn} + n]d$$

$$\Rightarrow 2a\cancel{(n-m)} = \cancel{(n-m)}d$$

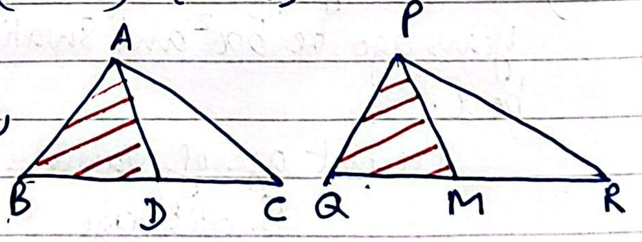
$$\boxed{2a = d}$$

$$\therefore \frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+(m-1)2a}{a+(n-1)2a} = \frac{a+2am-2a}{a+2an-2a}$$

$$= \frac{2am-a}{2an-a} = \frac{\cancel{a}(2m-1)}{\cancel{a}(2n-1)} = \frac{2m-1}{2n-1}$$

$\therefore$ The required ratio is $(2m-1):(2n-1)$

35)



Given:- in ΔABC and ΔPQR, AD and PM are the medians.

$\Delta ABC \sim \Delta PQR$

To prove:- $\frac{AB}{PQ} = \frac{AD}{PM}$

Proof:- Since $\Delta ABC \sim \Delta PQR$,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \rightarrow (1)$$

Also, $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R \rightarrow (2)$

In $\triangle ABD$ and $\triangle PQM$, $\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}$ [$\because$ D and M are the mid-points]

Also, $\angle ABD = \angle PQM$ [$\because \angle B = \angle Q$]

$\therefore \triangle ABD \sim \triangle PQM$ (SAS similarity)

Thus, $\frac{AB}{PQ} = \frac{AD}{PM}$ (Corresponding sides of Similar $\triangle$s are proportional)

Hence Proved.

36) (i) length of each side of a tile = x units.

area of 1 tile = x^2 sq. units

Area of floor = $200 \times x^2 = 200x^2$ sq. units

New length of each tile = $(x+1)$ units

area = $(x+1)^2$

Area of floor = $128 \times (x+1)^2$ sq. units

By Equating the area of floor,

$\cancel{200}^{25} x^2 = \cancel{128}^{16}(x+1)^2$

$25x^2 = 16x^2 + 16 + 32x$

$9x^2 - 32x - 16 = 0$ is the required quadratic equation.

(ii) $9x^2 - 32x - 16 = 0$ is of the form $ax^2 + bx + c = 0$.

(iii) $9x^2 - 36x + 4x - 16 = 0$

$9x(x-4) + 4(x-4) = 0$

$(9x+4)(x-4) = 0$

$\therefore x = -\frac{4}{9}, 4$

S	P
-32	-144
	$-36, 4$

$$\begin{array}{r|l} 3 & 144 \\ 3 & 48 \\ 2 & 16 \\ 2 & 8 \\ & 4 \end{array}$$

x cannot be $-$ve, $\therefore$ required value of $x = \underline{4}$.

(iv) $a = 9, b = -32, c = -16$

$b^2 - 4ac = 32^2 + 4 \times 9 \times 16 = 1024 + 576 = 1600$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{32 \pm 40}{18}$

$x = \frac{72}{18} = 4$ or $x = \frac{-8}{18} = \frac{-4}{9}$

x cannot be $-$ve

$\therefore$ required value of $x = \underline{4}$

37) $a = 3000$
$a_n = 3900$
$d = 5$

(i) 3000, 3005, 3010, ... 3900 forms an A.P with $a = 3000$ and $d = 5$

$a_n = a + (n-1)d$

$3900 = 3000 + (n-1)5$

$\frac{900}{5} = n-1$

$n - 1 = 180$ days

(ii) $n = 181$

$S_{181} = \frac{n}{2}[a + a_n]$

$= \frac{181}{2}(3000 + 3900)$

$= \frac{181}{2} \times 6900 = 181 \times 3450 = 624450$ push-ups

38) (i) $h(t) = -16t^2 + 8t + k$

when $t = 0$, $h = 48$ ft

$h(0) = 0 + 0 + k = 48$

$k = 48$

(ii) $h = 0$

$0 = -16t^2 + 8t + 48$

$\div 8 \Rightarrow -2t^2 + t + 6 = 0$

$\Rightarrow 2t^2 - t - 6 = 0$

$\Rightarrow 2t^2 - 4t + 3t - 6 = 0$

$\Rightarrow 2t(t-2) + 3(t-2) = 0$

$\Rightarrow (2t+3)(t-2) = 0$

$\therefore t = -\frac{3}{2}, 2$

S P
-1 -12 < -4, 3

$\therefore$ She touches the water in 2 seconds

(iii) $\alpha+\beta = 1$

$\alpha\beta = -6$

$q(t) = k[t^2 - t - 6]$

When $t=0$, $q(0) = 48$

$\Rightarrow k[t^2 - t - 6] = 48$

$\Rightarrow k(-6) = 48$

$k = -8$ //

$\therefore q(t) = -8t^2 + 8t + 48$ //

(iv) Let the zeroes be α and $(-\alpha)$

$a = -12$

$b = k-3$

$c = 48$

Sum of zeroes $= \alpha + (-\alpha) = 0 = \dfrac{-b}{a}$

$\Rightarrow 0 = \dfrac{-(k-3)}{-12}$

$\Rightarrow k-3 = 0$

$\underline{\underline{k = 3}}$