Half-Yearly Examination (2024-'25)

Class : X Marks: 80

Date : 31-08-2024 Time: 3 hours

SECTION-A (1 mark each)

1) Let a and b be two positive integers such that $a = p^3q^4$ and $b = p^2q^3$, where p and q are prime numbers. If HCF(a, b) $= p^m q^n$ and LCM(a, b) $= p^r q^s$, then $(m+n)(r+s) =$ —
(a) 15 (b) 30 (c) 35 (d) 72

2) Let p be a prime number. The quadratic equation having its roots as factors of p is —
(a) $x^2 - px + p = 0$ (b) $x^2 - (p+1)x + p = 0$
(c) $x^2 + (p+1)x + p = 0$ (d) $x^2 - px + p + 1 = 0$

3) If $p(x) = ax^2 + bx + c$ and $a+b+c = 0$, then one zero is
(a) $-\frac{b}{a}$ (b) $\frac{c}{a}$ (c) $\frac{b}{c}$ (d) none of these

4) $\triangle ABC \sim \triangle PQR$. If AM and PN are altitudes of $\triangle ABC$ and $\triangle PQR$ respectively and $AB^2 : PQ^2 = 4:9$, then AM : PN = —
(a) 3:2 (b) 16:81 (c) 4:9 (d) 2:3

5) The distance between the points $(a\cos\theta + b\sin\theta, 0)$ and $(0, a\sin\theta - b\cos\theta)$ is —
(a) a^2+b^2 (b) a^2-b^2 (c) $\sqrt{a^2+b^2}$ (d) $\sqrt{a^2-b^2}$

6) Three vertices of a ||gm ABCD are A(1, 4), B(−2, 3) and C(5, 8). The ordinate of the fourth vertex D is —
(a) 8 (b) 9 (c) 7 (d) 6

7) Two lines are given to be parallel. The equation of one of the lines is $3x - 2y = 5$. The equation of the second line can be
(a) $9x + 8y = 7$ (b) $-12x - 8y = 7$ (c) $-12x + 8y = 7$ (d) $12x + 8y = 7$

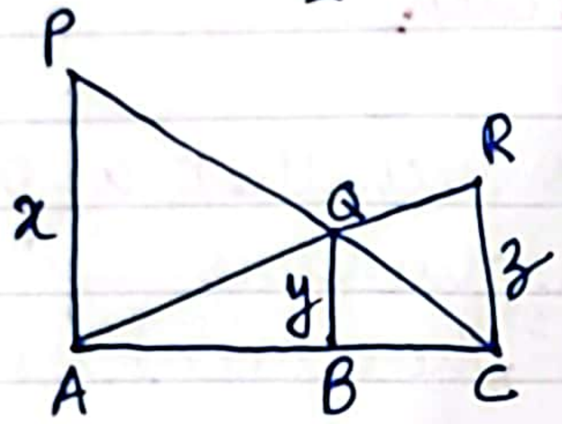
8) The value of x for which $2x$, $(x+10)$ and $(3x+2)$ are the three consecutive terms of an AP is —
(a) 6 (b) −6 (c) 18 (d) −18

9) The graph of $p(x)$ cuts the x-axis at 3 points and touches it at 2 other points. The no. of zeroes of $p(x)$ is
(a) 1 (b) 2 (c) 3 (d) 5

10) Points A(−1, y) and B(5, 7) lie on a circle with centre O(2, −3y). The values of y are
(a) 1, −7 (b) −1, 7 (c) 2, 7 (d) −2, −7

11) If θ is an acute angle and $\tan\theta + \cot\theta = 2$, then the value of $\sin^3\theta + \cos^3\theta = $ __

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\sqrt{2}$

12) 

PA, QB and RC are each perpendicular to AC. If $x = 8$ cm, $z = 6$ cm, then $y = $ __

(a) $\frac{56}{7}$ cm (b) $\frac{7}{56}$ cm (c) $\frac{25}{7}$ cm (d) $\frac{24}{7}$ cm

13) The LCM of two numbers is 2400. Which of the following cannot be their HCF?

(a) 300 (b) 400 (c) 500 (d) 600

14) If α, β are the zeroes of the polynomial $p(x) = x^2 - (k+6)x + 2(2k-1)$, then the value of k, if $\alpha + \beta = \frac{1}{2}\alpha\beta$ is

(a) -7 (b) 7 (c) -3 (d) 3

15) If $A(4,-2)$, $B(7,-2)$ and $C(7,9)$ are the vertices of a ΔABC, then ΔABC is

(a) equilateral Δ (b) isosceles Δ (c) rt. angled Δ (d) isosceles rt. angled Δ

16) In ΔABC, $DE \parallel BC$. If $AD = 2$cm, $BD = 3$cm, $BC = 7.5$cm, then length of $DE = $ __

(a) 2.5cm (b) 3cm (c) 5cm (d) 6cm

17) In ΔABC and ΔDEF, $\angle F = \angle C$, $\angle B = \angle E$ and $AB = \frac{1}{2} DE$. Then, the two triangles are

(a) congruent, but not similar (c) neither congruent nor similar
(b) similar, but not congruent (d) congruent as well as similar

18) The number of terms in the A.P: $18, 15\frac{1}{2}, 13, \ldots, -47$ is

(a) 72 (b) 27 (c) 25 (d) 29

19) Assertion:- If product of two ~~terms~~ numbers is 5780 and their HCF is 17, then their LCM is 340

Reason:- HCF is always a factor of LCM

(a) (b) (c) (d)

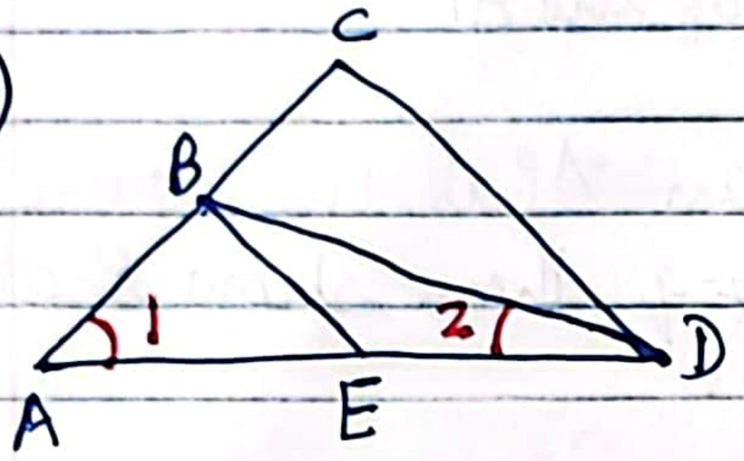
20) Assertion:- If the coordinates of the mid-points of the sides AB and AC of ΔABC are $D(3,5)$ and $E(-3,-3)$ respectively, then $BC = 20$ units.

Reason (R): The line joining the mid-points of two sides of a triangle is parallel to the third side and equal to half of it.

(a) (b) (c) (d)

SECTION-B (2 marks each)

21) If $49x + 51y = 499$; $51x + 49y = 501$, then find x and y

22) 

$\frac{AD}{AE} = \frac{AC}{BD}$ and $\angle 1 = \angle 2$

Show that $\triangle BAE \sim \triangle CAD$

23) If one root of the quadratic equation $x^2 + 12x - k = 0$ is thrice the other root, then find the value of k.

24) The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

25) Find the value of x:

$$2\,\mathrm{cosec}^2 30^\circ + x \sin^2 60^\circ - \frac{3}{4} \tan^2 30^\circ = 10$$

SECTION-C (3 marks each)

26) Prove that $\sqrt{2}$ is an irrational number.

27) Places A and B are 160 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 8 hrs but if they travel towards each other, they meet in 2 hrs. What are the speeds of the two cars?

(Or)

Vijay had some bananas and he divided them into two lots A and B. He sold the first lot at the rate of ₹5 for 6 bananas and the second lot at the rate of ₹1 per banana, and got a total of ₹675. If he had sold the first lot at the rate of ₹1 per banana and the second lot at the rate of ₹4 for 5 bananas, his total collection would have been ₹690. Find the total number of bananas he had.

28) Solve for x: $\dfrac{1}{2a+b+2x} = \dfrac{1}{2a} + \dfrac{1}{b} + \dfrac{1}{2x}$ (Or) $9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$

29) Find the value of $\dfrac{\sin\theta}{\sec\theta + \tan\theta - 1} + \dfrac{\cos\theta}{\operatorname{cosec}\theta + \cot\theta - 1}$

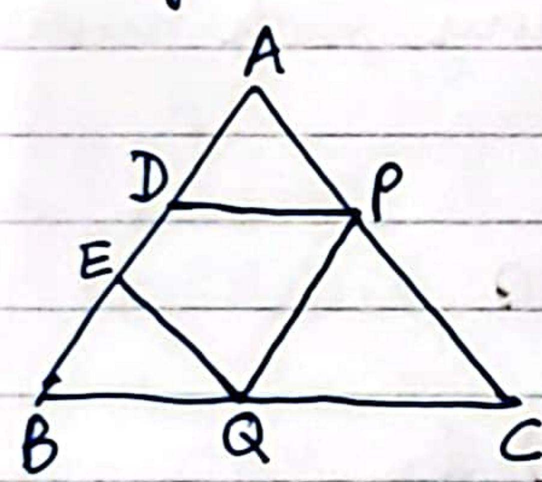
30) The first term of an AP is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the AP.

(Or)

If the sum of first p terms of an AP is the same as the sum of its first q terms, $p \neq q$, then show that sum of first $(p+q)$ terms is zero.

31) A street light bulb is fixed on a pole 6m above the level of the Street. If a woman of height 1.5m casts of a shadow of 3m, find how far she is away from the base of the pole.

(Or)



D and E are two points lying on side AB, such that $AD = BE$. If $DP \parallel BC$ and $EQ \parallel AC$, then prove that $PQ \parallel AB$.

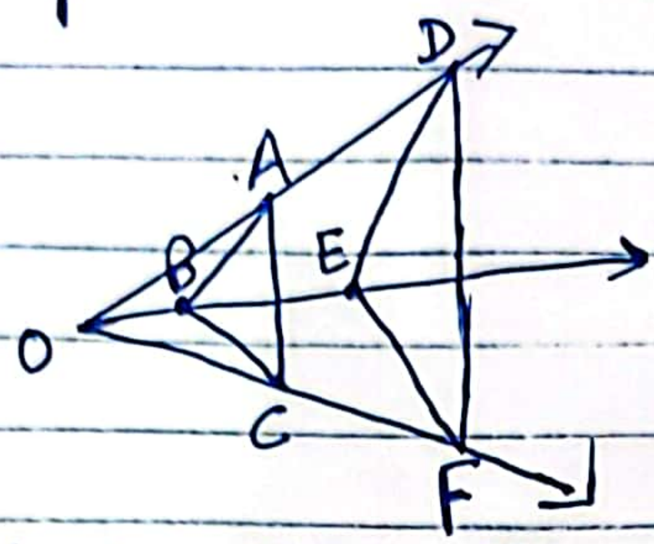
SECTION-D (5 marks each)

32) To fill a swimming pool, two pipes are used. If the pipe of larger diameter used for 4 hours and the pipe of smaller diameter of 9 hours, only half of the pool can be filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool?

(Or)

A motor boat whose speed is 18 Km/hr in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of stream.

33) State and prove Basic Proportionality Theorem.
Hence, prove that $AC \parallel DF$ if $AB \parallel DE$ and $BC \parallel EF$.

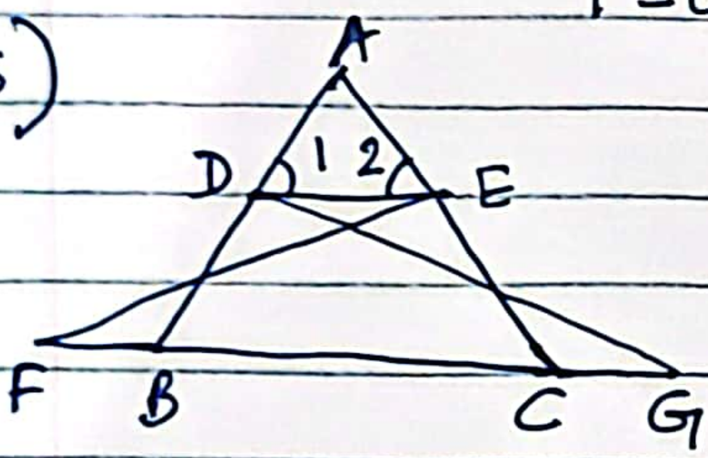


34) Prove that $\dfrac{\sin\theta - \cos\theta + 1}{\cos\theta + \sin\theta - 1} = \dfrac{1}{\sec\theta - \tan\theta}$

Or

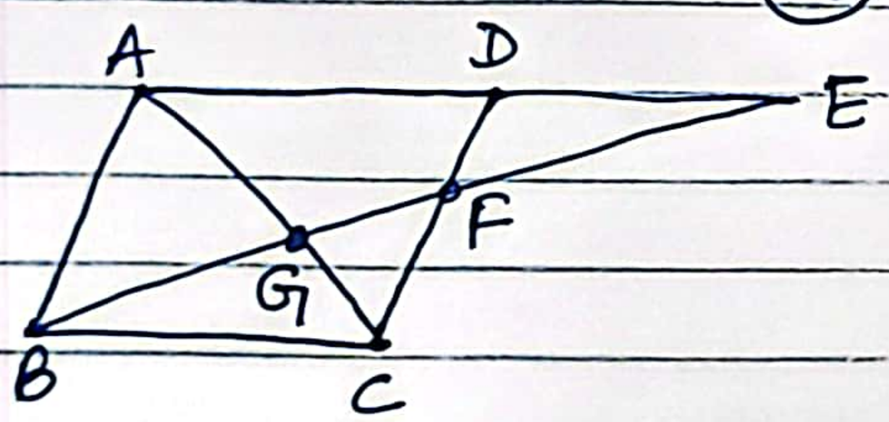
Prove that $\dfrac{\tan A}{1 - \cot A} + \dfrac{\cot A}{1 - \tan A} = 1 + \sec A \operatorname{cosec} A$

35)



$\triangle FEC \cong \triangle GBD$
$\angle 1 = \angle 2$
Prove that $\triangle ADE \sim \triangle ABC$

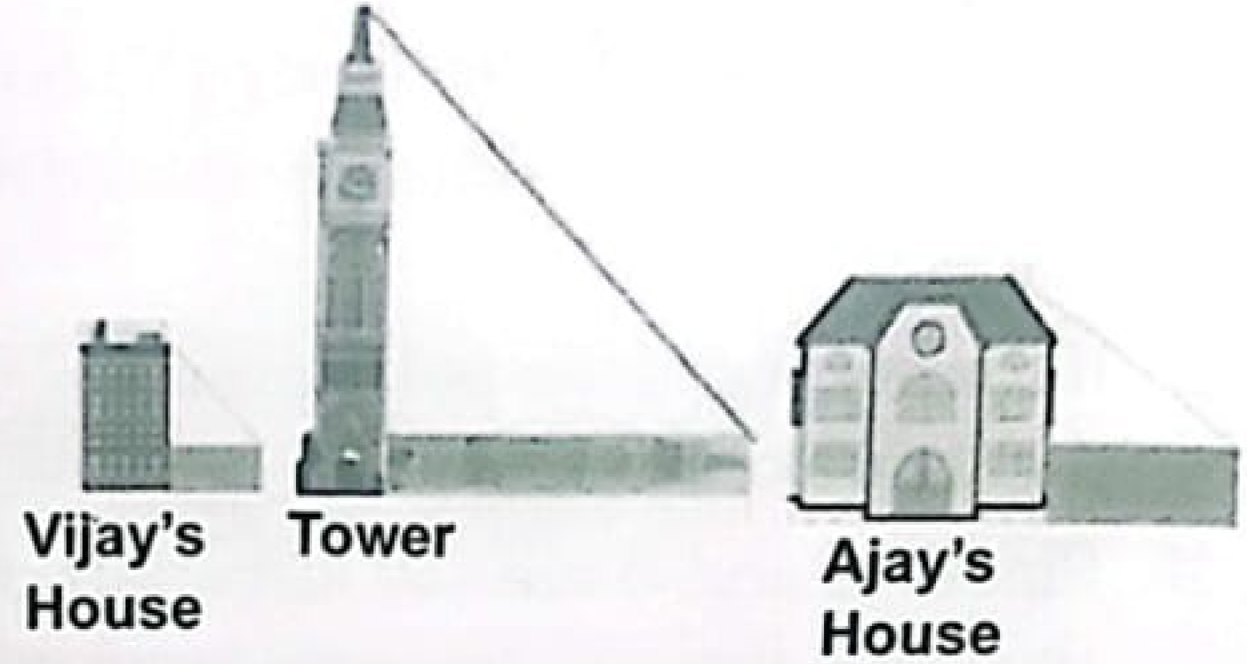
Or



Through the mid-point F of the side CD of a ||gm ABCD, the line BF is drawn intersecting AC at G and AD produced at E. Prove that $EG = 2BG$

SECTION–E (4 marks each)

36. Vijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Vijay's house if 20 m when Vijay's house casts a shadow 10 m long on the ground. At the same time, the tower casts a shadow 50 m long on the ground and the house of Ajay casts 20 m shadow on the ground.



(i) **What is the height of the tower?** 1

(ii) **What will be the length of the shadow of the tower when Vijay's house casts a shadow of 12 m?**

(iii) **What is the height of Ajay's house?**

(IV) **When the tower casts a shadow of 40 m. At same time what will be the length of the shadow of Ajay's house?**

37. India is competitive manufacturing location due to the lowcost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16,000 sets in 6^{th} year and 22,600 sets in 9^{th} year.

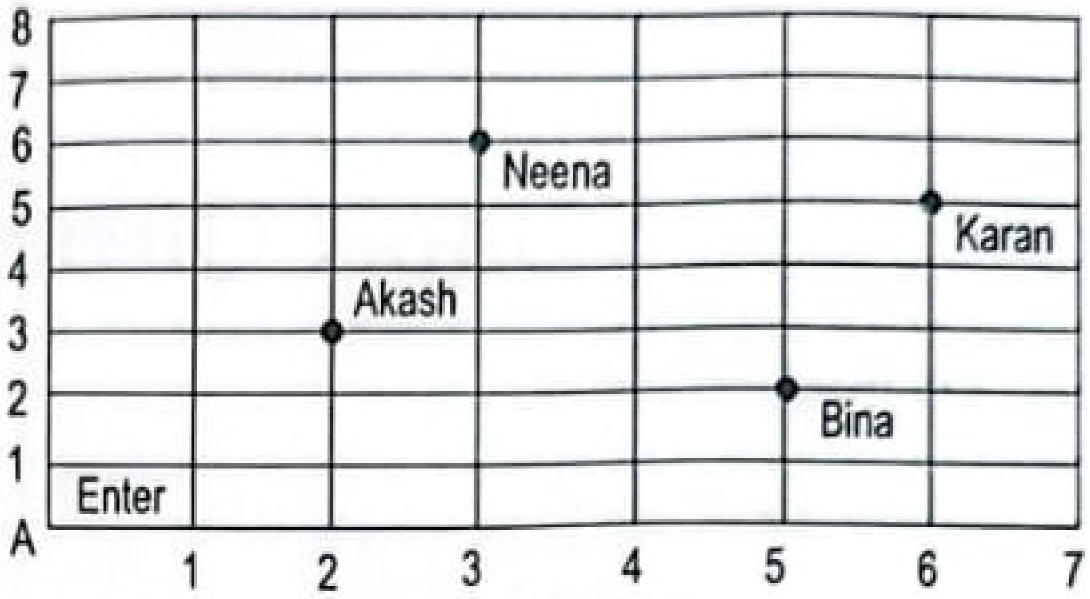
(i) **Find the production during the first year.**

(ii) **The increase in production every year is a fixed number. What is that number?**

(iii) **Find the production during first 3 years.**

(IV) **In which year the production reached at ₹29,200.**

38)



Karan went to the Lab near to his home for COVID 19 test along with his family members. The seats in the waiting area were as per the norms of distancing during this pandemic (as shown in the given figure). His family members took their seats surrounded by the circular do area.

(*i*) Considering A as the origin, what are the coordinates of A? What are the coordinates of seat of Akash?

(*ii*) What is the distance between Neena (N) and Karan (K)?

III Determine the shape of the figure we get on joining the points where Karan's family members are seated.

IV. What will be the coordinates of a point exactly between Akash (A) and Binu (B) where a person can be seated?

X Half yearly Exam - Answers

SECTION-A

1) $HCF(a,b) = p^2q^3$

$LCM(a,b) = p^3q^4$

$\therefore m=2, n=3, r=3, s=4$

$(m+n)(r+s) = (2+3)(3+4) = 5\times 7 = 35$ (c)

2) Factors of p are 1 and p.

$x^2-(p+1)x+p=0$

$\Rightarrow x^2-px-x+p=0$

$\Rightarrow x(x-p)-1(x-p)=0$

$\Rightarrow (x-1)(x-p)=0$

$x = 1, p$ (b)

3) $p(1) = a+b+c = 0$

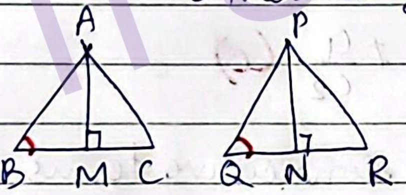
$\therefore$ one zero is 1

Let $\alpha = 1$

$\alpha\beta = \frac{c}{a}$

$\Rightarrow 1\times\beta = \frac{c}{a}$

$\therefore$ other ~~roo~~ zero, $\beta = \frac{c}{a}$ (b)

4) 

Since $\Delta ABC \sim \Delta PQR$, ΔABM is also similar to ΔPQN by AA similarity

$\therefore \frac{AB}{PQ} = \frac{AM}{PN} = \frac{2}{3}$ (d)

$\frac{AB^2}{PQ^2} = \frac{4}{9}$

$\Rightarrow \frac{AB}{PQ} = \frac{2}{3}$

5) $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{[0-(a\cos\theta+b\sin\theta)]^2+[a\sin\theta-b\cos\theta-0]^2}$

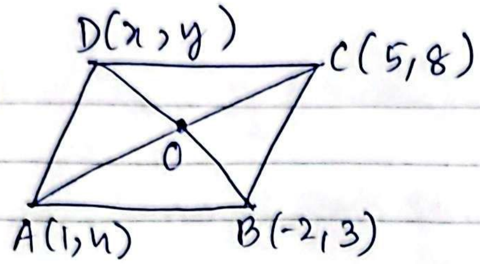
$= \sqrt{[-(a\cos\theta+b\sin\theta)]^2+(a\sin\theta-b\cos\theta)^2}$

$= \sqrt{a^2\cos^2\theta+b^2\sin^2\theta+\cancel{2ab\sin\theta\cos\theta}+a^2\sin^2\theta+b^2\cos^2\theta-\cancel{2ab\sin\theta\cos\theta}}$

$= \sqrt{a^2(\sin^2\theta+\cos^2\theta)+b^2(\sin^2\theta+\cos^2\theta)}$

$= \sqrt{a^2+b^2}$ (c)

6)



Since diagonals of a parallelogram bisect each other,

mid-point of AC = mid-point of BD

$$\left(\frac{1+5}{2}, \frac{4+8}{2}\right) = \left(\frac{x-2}{2}, \frac{y+3}{2}\right)$$

~~On equating x-coordinates and~~ y-coordinates,

$$\frac{12}{\cancel{2}_1} = \frac{y+3}{\cancel{2}_1} \Rightarrow y = 9$$

$\therefore$ The ordinate is 9 (b)

7) For parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$a_1 = 3, b_1 = -2, c_1 = -5$

For $-12x + 8y - 7 = 0$, $a_2 = -12, b_2 = 8, c_2 = -7$

$$\frac{a_1}{a_2} = \frac{-1}{4}$$

$$\frac{b_1}{b_2} = \frac{-1}{4}$$

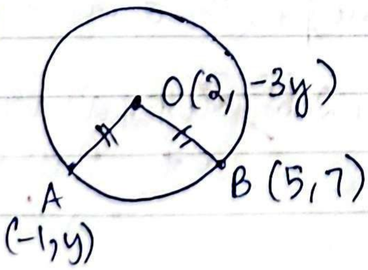
$$\frac{c_1}{c_2} = \frac{-5}{-7} = \frac{5}{7} \qquad \therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \text{ (c)}$$

8) Since the difference between two consecutive terms in an AP is a constant,

$$x + 10 - 2x = 3x + 2 - x - 10$$
$$-x + 10 = 2x - 8$$
$$-3x = -18$$
$$x = 6 \text{ (a)}$$

9) 5 (d)

10)



$$OA^2 = OB^2$$
$$(-1-2)^2 + (y+3y)^2 = (5-2)^2 + (7+3y)^2$$
$$\cancel{9} + 16y^2 = \cancel{9} + 49 + 9y^2 + 42y$$
$$7y^2 - 42y - 49 = 0$$
$$y^2 - 6y - 7 = 0$$
$$(y-7)(y+1) = 0$$

$y = 7, -1$

(b)

11) $\tan\theta + \cot\theta = 2 \Rightarrow \theta = 45^\circ$

$[\because \tan 45^\circ + \cot 45^\circ = 1 + 1 = 2]$

$\therefore \sin^3 45^\circ + \cos^3 45^\circ = \left(\frac{1}{\sqrt{2}}\right)^3 + \left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ (c)

12) we know that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$

$\Rightarrow \frac{1\times3}{8\times3} + \frac{1\times4}{6\times4} = \frac{1}{y}$

$\Rightarrow \frac{7}{24} = \frac{1}{y}$

$\therefore y = \frac{24}{7}$ cm (d)

13) Since HCF should be a factor of LCM.
500 is not a factor of 2400 (c)

14) $a = 1, b = -(k+6), c = 2(2k-1)$

$\alpha + \beta = \frac{-b}{a} = k+6$

$\alpha\beta = \frac{c}{a} = 2(2k-1)$

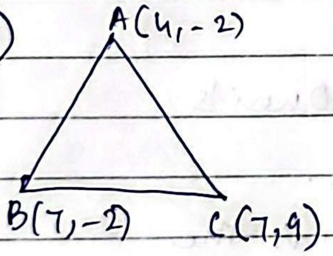
$\alpha + \beta = \frac{1}{2}\alpha\beta$

$\Rightarrow k+6 = \frac{1}{\cancel{2}_1} \times \cancel{2}^1 (2k-1)$

$\Rightarrow -k = -7$

$k = 7$ (b)

15)



$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$AB = \sqrt{(7-4)^2 + (-2+2)^2} = \sqrt{9} = 3$ units

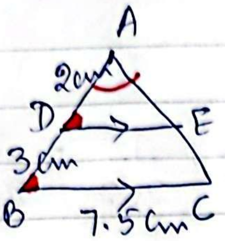
$BC = \sqrt{(7-7)^2 + (9+2)^2} = \sqrt{121} = 11$ units

$AC = \sqrt{(7-4)^2 + (9+2)^2} = \sqrt{9+121} = \sqrt{130}$ units

$AB^2 + BC^2 = 3^2 + 11^2 = 9 + 121 = 130$

$AC^2 = (\sqrt{130})^2 = 130$

$\therefore AC^2 = AB^2 + BC^2 \Rightarrow$ right Δ (c)

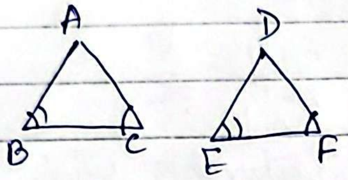
16) 

$\triangle ADE \sim \triangle ABC$ (AA Similarity)

$$\frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \frac{2}{5} = \frac{DE}{7.5}$$

$$\therefore DE = \frac{2 \times \cancel{7.5}^{1.5}}{\cancel{5}_{1}} = 3\text{cm}$$ (b)

17) 

Similar, but not congruent (b)

18) $a = 18,\ d = \frac{31}{2} - 18 = \frac{31-36}{2} = -\frac{5}{2}$

$a_n = a + (n-1)d = -47$

$\Rightarrow 18 - \frac{5}{2}(n-1) = -47$

$\Rightarrow -\frac{5}{2}(n-1) = -65$

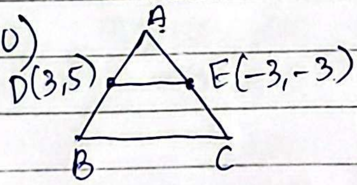
$n - 1 = -\cancel{65}^{13} \times -\frac{2}{\cancel{5}_{1}} = 26$

$n = 27$ (b)

19) $17 \times 340 = 5780$

Assertion is true

Reason is also true but not the correct explanation (b)

20) 

$DE = \sqrt{(-3-3)^2 + (-3-5)^2}$

$= \sqrt{36+64} = \sqrt{100} = 10$ units

$BC = 2 \times 10 = 20$ units.

(a) assertion and reason are true and reason is the correct explanation of assertion.

SECTION-B

21) $49x + 51y = 499 \rightarrow (1)$

$51x + 49y = 501 \rightarrow (2)$

$(1)+(2) \Rightarrow 100x + 100y = 1000$

$\div 100 \Rightarrow x + y = 10 \rightarrow (3)$

$(1)-(2) \Rightarrow -2x + 2y = -2$

$(\div -2) \quad x - y = 1 \rightarrow (4)$

$(3)+(4), \quad 2x = 11 \Rightarrow x = \frac{11}{2}$

From eq:(3),

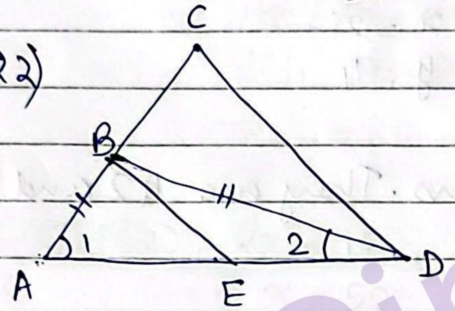
$\frac{11}{2} + y = 10$

$y = 10 - \frac{11}{2}$

$= \frac{9}{2}$

$\boxed{x = 11/2}$

$\boxed{y = 9/2}$

22) 

Given:- $\frac{AD}{AE} = \frac{AC}{BD}$

$\angle 1 = \angle 2$

To prove:- $\triangle BAE \sim \triangle CAD$

Proof:- Since $\angle 1 = \angle 2$,

$AB = BD$ (sides opposite to equal angles) $\rightarrow (1)$

$\frac{AD}{AE} = \frac{AC}{BD}$ (given)

$\Rightarrow \frac{AD}{AE} = \frac{AC}{AB}$ [from eq: (1)]

Also, $\angle BAE = \angle CAD$ (common angle)

$\therefore \triangle BAE \sim \triangle CAD$ (SAS similarity)

Hence Proved.

23) Let the given equation be of the form $ax^2 + bx + c = 0$; where $a = 1, b = 12$ and $c = -k$ and α and 3α be the zeroes.

Then, $\alpha + 3\alpha = -\frac{b}{a} = -12$

$\Rightarrow 4\alpha = -12$

$\boxed{\alpha = -3}$

Also, $\alpha \times 3\alpha = \frac{c}{a} = -k$

$\Rightarrow 3\alpha^2 = -k$

$\Rightarrow 3(-3)^2 = -k$

$\therefore -k = 3 \times 9 = 27$

$\therefore k = -27$

24) Let the digit in the ten's place be x and that in the ones place be y.

T	O
x	y

Original no. is $10x+y$

Reversed no. is $10y+x$

ATQ, $10x+y+10y+x=66$

$11x+11y=66$

$x+y=6 \rightarrow (1)$

Also, $x-y=\pm 2$

Case 1:- $x+y=6$

$x-y=2$

(+), $2x=8$

$x=4$

$y=2$

Case 2:- $x+y=6$

$x-y=-2$

(+), $2x=4$

$x=2$

$y=4$

Hence, there are 2 such numbers. They are 42 and 24.

25) $\operatorname{cosec} 30^\circ = 2$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$

$$2\times 4 + x\times\frac{3}{4} - \frac{\cancel{3}^{1}}{4}\times\frac{1}{\cancel{3}_{1}} = 10$$

$$8+\frac{3x}{4}-\frac{1}{4}=10$$

$$\frac{3x}{4}+\frac{32-1}{4}=10$$

$$3x+31=40$$

$$3x=9$$

$$\boxed{x=3}$$

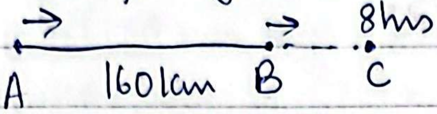
SECTION - C

26) **Do yourself**

27) Let the speed of the car starting from place A be x km/hr and that from B be y km/hr

Distance = speed × time

Case 1 :-

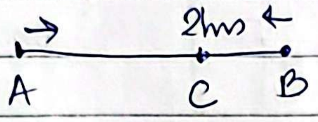


$AB = AC - BC$

$160 = 8x - 8y$

$x - y = 20 \rightarrow (1)$

Case 2 :-



$AB = AC + BC$

$160 = 2x + 2y$

$x + y = 80 \rightarrow (2)$

$(1) + (2)$, $2x = 100$

$x = 50$

$y = 30$

Hence, the speeds of the two cars are 50km/hr and 30km/hr.

OR Let the no. of bananas in lot A be x and that in lot B be y.

Cost of 1 banana in lot A = ₹ $\frac{5}{6}$

Cost of 1 banana in lot B = ₹1

Thus, $\frac{5}{6}x + 1y = 675 \rightarrow (1)$

Also, cost of 1 banana in lot A = ₹ 1

Cost of 1 banana in lot B = ₹ $\frac{4}{5}$

Now, $1x + \frac{4}{5}y = 690 \rightarrow (2)$

From eq:(1), $5x + 6y = 4050$

From eq:(2), $5x + 4y = 3450$

$(-)$, $2y = 600$

$\boxed{y = 300}$

From eq: $5x + 6y = 4050$,

$5x + 1800 = 4050$

$5x = 2250$

$\boxed{x = 450}$

Hence, the total no. of bananas $= x+y = 300+450 = \underline{750 \text{ bananas}}$

28) $\dfrac{1}{2a+b+2x} = \dfrac{1}{2a} + \dfrac{1}{b} + \dfrac{1}{2x}$

$\Rightarrow \dfrac{1}{2a+b+2x} - \dfrac{1}{2x} = \dfrac{1}{2a} + \dfrac{1}{b}$

$\Rightarrow \dfrac{\cancel{2x} - 2a - b - \cancel{2x}}{2x(2a+b+2x)} = \dfrac{b+2a}{2ab}$

$\Rightarrow \dfrac{-\cancel{(2a+b)}}{4ax+2bx+4x^2} = \dfrac{\cancel{2a+b}}{2ab}$

$\Rightarrow -2ab = 4x^2 + 4ax + 2bx$

$\Rightarrow 4x^2 + 4ax + 2bx + 2ab = 0$

$\Rightarrow 4x(x+a) + 2b(x+a) = 0$

$(4x+2b)(x+a) = 0$

$\therefore x = \dfrac{-2b}{4} = \dfrac{-b}{2}$

or $x = -a$

Or Let $9x^2 - 9(a+b)x + (2a^2+5ab+2b^2) = 0$ be of the form $Ax^2+Bx+C=0$; where $A = 9$

$B = -9(a+b)$

$C = 2a^2+5ab+2b^2$

$B^2 - 4AC = [-9(a+b)]^2 - 4\times 9(2a^2+5ab+2b^2)$

$= 81(a+b)^2 - 36(2a^2+5ab+2b^2)$

$= 81(a^2+b^2+2ab) - 72a^2 - 180ab - 72b^2$

$= 81a^2+81b^2+162ab-72a^2-180ab-72b^2$

$= 9a^2+9b^2-18ab$

$= (3a-3b)^2$

$\therefore x = \dfrac{-B \pm \sqrt{B^2-4AC}}{2A} = \dfrac{9(a+b) \pm (3a-3b)}{18}$

$x = \dfrac{9a+9b+3a-3b}{18} = \dfrac{12a+6b}{18} = \dfrac{2a+b}{3}$

or $x = \dfrac{9a+9b-3a+3b}{18} = \dfrac{6a+12b}{18} = \dfrac{a+2b}{3}$

29) $\dfrac{\sin\theta}{\dfrac{1}{\cos\theta}+\dfrac{\sin\theta}{\cos\theta}-1} + \dfrac{\cos\theta}{\dfrac{1}{\sin\theta}+\dfrac{\cos\theta}{\sin\theta}-1}$

$= \dfrac{\sin\theta.\cos\theta}{1+\sin\theta-\cos\theta} + \dfrac{\cos\theta.\sin\theta}{1+\cos\theta-\sin\theta}$

$= \dfrac{\sin\theta\cos\theta}{1+(\sin\theta-\cos\theta)} + \dfrac{\sin\theta.\cos\theta}{1-(\sin\theta-\cos\theta)}$

$= \sin\theta\cos\theta\left[\dfrac{1}{1+(\sin\theta-\cos\theta)} + \dfrac{1}{1-(\sin\theta-\cos\theta)}\right]$

$= \sin\theta\cos\theta\left[\dfrac{1-(\sin\theta-\cos\theta)+1+(\sin\theta-\cos\theta)}{1-(\sin\theta-\cos\theta)^2}\right]$

$= \sin\theta\cos\theta\left[\dfrac{1-\cancel{\sin\theta}+\cancel{\cos\theta}+1+\cancel{\sin\theta}-\cancel{\cos\theta}}{1-\sin^2\theta-\cos^2\theta+2\sin\theta\cos\theta}\right]$

$= \sin\theta\cos\theta\left[\dfrac{2}{1-(\sin^2\theta+\cos^2\theta)+2\sin\theta\cos\theta}\right]$

$= \sin\theta\cos\theta\left[\dfrac{2}{\cancel{1}-\cancel{1}+2\sin\theta\cos\theta}\right]$

$= \dfrac{2\sin\theta\cos\theta}{2\sin\theta\cos\theta} = \underline{1}$

30) $a = 5$

$a_n = 45$

$S_n = 400 = \dfrac{n}{2}[a+a_n]$

$\Rightarrow 400 = \dfrac{n}{2}[5+45]$

$\Rightarrow 40\cancel{0} = \dfrac{n}{2}\times 5\cancel{0}$

$\Rightarrow \dfrac{n}{2} = 8$

$n = 16$

$a_n = a+(n-1)d = 45$

$\Rightarrow 5+15\times d = 45$

$15d = 40$

$d = \dfrac{40}{15} = \underline{\underline{\dfrac{8}{3}}}$

(Or) Let the first term and common difference be a and d respectively.

ATQ, $S_p = S_q$

$\Rightarrow \frac{p}{\cancel{2}}[2a+(p-1)d] = \frac{q}{\cancel{2}}[2a+(q-1)d]$

$\Rightarrow 2ap+(p^2-p)d = 2aq+(q^2-q)d$

$\Rightarrow 2ap-2aq = (q^2-q)d-(p^2-p)d$

$\Rightarrow 2a(p-q) = (q^2-q-p^2+p)d$

$\Rightarrow 2a(p-q) = [\underbrace{(q^2-p^2)}_{(q+p)(q-p)}-(q-p)]d$

$\Rightarrow 2a(\cancel{p-q}) = \cancel{(q-p)}_{-1}[q+p-1]d$

$\Rightarrow 2a = -[q+p-1]d$

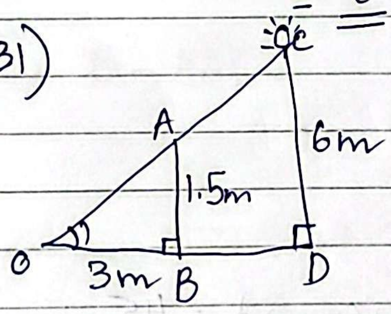
$\Rightarrow 2a+(p+q-1)d = 0 \longrightarrow (1)$

$\therefore S_{p+q} = \frac{p+q}{2}[2a+(p+q-1)d]$

$= \frac{p+q}{2} \times 0$ [from eq:(1)]

$= \underline{\underline{0}}$

31)



Let CD be the height of the pole and AB be the height of Women.

In $\triangle ABO$ and $\triangle CDO$,

$\angle ABO = \angle CDO$ (each 90°)

$\angle AOB = \angle COD$ (common angle)

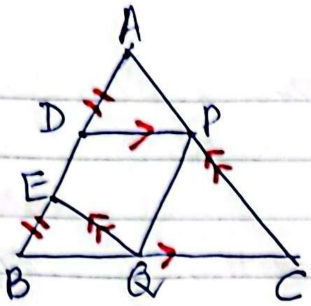
$\therefore \triangle ABO \sim \triangle CDO$ (AA Similarity)

Thus, $\frac{AB}{CD} = \frac{OB}{OD}$ [corresponding sides of similar $\triangle$s are proportional]

$\Rightarrow \frac{\cancel{1.5}^{0.5}}{\cancel{6}_{2}} = \frac{3}{3+BD} \Rightarrow 3+BD = \frac{6}{0.5} = \frac{60}{5} = 12$

Hence woman is 9m away from the pole. $BD = \underline{\underline{9m}}$

Or



Given:- In ΔABC, $AD = BE$
$DP \parallel BC$ and $EQ \parallel AC$.
To prove: $PQ \parallel AB$.
Proof:- Since $DP \parallel BC$, using Thales theorem,

$$\frac{AD}{DB} = \frac{AP}{PC} \longrightarrow (1)$$

Similarly, since $EQ \parallel AC$, $\frac{BE}{EA} = \frac{BQ}{QC} \longrightarrow (2)$

But $\frac{BE}{EA} = \frac{AD}{ED + AD} = \frac{AD}{ED + BE} = \frac{AD}{BD}$ $[\because AD = BE]$

$$\therefore \frac{BE}{EA} = \frac{AD}{DB} \rightarrow (3)$$

From (1), (2) and (3), $\frac{AP}{PC} = \frac{BQ}{QC}$

$\Rightarrow PQ \parallel AB$ using converse of Thales theorem.

Hence proved.

SECTION-D

32) Let the time taken by larger diameter pipe to fill the pool alone be x hrs and that by smaller diameter pipe be $(x+10)$ hrs.

ATQ, $\frac{4}{x} + \frac{9}{x+10} = \frac{1}{2}$

$\Rightarrow \frac{4x + 40 + 9x}{x^2 + 10x} = \frac{1}{2}$

$\Rightarrow (13x + 40)2 = x^2 + 10x$

$\Rightarrow 26x + 80 = x^2 + 10x$

$\Rightarrow x^2 - 16x - 80 = 0$

$\Rightarrow (x - 20)(x + 4) = 0$

$x = 20, -4$

S	P
-16	-80
	$-20, 4$

x cannot be $-$ve, $\therefore$ required value of $x = 20$ hrs

Hence, time taken by the two pipes to fill the pool separately are 20 hrs and 30 hrs.

(Or) Let the speed of stream be x km/hr

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$24\left[\frac{1}{18-x} - \frac{1}{18+x}\right] = 1$$

$$24\left[\frac{\cancel{18}+x-\cancel{18}+x}{18^2-x^2}\right] = 1$$

$$24 \times 2x = 324 - x^2$$

$$x^2 + 48x - 324 = 0$$

$$(x+54)(x-6) = 0$$

$$x = -54, 6$$

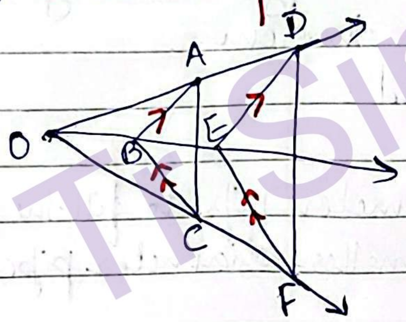
S: 48 P: −324 → 54, −6

$3 \,|\, 324$, $3 \,|\, 108$, $3 \,|\, 36$, $3 \,|\, 12$, $2 \,|\, 4$, 2

x cannot be −ve, ∴ required value of $x = 6$ km/hr

Hence, the speed of stream = 6 km/hr.

33) State and prove Basic Proportionality theorem



Given: $AB \parallel DE$, $BC \parallel EF$

To prove: $AC \parallel DF$

Proof:-

Since $BA \parallel ED$ in $\triangle OED$, using Thales theorem,

$$\frac{OB}{BE} = \frac{OA}{AD} \rightarrow (1)$$

Similarly, in $\triangle OFE$, $\frac{OB}{BE} = \frac{OC}{CF} \rightarrow (2)$

From eq's (1) and (2), $\frac{OA}{OD} = \frac{OC}{CF}$

$\Rightarrow AC \parallel DF$ using converse of Thales theorem.

Hence Proved.

34) LHS, $\frac{\sin\theta-\cos\theta+1}{\cos\theta+\sin\theta-1}$

($\div\cos\theta$) $=\frac{\tan\theta-1+\sec\theta}{1+\tan\theta-\sec\theta}$

$=\frac{\sec\theta+\tan\theta-1}{1+\tan\theta-\sec\theta}$

$=\frac{(\sec\theta+\tan\theta)-(\sec^2\theta-\tan^2\theta)}{1+\tan\theta-\sec\theta}$

$=\frac{(\sec\theta+\tan\theta)-(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{1+\tan\theta-\sec\theta}$

$=\frac{(\sec\theta+\tan\theta)[1-(\sec\theta-\tan\theta)]}{1+\tan\theta-\sec\theta}$

$=\frac{(\sec\theta+\tan\theta)\overset{1}{\cancel{(1-\sec\theta+\tan\theta)}}}{\underset{1}{\cancel{1+\tan\theta-\sec\theta}}}$

$=\frac{\sec\theta+\tan\theta}{1}$

$=\frac{\sec\theta+\tan\theta}{\sec^2\theta-\tan^2\theta}$

$=\frac{\overset{1}{\cancel{\sec\theta+\tan\theta}}}{\underset{1}{\cancel{(\sec\theta+\tan\theta)}}(\sec\theta-\tan\theta)}$

$=\frac{1}{\sec\theta-\tan\theta}$, RHS

Or LHS, $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$

$=\frac{\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}+\frac{\frac{\cos A}{\sin A}}{1-\frac{\sin A}{\cos A}}=\frac{\frac{\sin A}{\cos A}}{\frac{\sin A-\cos A}{\sin A}}+\frac{\frac{\cos A}{\sin A}}{\frac{\cos A-\sin A}{\cos A}}$

$$= \frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} + \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A}$$

$$= \frac{\sin^2 A}{\cos A(\sin A - \cos A)} - \frac{\cos^2 A}{\sin A(\sin A - \cos A)}$$

$$= \frac{\sin^3 A - \cos^3 A}{(\sin A - \cos A)\sin A \cos A} \qquad a^3 - b^3 = (a-b)(a^2+b^2+ab)$$

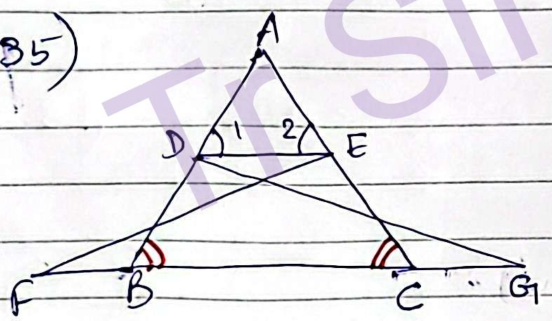
$$= \frac{\cancel{(\sin A - \cos A)}(\sin^2 A + \cos^2 A + \sin A \cos A)}{\cancel{(\sin A - \cos A)}\sin A \cos A}$$

$$= \frac{1 + \sin A \cos A}{\sin A \cos A} \qquad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{1}{\sin A \cos A} + \frac{\sin A \cos A}{\sin A \cos A}$$

$$= \operatorname{cosec} A \sec A + 1 \text{ , RHS}$$

35)



Given :- $\triangle FEC \cong \triangle GDB$

$\angle 1 = \angle 2$

To prove :- $\triangle ADE \sim \triangle ABC$

Proof :- Since $\angle 1 = \angle 2$, in $\triangle ADE$

$AD = AE$ (sides opposite to equal angles) $\rightarrow (1)$

Since $\triangle FEC \cong \triangle GDB$, $\angle ECF = \angle GBD$ (cpct)

$\Rightarrow \angle C = \angle B$

$\Rightarrow AB = AC \rightarrow (2)$

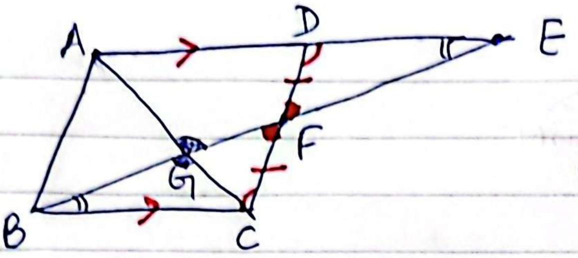
Also, $\angle DAE = \angle BAC$ (common angle)

~~$\therefore \triangle ADE \sim \triangle ABC$ (~~

From (1) and (2), $\frac{AD}{AB} = \frac{AE}{AC}$

$\therefore \triangle ADE \sim \triangle ABC$ (SAS similarity)

Hence Proved.

32) 

Given:- in parallelogram ABCD, FD = CF

To prove:- EG = 2BG

Proof:- In $\triangle BFC$ and $\triangle EFD$, $\angle BFC = \angle EFD$ (VOA)

$FC = DF$ ($\because$ F is the mid-point)

$\angle BCF = \angle EDF$ (alternate interior angles)

$\therefore \triangle BFC \cong \triangle EFD$ (ASA congruency)

Thus, $BC = DE$ (by cpct) $\rightarrow (1)$

In $\triangle BGC$ and $\triangle EGA$, $\angle BGC = \angle EGA$ (VOA)

$\angle GBC = \angle GEA$ (alternate interior angles)

$\therefore \triangle BGC \sim \triangle EGA$ (AA similarity)

Thus $\frac{BG}{EG} = \frac{BC}{AE}$ ($\because$ corresponding sides of similar $\triangle$s are in proportion)

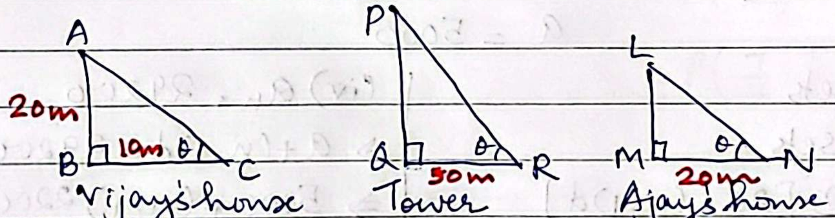
$\Rightarrow \frac{BG}{EG} = \frac{BC}{AD+DE}$

$= \frac{BC}{BC+BC}$ [from eq:(1) and opposite sides of a ||gm]

$= \frac{\cancel{BC}\,1}{2\cancel{BC}\,1}$

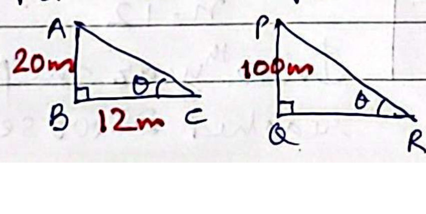
$\therefore \frac{BG}{EG} = \frac{1}{2} \Rightarrow \underline{EG = 2BG}$ Hence Proved

SECTION-E

36) 

All three triangles are similar to each other by AA similarity

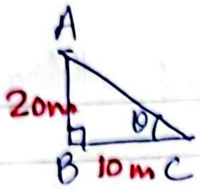
(i) $\frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \frac{20}{PQ} = \frac{10}{50} \Rightarrow PQ = \frac{20 \times 50}{10} = \underline{100m}$

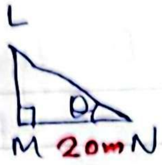
(ii) 

$\frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \frac{20}{100} = \frac{12}{QR}$

$\therefore QR = \frac{12 \times 100}{20} = \underline{60m}$

(iii)



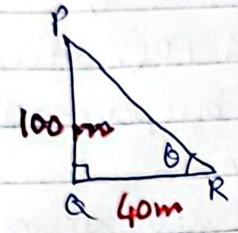


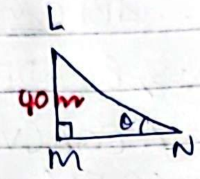
$\frac{AB}{LM} = \frac{BC}{MN}$

$\frac{20}{LM} = \frac{10}{20}$

$\therefore LM = \frac{20 \times 20}{10} = \underline{\underline{40m}}$

(iv)





$\frac{PQ}{LM} = \frac{QR}{MN}$

$\Rightarrow \frac{100}{40} = \frac{40}{MN}$

$\therefore MN = \frac{40 \times 40}{100} = \underline{\underline{16m}}$

37) $a_6 = 16000 \Rightarrow a + 5d = 16000 \rightarrow (1)$

$a_9 = 22600 \Rightarrow a + 8d = 22600 \rightarrow (2)$

$(-), \; -3d = {}^{-}6600$

$d = 2200$

From eq: (1), $a + 11000 = 16000$

$a = 5000$

(i) 5000 TV sets

(ii) 2200 TV sets.

(iii) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_3 = \frac{3}{2}[10000 + 2 \times 2200]$

$= \frac{3}{\cancel{2}_1} \times \cancel{14400}^{7200}$

$= \underline{\underline{21,600}}$ TV sets

(iv) $a_n = 29200$

$\Rightarrow a + (n-1)d = 29200$

$\Rightarrow 5000 + (n-1) \cdot 2200 = 29200$

$\Rightarrow (n-1)2200 = 24200$

$n - 1 = 11$

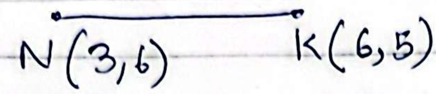
$n = 12$

In 12^{th} year the production reached 29200 sets.

38) (i) A(0, 0)

Coordinates of Akash = (2, 3)

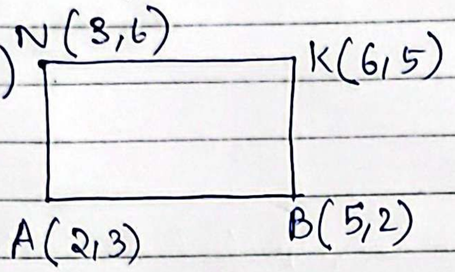
(ii)



$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(6-3)^2 + (5-6)^2} = \sqrt{9+1}$

$= \sqrt{10}$ units

(iii)



$AB = \sqrt{(5-2)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$ units

$BK = \sqrt{(6-5)^2 + (5-2)^2} = \sqrt{1+9} = \sqrt{10}$ units

$NK = \sqrt{10}$ units

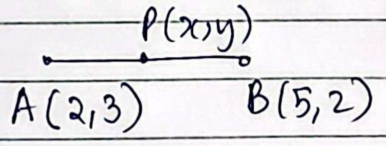
$AN = \sqrt{(3-2)^2 + (6-3)^2} = \sqrt{1+9} = \sqrt{10}$ units

Diagonal $AK = \sqrt{(6-2)^2 + (5-3)^2} = \sqrt{16+4} = \sqrt{20}$ units

Diagonal $BN = \sqrt{(3-5)^2 + (6-2)^2} = \sqrt{4+16} = \sqrt{20}$ units

Since all sides are equal and diagonals too are equal, the figure formed is a square.

(iv)



$P(x, y) = P\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= P\left(\frac{2+5}{2}, \frac{3+2}{2}\right)$

$= P\left(\frac{7}{2}, \frac{5}{2}\right)$