23. Sides of a triangle are in the ratio 12:17:25 and its perimeter is 540 cm. Find the semi perimeter and its sides.

(OR)

The difference between the semi-perimeter and the sides of a $\triangle ABC$ are 8 cm, 7 cm and 5 cm respectively. Find the area of the triangle.

1) Let the sides of the triangle be $12x, 17x$ and $25x$

   Perimeter $= 540$ cm

   $\Rightarrow 12x + 17x + 25x = 540$

   $\Rightarrow \qquad\qquad 54x = 540$

   $\qquad\qquad\qquad x = 10$ cm

   $\therefore$ The sides are 120 cm, 170 cm and 250 cm.

   Semi-perimeter $= \dfrac{\text{perimeter}}{2} = \dfrac{540}{2} = 270$ cm.

2) Let the sides be a cm, b cm and c cm and $S = \dfrac{a+b+c}{2}$

   Given, $S - a = 8$ cm

   $\qquad\quad S - b = 7$ cm

   $\qquad\quad S - c = 5$ cm

   On adding, $3S - (a+b+c) = 8 + 7 + 5$

   $\qquad \Rightarrow 3S - 2S = 20$

   $\qquad\qquad \therefore \quad S = 20$ cm

   Then, area of the $\triangle = \sqrt{S(S-a)(S-b)(S-c)}$

   $\qquad\qquad\qquad\qquad = \sqrt{\underset{\substack{\wedge \\ 4\times5}}{20} \times \underset{\substack{\wedge \\ 4\times2}}{8} \times 7 \times 5}$

   $\qquad\qquad\qquad\qquad = 4 \times 5 \times \sqrt{14}$

   $\qquad\qquad\qquad\qquad = 20\sqrt{14} \text{ cm}^2$