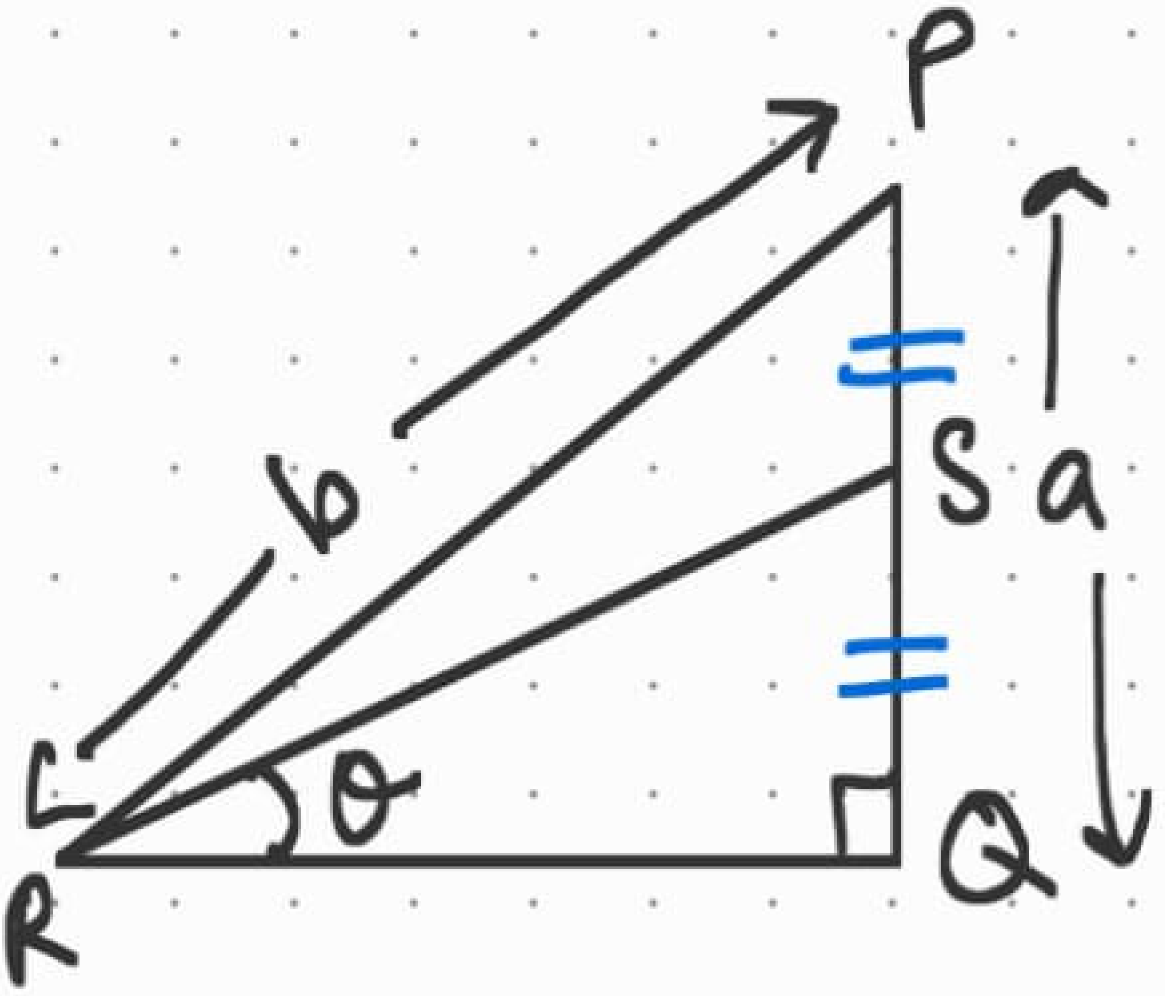
Test-16

1) Find $\tan\theta$ if $PS = SQ$



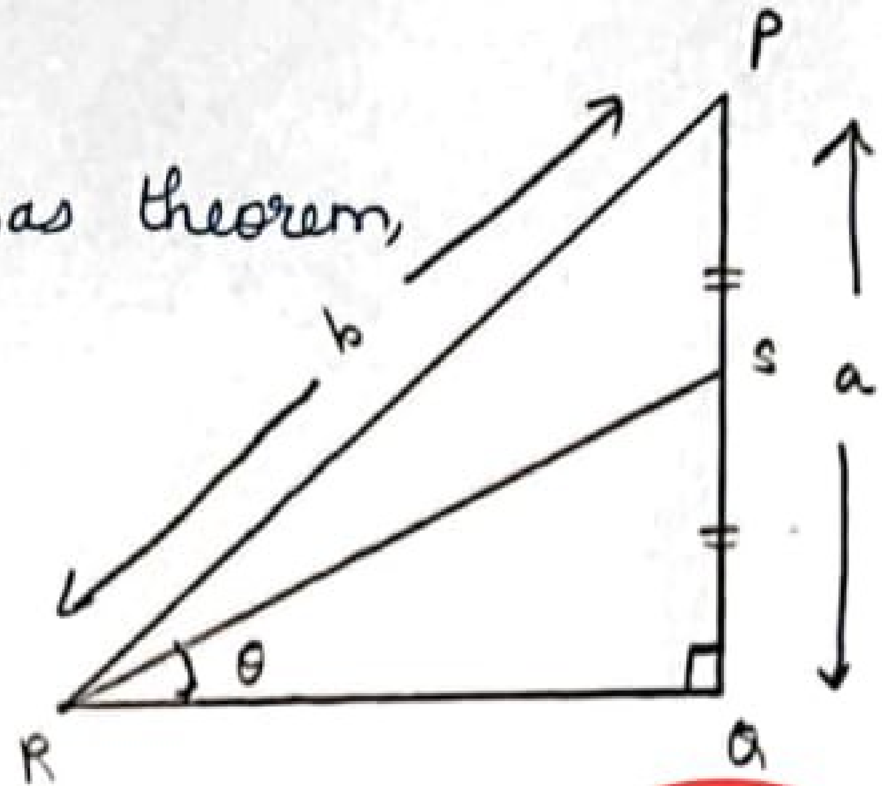
2) If $5\tan\theta = 4$, find $\dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}$

3) Evaluate:- $4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \cos^2 0^\circ)$

4) P.T $(\sqrt{3}+1)(3 - \tan 60^\circ) = \cot^3 30^\circ - 2\sin 60^\circ$

5) P.T $\sqrt{\dfrac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\dfrac{\sec\theta + 1}{\sec\theta - 1}} = \dfrac{2}{\sin\theta}$

24/8/24 Test - 16).



Q1). In rt. ΔPQR, using pythagoras theorem,

$QR = \sqrt{PR^2 - PQ^2}$

$QR = \sqrt{b^2 - a^2}$

Given,

$PS = SQ$ —(1)

$PS + SQ = a$

$\Rightarrow$ $2SQ = a$ [From eq: (1)]

$SQ = \frac{a}{2}$

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$\tan\theta = \frac{SQ}{QR}$

$\tan\theta = \dfrac{\frac{a}{2}}{\sqrt{b^2-a^2}}$ $\cancel{a\sqrt{b^2}}$

$\tan\theta = \frac{a}{2} \times \frac{1}{\sqrt{b^2-a^2}} = \frac{a}{2\sqrt{b^2-a^2}}$

$\tan\theta = \frac{a}{2\sqrt{b^2-a^2}}$

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Q2). $5\tan\theta = 4$

$\Rightarrow \tan\theta = \frac{4}{5}$

To find, $\dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}$

$(\div \cos\theta), = \dfrac{5\tan\theta - 3}{5\tan\theta + 3}$

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$= \dfrac{5 \times \frac{4}{5} - 3}{5 \times \frac{4}{5} + 3} = \frac{1}{7}$ //

Q3).

$$4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \cos^2 0^\circ)$$

$$= 4\left(\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right) - 3\left(\left(\frac{1}{\sqrt{2}}\right)^2 - 1^2\right)$$

$\sin 30^\circ = \frac{1}{2}$

$\cos 60^\circ = \frac{1}{2}$

$\cos 45^\circ = \frac{1}{\sqrt{2}}$

$\cos 0^\circ = 1$

$$= 4\left(\frac{1}{16} + \frac{1}{16}\right) - 3\left(\frac{1}{2} - 1\right)$$

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$$= \frac{4 \times 2}{16} + \frac{3 \times 1 \times 8}{2 \times 8}$$

$$= \frac{8 + 24}{16} = \frac{32}{16} = \frac{4}{2} = 2$$

Q4). P.T. $(\sqrt{3}+1)(3 - \tan 60^\circ) = \cot^3 30^\circ - 2\sin 60^\circ$

LHS

$$= (\sqrt{3}+1)(3 - \sqrt{3})$$

$\tan 60^\circ = \sqrt{3}$

$\cot 30^\circ = \sqrt{3}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

$$= 3\sqrt{3} - \cancel{3} + \cancel{3} - \sqrt{3}$$

$$= 2\sqrt{3}$$

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RHS, $(\sqrt{3})^3 - \frac{2 \times \sqrt{3}}{2} = 3\sqrt{3} - \frac{2\sqrt{3}}{2}$

$$= 2\sqrt{3}$$

$\therefore$ LHS = RHS.

Q5). P.T. $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = \frac{2}{\sin\theta}$

LHS

$$= \sqrt{\frac{(\sec\theta - 1)^2}{(\sec\theta + 1)(\sec\theta - 1)}} + \sqrt{\frac{(\sec\theta + 1)^2}{(\sec\theta - 1)(\sec\theta + 1)}}$$

$$= \sqrt{\frac{(\sec\theta - 1)^2}{\sec^2\theta - 1}} + \sqrt{\frac{(\sec\theta + 1)^2}{\sec^2\theta - 1}} = \sqrt{\frac{(\sec\theta - 1)^2}{\tan^2\theta}} + \sqrt{\frac{(\sec\theta + 1)^2}{\tan^2\theta}}$$

$[\because \sec^2\theta - 1 = \tan^2\theta]$

$$= \frac{\sec\theta - 1}{\tan\theta} + \frac{\sec\theta + 1}{\tan\theta}$$

$$= \frac{\sec\theta - \cancel{1} + \sec\theta + \cancel{1}}{\tan\theta}$$

$$= \frac{2\sec\theta}{\tan\theta} = \frac{2 \times \frac{1}{\cancel{\cos\theta}}}{\frac{\sin\theta}{\cancel{\cos\theta}}} = 2 \times \frac{1}{\sin\theta} = \frac{2}{\sin\theta}, \text{RHS.}$$

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