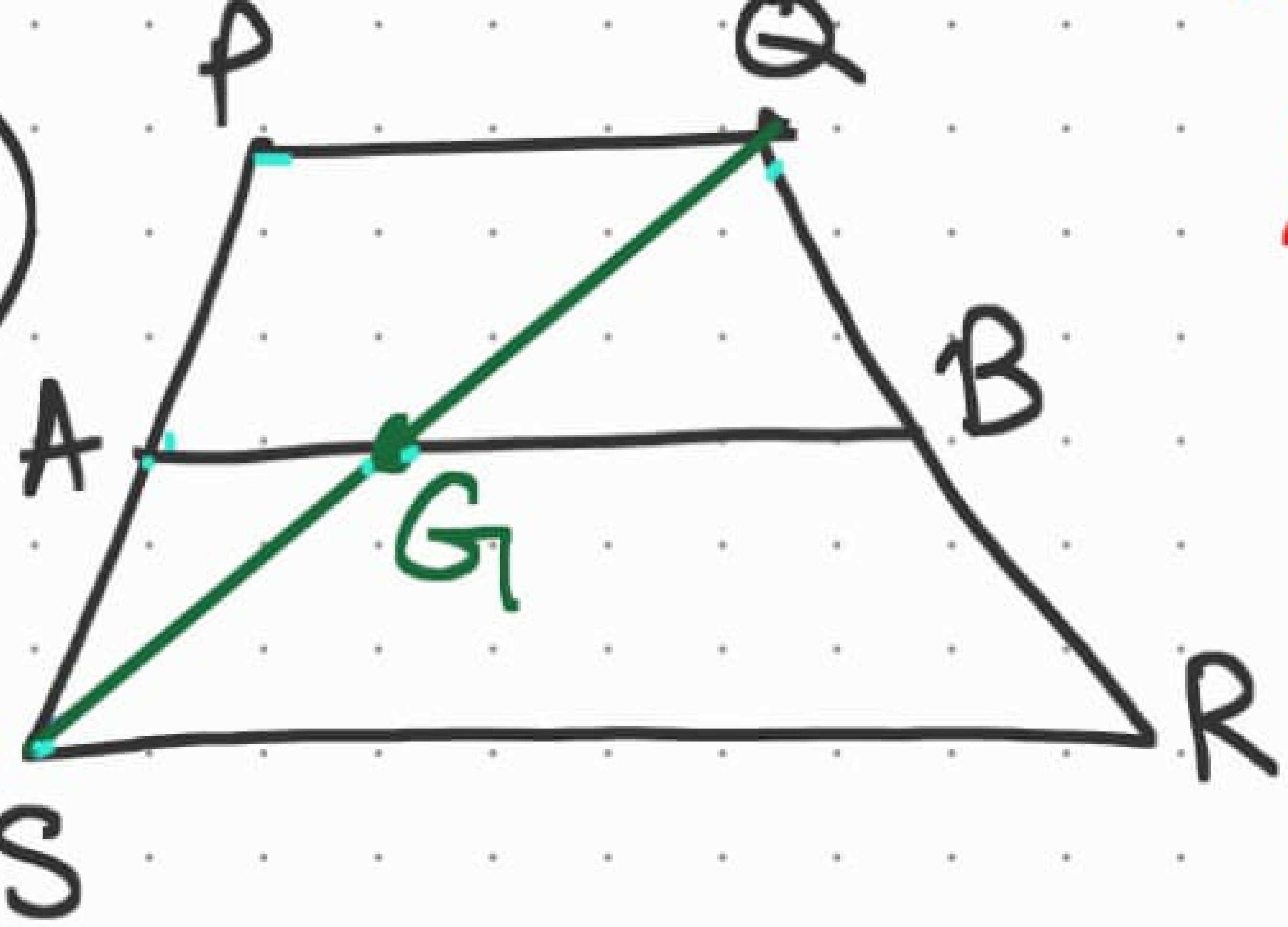
Test-15 (Submit in WhatsApp as PDF only)

1)



Given :-
$PQ \parallel SR$
$SR = 2PQ$
$AB \parallel SR$
$\frac{QB}{BR} = \frac{3}{4}$

(5)

To prove :- $7AB = 10PQ$

construction :-

2) $S_{17} = 289,\ S_7 = 49$

(5) find S_{20}

3) $\frac{a_{10}}{a_{30}} = \frac{1}{3},\ S_6 = 42$

(5) find first term and common difference.

Test-15 Answers

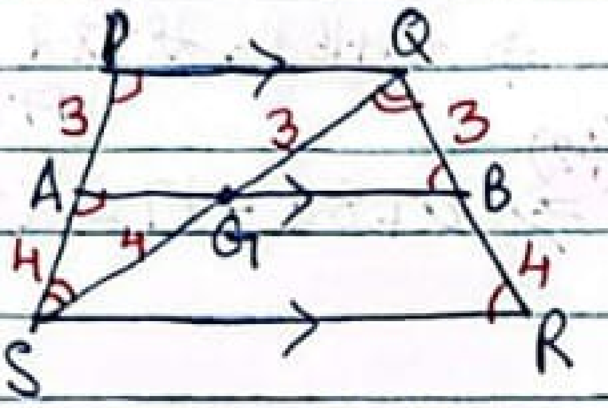
Given:- $PQ \parallel SR$

$AB \parallel SR$

$SR = 2PQ$

$\frac{QB}{BR} = \frac{3}{4}$

1)



To prove:- $7AB = 10PQ$

Construction: Join QS to meet AB at G

Proof:- Since $PQ \parallel SR$ and $AB \parallel SR$, $PQ \parallel AB \parallel SR$

Using Thales theorem in ΔQSR, since $GB \parallel SR$, $\frac{QB}{BR} = \frac{QG}{GS} = \frac{3}{4} \rightarrow (1)$

Similarly, in ΔPSQ, since $AG \parallel PQ$, $\frac{QG}{GS} = \frac{PA}{AS} = \frac{3}{4} \rightarrow (2)$

$$\therefore \boxed{\frac{QB}{BR} = \frac{PA}{AS} = \frac{3}{4}}$$

In ΔQGB and ΔQSR, $\angle GQB = \angle SQR$ (common angle)

$\angle QBG = \angle QRS$ (corresponding angles)

$\therefore \Delta QGB \sim \Delta QSR$ (AA similarity)

Thus, $\frac{QB}{QR} = \frac{GB}{SR} = \frac{3}{7} \rightarrow (3)$ [$\because$ corresponding sides of similar Δs are in proportion.

Similarly, in ΔASG and ΔPSQ, $\angle ASG = \angle PSQ$ (common angle)

$\angle SAG = \angle SPQ$ (corresponding angles)

$\therefore \Delta ASG \sim \Delta PSQ$ (AA Similarity)

Thus, $\frac{SA}{SP} = \frac{AG}{PQ} = \frac{4}{7} \rightarrow (4)$

From eq: (3), $GB = \frac{3}{7} SR$

From eq: (4), $AG = \frac{4}{7} PQ$

On adding, $GB + AG = \frac{3}{7} SR + \frac{4}{7} PQ$

$\Rightarrow AB = \frac{3}{7}(2PQ) + \frac{4}{7} PQ$ [$\because$ given, $SR = 2PQ$]

$= \frac{6PQ + 4PQ}{7}$

$\therefore 7AB = 10PQ$. Hence Proved

$S_n = \frac{n}{2}[2a+(n-1)d]$

2) $S_7 = 49 \Rightarrow \frac{7}{2}[2a+6d] = 49$

$\Rightarrow 2a+6d = \cancel{49}^{7} \times \frac{2}{\cancel{7}_1}$

$(\div 2)$

$\Rightarrow a+3d = 7 \rightarrow (1)$

$S_{17} = 289 \Rightarrow \frac{17}{2}[2a+16d] = 289$

$\Rightarrow 2a+16d = \cancel{289}^{17} \times \frac{2}{\cancel{17}_1}$

$(\div 2)$

$\Rightarrow a+8d = 17 \rightarrow (2)$

$(1)-(2), \; -5d = -10$

$\boxed{d=2}$

from eq: (1), $a+6=7$

$\boxed{a=1}$

$\therefore S_{20} = \frac{20}{2}[2a+19d] = 10[2+38] = 10 \times 40 = \underline{\underline{400}}$

3) $\frac{a_{10}}{a_{30}} = \frac{1}{3}$

$\Rightarrow \frac{a+9d}{a+29d} = \frac{1}{3}$ $[\because a_n = a+(n-1)d]$

$\Rightarrow 3(a+9d) = a+29d$

$\Rightarrow 3a+27d = a+29d$

$\Rightarrow 2a = 2d$

$\boxed{a=d}$

$S_6 = 42$ $[\because S_n = \frac{n}{2}[2a+(n-1)d]]$

$\Rightarrow \frac{6}{2}[2a+5d] = 42$

$\Rightarrow 2a+5d = \cancel{42}^{7} \times \frac{2}{\cancel{6}_1}$

$\Rightarrow 2a+5d = 14$

$\Rightarrow 2a+5a = 14$ $[\because a=d]$

$\Rightarrow 7a = 14$

$\boxed{\begin{array}{l} a=2 \\ d=2 \end{array}}$