Test_14 (Q.E) WhatsApp Test

1) A bird, travelling at a uniform speed for 360 km, would have taken 48 min less to travel the same distance, if its speed were 5 km/hr more. Find the original speed of the bird.

(Use Q.F)

2) Find the roots: $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$

3) If $(1+p^2)x^2 + 2pqx + q^2 - a^2 = 0$ has equal roots, S.T $q^2 = a^2(1+p^2)$

TEST - 14:—

1. Let uniform speed of bird be x km/hr

distance travelled by bird = 360 km

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

time for covering 360km in x km/hr = $\frac{360}{x}$ hr $\rightarrow$ (1)

time for covering 360km in $(x+5)$ km/hr = $\frac{360}{x+5}$ hr $\rightarrow$ (2)

time difference = 48 min

$$= \frac{48}{60} = \frac{8}{10} = \frac{4}{5} \text{ hr}$$

So, according to the question,

$$\frac{360}{x} - \frac{360}{x+5} = \frac{4}{5}$$

$$\Rightarrow 360\left[\frac{1}{x} - \frac{1}{x+5}\right] = \frac{4}{5}$$

$$\Rightarrow \frac{1}{x} - \frac{1}{x+5} = \frac{4}{5\times 360}$$

$$\Rightarrow \frac{1}{x} - \frac{1}{x+5} = \frac{1}{5\times 90}$$

$$\Rightarrow \frac{1}{x} - \frac{1}{x+5} = \frac{1}{450}$$

$$\Rightarrow \frac{5}{x^2+5x} = \frac{1}{450}$$

$$\Rightarrow x^2 + 5x = 2250$$

$$\Rightarrow x^2 + 5x - 2250 = 0$$

$$\Rightarrow x(x-45) + 50(x-45) = 0$$

$\Rightarrow (x - 45)(x + 50) = 0$

$\Rightarrow x = 45, -50$

Since speed can't be negative, $x = \underline{\underline{45}}$

$\therefore$ The original speed of the bird is 45 km/hr.

2. $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$

$\Rightarrow \frac{x^2 - 2\sqrt{11}x + 2}{2} = 0$

$\Rightarrow x^2 - 2\sqrt{11}x + 2 = 0$; where $a = 1, b = -2\sqrt{11}, c = 2$

Using quadratic formula,

$D = b^2 - 4ac$

$= (-2\sqrt{11})^2 - 4(1)(2)$

$= 44 - 8 = \underline{\underline{36}}$

$\therefore \underline{\underline{D > 0}}$

Now, $x = \frac{-b \pm \sqrt{D}}{2a}$

$\Rightarrow x = \frac{-(-2\sqrt{11}) \pm \sqrt{36}}{2(1)}$

$= \frac{2\sqrt{11} \pm 6}{2}$

$= \frac{2(\sqrt{11} \pm 3)}{2}$

$= \underline{\underline{\sqrt{11} \pm 3}}$

$\therefore$ The two roots are $\underline{\underline{\sqrt{11} + 3}}$ and $\underline{\underline{\sqrt{11} - 3}}$.

3. $(1+p^2)x^2 + 2pqx + q^2 - a^2 = 0$

Since it has equal roots,

$D = 0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (2pq)^2 - 4(1+p^2)(q^2-a^2) = 0$

$\Rightarrow 4p^2q^2 - 4(q^2 - a^2 + p^2q^2 - p^2a^2) = 0$

$\Rightarrow 4[p^2q^2 - (q^2 - a^2 + p^2q^2 - p^2a^2) = 0$

$\Rightarrow p^2q^2 - (q^2 - a^2 + p^2q^2 - p^2a^2) = 0$

$\Rightarrow \cancel{p^2q^2} - q^2 + a^2 - \cancel{p^2q^2} + p^2a^2 = 0$

$\Rightarrow a^2 + p^2a^2 = q^2$

$\Rightarrow a^2(1+p^2) = q^2$

$\Rightarrow q^2 = a^2(1+p^2)$

Hence, proved.