Revision - 2 (SECTION C & D)

21) Find the ratio in which the point $\left(\frac{8}{5}, y\right)$ divides the line segment joining the points (1,2) and (2,3). Also, find the value of y.

22) ABCD is a rectangle formed by the points A(-1,-1), B(-1,6), C(3,6) and D(3,-1). P, Q, R and S are mid-points of sides AB, BC, CD and DA respectively. Show that diagonals of the quadrilateral PQRS bisect each other.

23) In a teacher's workshop, the number of teachers teaching French, Hindi and English are 48, 80 and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject.

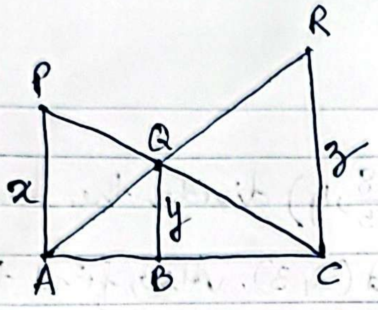
24) Prove that : $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1+\sec\theta\,\mathrm{cosec}\,\theta$

25) Three years ago, Rashmi was thrice as old as Nazma. Ten years later, Rashmi will be twice as old as Nazma. How old are Rashmi and Nazma now?

26) The sum of first and eighth terms of an AP is 32 and their product is 60. Find the first term and common difference of the AP. Hence, also find the sum of its first 20 terms.

27) In an AP of 40 terms, the sum of first 9 terms is 153 and the sum of last 6 terms is 687. Determine the first term and common difference of AP. Also, find the sum of all the terms of the AP.

28) If a line is drawn parallel to one side of a Δ to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio

29) 

PA, QB and RC are each perpendicular to AC. If $AP = x$, $BQ = y$ and $CR = z$, then prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$.

30) How many terms of the AP 27, 24, 21, ... must be taken so that their sum is 105? Which term of the AP is zero?

EWS-2

Revision - 2 (Answers)

21) $\underset{A(1,2)}{\bullet}\overset{k}{\text{———}}\underset{P(\frac{8}{5},y)}{\bullet}\overset{1}{\text{———}}\underset{B(2,3)}{\bullet}$

Let the ratio in which $P(x,y)$ divides AB in the ratio $k:1$

$\therefore P(x,y) = P\left(\dfrac{m_1x_2+m_2x_1}{m_1+m_2}, \dfrac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$$\left(\frac{8}{5}, y\right) = \left(\frac{2k+1}{k+1}, \frac{3k+2}{k+1}\right)$$

On equating the x-coordinates,

$$\frac{8}{5} = \frac{2k+1}{k+1}$$

$\Rightarrow 8(k+1) = 5(2k+1)$

$\Rightarrow 8k+8 = 10k+5$

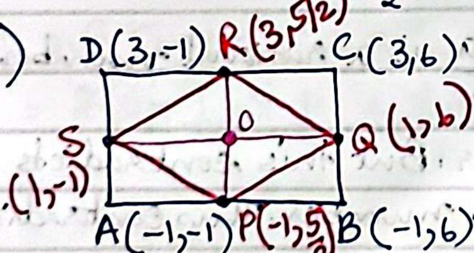
$\Rightarrow -2k = -3$

$k = \frac{3}{2}$

$\therefore$ The required ratio is $3:2$

On equating the y-coordinates and substituting the value of k,

$$y = \frac{3\times\frac{3}{2}+2}{\frac{3}{2}+1} = \frac{\frac{9+4}{2}}{\frac{3+2}{2}} = \underline{\underline{\frac{13}{5}}}$$

22) 

mid-point $P(x,y) = P\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$P(x,y) = \left(\dfrac{-1-1}{2}, \dfrac{-1+6}{2}\right) = \left(-1, \dfrac{5}{2}\right)$

$Q(x,y) = \left(\dfrac{3-1}{2}, \dfrac{6+6}{2}\right) = (1,6)$

$R(x,y) = \left(\dfrac{3+3}{2}, \dfrac{-1+6}{2}\right) = \left(3, \dfrac{5}{2}\right)$

$S(x,y) = \left(\dfrac{3-1}{2}, \dfrac{-1-1}{2}\right) = (1,-1)$

mid-point of PR $= \left(\frac{3-1}{2}, \frac{\frac{5}{2}+\frac{5}{2}}{2}\right) = \left(1, \frac{5}{2}\right)$

mid-point of QS $= \left(\frac{1+1}{2}, \frac{-1+6}{2}\right) = \left(1, \frac{5}{2}\right)$

Since coordinates of mid-point of PR is equal to the coordinates of mid-point of QS, diagonals PR and QS bisect each other.

23) $48 = 2^4 \times 3$

$80 = 2^4 \times 5$

$144 = 2^4 \times 3^2$

HCF $= 2^4 = \underline{\underline{16}}$ teachers in 1 room

$$\begin{array}{r|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array} \qquad \begin{array}{r|l} 2 & 80 \\ 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ & 5 \end{array} \qquad \begin{array}{r|l} 2 & 144 \\ 2 & 72 \\ 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ & 3 \end{array}$$

$\therefore$ Total no. of rooms required $= \frac{\cancel{48}^{3}}{\cancel{16}_{1}} + \frac{\cancel{80}^{5}}{\cancel{16}_{1}} + \frac{\cancel{144}^{9}}{\cancel{16}_{1}}$

$= \underline{17 \text{ rooms}}$

24) LHS, $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$\Rightarrow \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$ $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$= \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta-\cos\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta-\sin\theta}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$= \frac{\sin^3\theta - \cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$= \frac{\cancel{(\sin\theta-\cos\theta)}^{1}(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta\cancel{(\sin\theta-\cos\theta)}_{1}}$ $[a^3-b^3 = (a-b)(a^2+b^2+ab)]$

$= \dfrac{\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \dfrac{1 + \sin\theta\cos\theta}{\sin\theta\cos\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \dfrac{1}{\sin\theta\cos\theta} + \dfrac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \operatorname{cosec}\theta\sec\theta + 1$ $[\because \operatorname{cosec}\theta = 1/\sin\theta, \sec\theta = 1/\cos\theta]$

25) Let the present ages of Rashmi and Nazma be x and y years.

3 years ago, Rashmi's age $= (x-3)$ years

Nazma's age $= (y-3)$ years

ATQ, $x-3 = 3(y-3)$

$\Rightarrow x-3 = 3y-9$

$\Rightarrow x-3y = -6 \rightarrow (1)$

After 10 years, Rashmi's age $= (x+10)$ years

Nazma's age $= (y+10)$ years

Also, $x+10 = 2(y+10)$

$\Rightarrow x+10 = 2y+20$

$\Rightarrow x-2y = 10 \rightarrow (2)$

(1) − (2), $-3y+2y = -6-10$

$-y = -16$

$\boxed{y = 16}$

From eq: (1), $x - 48 = -6$

$\boxed{x = 42}$

Hence, present ages of Rashmi and Nazma are 42 years and 16 years.

26) $a + a_8 = 32 \Rightarrow a + a + 7d = 32 \Rightarrow 2a + 7d = 32 \rightarrow (1)$

$a \times a_8 = 60 \Rightarrow a(a+7d) = 60 \Rightarrow a^2 + 7ad = 60 \rightarrow (2)$

From eq: (1), $7d = 32 - 2a \rightarrow (3)$

On substituting eq: (3) in eq: (2), $a^2 + a(32-2a) = 60$

$\Rightarrow a^2 + 32a - 2a^2 = 60$

$\Rightarrow -a^2 + 32a - 60 = 0$

$\Rightarrow a^2 - 32a + 60 = 0$

$\Rightarrow (a-30)(a-2) = 0$

$\therefore a = 30$ or 2

When $a = 30$, $7d = 32 - 60$

$\Rightarrow 7d = -28$

$d = -4$

When $a = 2$, $7d = 32 - 4$

$\Rightarrow 7d = 28$

$\therefore d = 4$

Hence, the first term and c.d. are $(30, -4)$ or $(2, 4)$

$S_n = \frac{n}{2}[2a + (n-1)d]$

When $a = 30$ and $d = -4$,

$S_{20} = \frac{20}{2}[60 - 4 \times 19]$

$= 10 \times -16 = \underline{-160}$

When $a = 2$, $d = 4$

$S_{20} = \frac{20}{2}[4 + 19 \times 4]$

$= 10 \times 80$

$= \underline{\underline{800}}$

27) $n = 40$; $S_n = n/2[2a + (n-1)d]$

$S_9 = 153 \Rightarrow \frac{9}{2}[2a + 8d] = 153$

$\Rightarrow a + 4d = 17 \rightarrow (1)$

$S_{40} - S_{34} = 687$

$\Rightarrow \frac{40}{2}[2a + 39d] - \frac{34}{2}[2a + 33d] = 687$

$\Rightarrow 40a + 780d - 34a - 561d = 687$

$\Rightarrow 6a + 219d = 687$

$(\div 3) \Rightarrow 2a + 73d = 229 \rightarrow (2)$

$(1) \times 2 \Rightarrow 2a + 8d = 34$

$(2) \Rightarrow 2a + 73d = 229$

$(-)$, $-65d = -195$

$\boxed{d = 3}$

From eq: (1),

$a + 12 = 17$

$\boxed{a = 5}$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{40} = \frac{40}{2}[10 + 39 \times 3]$

$= 20 \times 127$

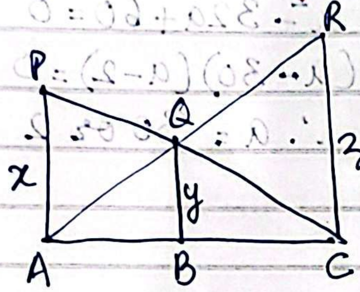
$= \underline{\underline{2540}}$

28) draw correct figure :

Write given, to prove and construction

Write correct proof of basic proportionality theorem.

29)



Given :- $PA \perp AC$, $QB \perp AC$, $RC \perp AC$.

To prove :- $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$

Proof :- In $\triangle PAC$ and $\triangle QBC$, $\angle PAC = \angle QBC$ (each $90°$)

$\angle PCA = \angle QCB$ (common angle)

$\therefore \triangle PAC \sim \triangle QBC$ (AA Similarity)

Thus, $\frac{QB}{PA} = \frac{BC}{AC}$ (corresponding sides of similar $\triangle$s are in proportion)

$\Rightarrow \frac{y}{x} = \frac{BC}{AC} \rightarrow (1)$

Similarly, in $\triangle QAB$ and $\triangle RAC$, $\angle QAB = \angle RAC$ (common angle)

$\angle QBA = \angle RCA$ (each $90°$)

$\therefore \triangle QAB \sim \triangle RAC$ (AA Similarity)

Thus, $\frac{QB}{RC} = \frac{AB}{AC}$ (Corresponding sides of similar $\triangle$s are proportional)

$\Rightarrow \frac{y}{z} = \frac{AB}{AC} \rightarrow (2)$

(1) + (2), $\frac{y}{x} + \frac{y}{z} = \frac{BC}{AC} + \frac{AB}{AC}$

$\Rightarrow y\left[\frac{1}{x} + \frac{1}{z}\right] = \frac{BC+AB}{AC}$

$\Rightarrow y\left[\frac{1}{x} + \frac{1}{z}\right] = \frac{AC}{AC} = 1$

$\therefore \frac{1}{x} + \frac{1}{z} = \frac{1}{y}$ $\therefore$ Hence Proved.

30) $a = 27$

$d = 24 - 27 = -3$

$S_n = 105$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 105$

$\Rightarrow \frac{n}{2}[54 - 3(n-1)] = 105$

$\Rightarrow n[54 - 3n + 3] = 210$

$\Rightarrow -3n^2 + 57n - 210 = 0$

$\Rightarrow n^2 - 19n + 70 = 0$

$\Rightarrow (n-14)(n-5) = 0$

$n = 14 \text{ or } 5$

$a_n = 0$

$\Rightarrow a + (n-1)d = 0$

$\Rightarrow 27 - 3(n-1) = 0$

$\Rightarrow -3(n-1) = -27$

$\Rightarrow n - 1 = 9$

$n = 10$

$\therefore$ 10^{th} term of the given AP is zero.