Test - 12 (pdf in whatsapp)

1) Prove that $3+2\sqrt{5}$ is irrational, given $\sqrt{5}$ is irrational.

2) If α and β are the zeroes of $p(x) = x^2+6x+9$, then form a polynomial whose zeroes are $-\alpha$ and $-\beta$.

3) If α and β are the zeroes of $p(x) = 3x^2-12x+k$ such that $\alpha-\beta = 2$, find the value of k

4) If both the zeroes of $p(x) = (k+2)x^2-(k-2)x-5$ are

equal in magnitude and opposite in sign, find k.

5) If one zero of $p(x) = 6x^2 + 37x - (k-2)$ is reciprocal of the other, then find k.

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Q1). Let's assume $3+2\sqrt{5}$ as rational,

then $3+2\sqrt{5} = \frac{a}{b}$; where a & b are co-primes integers and $b \neq 0$.

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$2\sqrt{5} = \frac{a-3b}{b}$$

$$\sqrt{5} = \frac{a-3b}{2b}$$

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Since a, b, 2, -3 are integers, $\frac{a-3b}{2b}$ is a rational number. Thus $\sqrt{5}$ is also rational number. But it contradicts the fact that $\sqrt{5}$ is irrational (which is given). This contradiction has arised due to our wrong assumption that $3+2\sqrt{5}$ is rational. Hence $3+2\sqrt{5}$ is irrational

Q2). $p(x) = x^2+6x+9$.

Let $p(x) = x^2+6x+9$ be of the general form ax^2+bx+c where $a = 1$

$b = 6$

$c = 9$. α & β are the zeroes of $p(x)$

~~Sum of roots,~~

~~$\alpha+\beta = \frac{b}{a}$~~

~~$\alpha+\beta = -6$~~

Sum of roots	Product of roots.
$\alpha+\beta = \frac{-b}{a}$	$\alpha\beta = \frac{c}{a}$
$\alpha+\beta = -6$	$\alpha\beta = 9$

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For the required quadratic polynomial
~~Sum~~ zeroes are $-\alpha$ & $-\beta$.
Sum of roots,

$$-\alpha - \beta = -(\alpha + \beta) = -(-6) = 6.$$

Product of roots,

$$-\alpha \times -\beta = \alpha\beta = 9.$$

$\therefore$ The required quadratic polynomial is,

$k\,(x^2 - (\text{Sum of roots})x + \text{Product of roots})$;
where k is any non zero real number.

$\Rightarrow k\,(x^2 - 6x + 9)$

$\Rightarrow x^2 - 6x + 9$ where $k = 1$

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Q3). Let $p(x) = 3x^2 - 12x + K$ be of the general form $ax^2 + bx + c$ where $a = 3$

$b = -12$

$c = K.$

& α & β are the zeroes of of $p(x)$.

Sum of roots, $\alpha + \beta = \frac{-b}{a}$

$\alpha + \beta = \frac{+12}{3}$

$\alpha + \beta = 4$

Product of roots, $\alpha\beta = \frac{c}{a}$

$\alpha\beta = \frac{K}{3}.$

Given, $\alpha - \beta = 2$

Squaring both sides,

$(\alpha - \beta)^2 = 4$

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$(\alpha+\beta)^2 - 4\alpha\beta = 4$ $\quad$ ($\because$ ~~$(\alpha-\beta)^2 = \alpha + \beta^2$~~) $\quad$ ($\because (a-b)^2 = (a+b)^2 - 4ab$)

$4^2 - 4 \times \frac{K}{3} = 4$

$16 - \frac{4K}{3} = 4$

$\frac{\cancel{4}K}{3} = \cancel{12}\,3$

$\boxed{K = 9}$

Q4) Let $p(x) = (K+2)x^2 - (K-2)x - 5$. See of the general form ax^2+bx+c where

$a = K+2$

$b = -(K-2)$

$c = -5$. The zeroes of $p(x)$ are α & $-\alpha$

Sum of roots,

$\alpha - \alpha = -\frac{b}{a}$

$0 = \frac{+(K-2)}{K+2}$

$K - 2 = 0$

$\boxed{K = 2}$

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Q5). Let $p(x) = 6x^2 + 37x - (K-2)$ be of the general form $ax^2 + bx + c$ where

$$a = 6$$
$$b = 37$$
$$c = -(K-2)$$

Let the zeroes of $p(x)$ be α & $\frac{1}{\alpha}$

Product of roots,

$$\alpha \times \frac{1}{\alpha} = \frac{c}{a}$$

$$\Rightarrow \quad 1 = \frac{-(K-2)}{6}$$

$$6 = -K + 2,$$

$$\boxed{K = -4}$$

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