Test - 6

1) Two students Anish and Biju were assigned a polynomial $p(x) = 12x^2 + 11x - 15$ to express as product of factors. Answer the following questions

(i) Find the value of $p(x)$ at $x = 100$

(ii) find the factors

(iii) find the value of x if the factors are equal.

2) Using suitable identity, find $(98)^3$

3) Factorise : $64p^3 - 27q^3 - 144p^2q + 108pq^2$

4) Factorise : $8a^3 - (2a-b)^3$

5) Evaluate : $\dfrac{83^3 + 17^3}{83^2 - 83 \times 17 + 17^2}$

6) Factorise : $x^3 - 23x^2 + 142x - 120$

7) Find the value of a if $p(x) = 2x^3 - ax^2 + 3x + 10$ is exactly divisible by $(x+2)$

8) Write the zeroes of $p(x) = x(x-2)(x-3)$

9) Find the degree of the polynomial $(x^3+5)(4-x^5)$

10) If $\frac{x}{y} + \frac{y}{x} = -1$, the value of $x^3 - y^3 =$ —

(a) 1 (b) -1 (c) 0 (d) $\frac{1}{2}$

Answers

1 i) $p(100) = 12 \times 100^2 + 11 \times 100 - 15$

$= 120000 + 1100 - 15$

$= \underline{\underline{121085}}$

(ii) $p(x) = 12x^2 + 11x - 15$

$= 12x^2 + 20x - 9x - 15$

$= 4x(3x+5) - 3(3x+5)$

$= \underline{\underline{(4x-3)(3x+5)}}$

S	P
11	-180
	$-9, 20$

(iii) $4x - 3 = 3x + 5$

$4x - 3x = 5 + 3$

$\underline{\underline{x = 8}}$

2) $(98)^3 = (100-2)^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$= 100^3 - 3 \times 100^2 \times 2 + 3 \times 100 \times 2^2 - 2^3$

$= 1000000 - 60000 + 1200 - 8$

$= 1001200 - 60008$

$= \underline{\underline{941192}}$

3) $64p^3 - 27q^3 - 144p^2q + 108pq^2$

$= (4p)^3 - (3q)^3 - 3 \times (4p)^2 \times 3q + 3 \times 4p \times (3q)^2$

$a^3 - b^3 - 3a^2b + 3ab^2 = (a-b)^3$

$= (4p - 3q)^3$

$= (4p-3q)(4p-3q)(4p-3q)$

4) $8a^3 - (2a-b)^3$

$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

$= (2a)^3 - (2a-b)^3$

$= (\cancel{2}a - \cancel{2}a + b)(4a^2 + 2a(2a-b) + (2a-b)^2)$

$= b(4a^2 + 4a^2 - 2ab + 4a^2 + b^2 - 4ab)$

$= b(12a^2 - 6ab + b^2)$

5) $a^3+b^3 = (a+b)(a^2-ab+b^2)$ 1

$$\frac{83^3+17^3}{83^2-83\times17+17^2} = \frac{(83+17)\cancel{(83^2-83\times17+17^2)}}{\cancel{83^2-83\times17\times17^2}}$$

1

$= 83+17$

$= \underline{\underline{100}}$

6) Let $p(x) = x^3-23x^2+142x-120$

Factors of 120 are $\pm1, \pm2, \pm3$ etc

$p(1) = 1-23+142-120 = 143-143 = 0$

$\therefore (x-1)$ is a factor of $p(x)$

On dividing

$$\begin{array}{r|l} & x^2-22x+120 \\ \hline x-1 & \cancel{x^3}-23x^2+142x-120 \\ & \overset{(-)}{}\cancel{x^3}\overset{(+)}{-}x^2 \\ \hline & -\cancel{22x^2}+142x-120 \\ & \overset{(+)}{-}\cancel{22x^2}\overset{(-)}{+}22x \\ \hline & \cancel{120x}-\cancel{120} \\ & \overset{(-)}{}\cancel{120x}\overset{(+)}{-}\cancel{120} \\ \hline & 0 \end{array}$$

Using division algorithm,

$p(x) = (x-1)(x^2-22x+120)$

$= (x-1)(x^2-12x-10x+120)$ S P

-22 120

$= (x-1)[x(x-12)-10(x-12)]$

$^-12, ^-10$

$= (x-1)(x-10)(x-12)$

7) since $p(x)$ is divisible by $(x+2)$,

$\boxed{p(-2) = 0}$

$\Rightarrow 2(-2)^3 - a(-2)^2 + 3(-2) + 10 = 0$

$\Rightarrow -16 - 4a - 6 + 10 = 0$

$\Rightarrow -12 - 4a = 0$

$\Rightarrow -4a = 12$

$\boxed{a = -3}$

8) put $p(x) = 0$

$\Rightarrow x(x-2)(x-3) = 0$

$\therefore$ the zeroes are 0, 2 and 3

9) $(x^3+5)(4-x^5)$

$= 4x^3 - x^8 + 20 - 5x^5$

$\therefore$ degree $= 8$

10) $\frac{x}{y} + \frac{y}{x} = -1$

$\Rightarrow \frac{x^2+y^2}{xy} = -1$

$\Rightarrow x^2+y^2 = -xy$

$\Rightarrow x^2+y^2+xy = 0 \rightarrow (1)$

$\therefore x^3 - y^3 = (x-y)(x^2+y^2+xy)$

$= (x-y) \times 0$

$= 0$ (c)