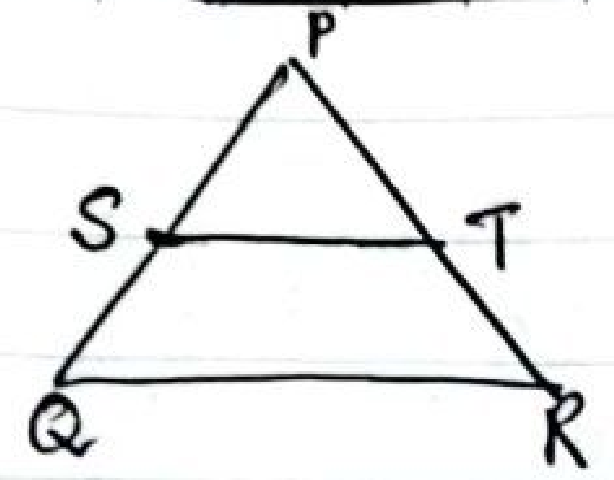
No.

Date X H.W-10

1)

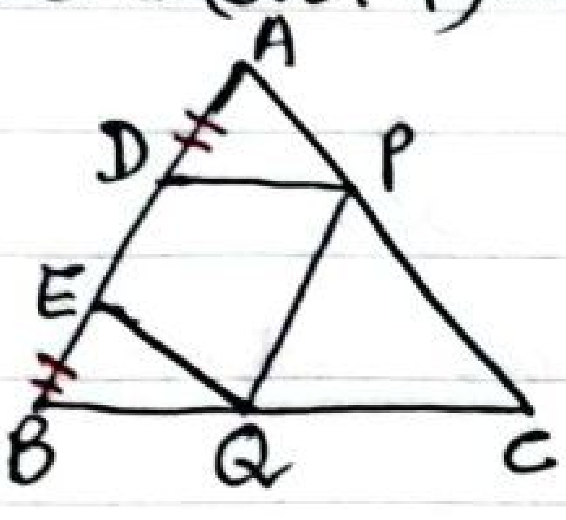


$PS = 4cm$, $SQ = 1cm$, $PT = 6cm$, $TR = 1.5cm$.

Prove that $ST \parallel QR$

2) In $\triangle ABC$, $DE \parallel BC$, so that $AD = (7x-4)cm$, $AE = (5x-2)cm$, $DB = (3x+4)cm$ and $EC = 3x\ cm$. Find the value of x.

3)



D and E are two points lying on side AB such that $AD = BE$. If $DP \parallel BC$ and $EQ \parallel AC$, then prove that $PQ \parallel AB$

4) The sum of a two digit number and the number formed by interchanging its digit is 110. If 10 is subtracted from the first number, the new number is 4 more than 5 times the sum of the digit. Find the first number.

5) Solve using substitution method: $x+2y=10$; $2x+y=3$

6) If α and β are the zeroes of $p(x) = 2x^2+3x+4$, then evaluate $\alpha^2\beta + \alpha\beta^2$

7) Find a quadratic polynomial, the sum and product of whose zeroes are -3 and 4, respectively

8) Find the sum of all natural numbers lying between 100 and 500, which are exactly divisible by 8.

9) Solve using quadratic formula:-

$a^2b^2x^2 - (4b^4 - 3a^4)x - 12a^2b^2 = 0$

10) Find value of k for which the pair of eq:s has infinitely many solutions

$(3k+1)x + 3y - 2 = 0$; $(k^2+1)x + (k-2)y - 5 = 0$

11) A motor boat whose speed 17 km/hr in still water goes 49 km downstream and comes back in a total of 9 hours. Find the speed of stream.

12) Solve for x: $(-4)+(-1)+2+\cdots+x = 437$

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13) The sum of four consecutive numbers in AP is 32 and the ratio of the product of the first and the last terms to the product of two middle terms is 7:15. Find the no.s

14) Sum of the first n terms of an AP is $5n^2 - 3n$. Find the AP and also find its 16^{th} term.

15) Find the sum of those integers between 1 and 500, which are multiples of 2 as well as 5.

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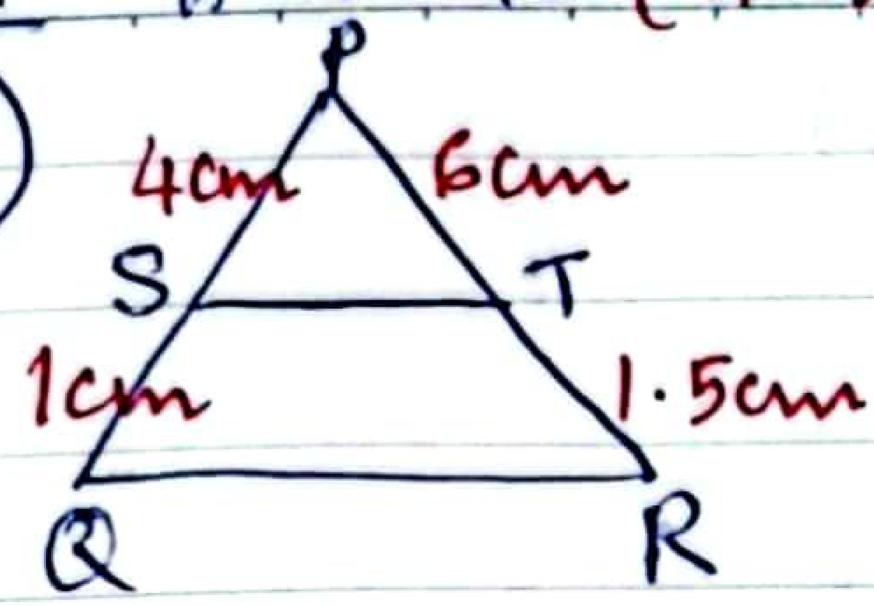
$\overline{X}$ H.W-10 (Answers)

1)

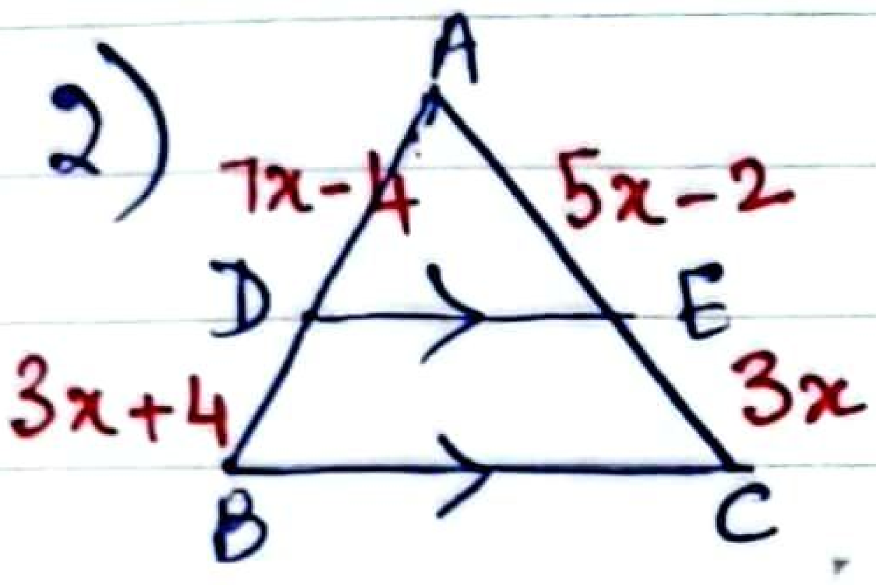


$$\frac{PS}{SQ} = \frac{4}{1}$$

$$\frac{PT}{TR} = \frac{6}{1.5} = \frac{60}{15} = \frac{4}{1}$$

$\therefore \frac{PS}{SQ} = \frac{PT}{TR} \Rightarrow ST \parallel QR$ using converse of Thales theorem

2)



Since $DE \parallel BC$, using Thales theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{7x-4}{3x+4} = \frac{5x-2}{3x}$$

$\Rightarrow 3x(7x-4) = (5x-2)(3x+4)$

$\Rightarrow 21x^2 - 12x = 15x^2 + 20x - 6x - 8$

$\Rightarrow 21x^2 - 12x = 15x^2 + 14x - 8$

$\Rightarrow 6x^2 - 26x + 8 = 0$

$\Rightarrow 3x^2 - 13x + 4 = 0$

S	P
-13	12 < -1, -12

$\Rightarrow 3x^2 - x - 12x + 4 = 0$

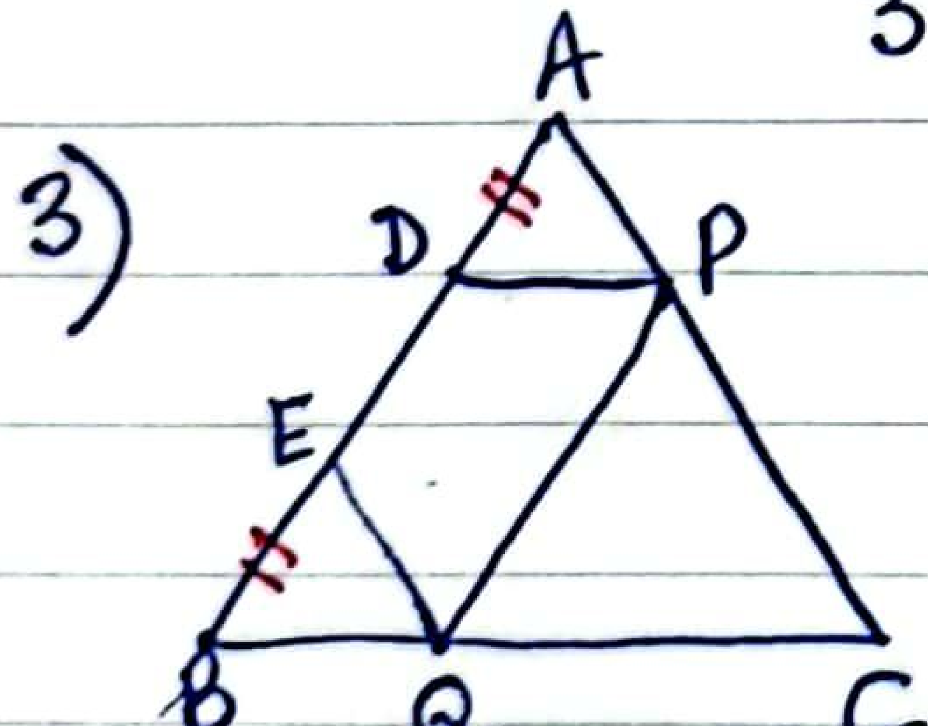
$\Rightarrow x(3x-1) - 4(3x-1) = 0$

$\Rightarrow (x-4)(3x-1) = 0$

$\therefore x = 4, \frac{1}{3}$

x cannot be $\frac{1}{3}$, $\therefore$ required value of $x = 4$

3)



Given :- in $\triangle ABC$, $AD = BE$
$DP \parallel BC$, $EQ \parallel AC$

To prove :- $PQ \parallel AB$

Proof :- In $\triangle ABC$, since $DP \parallel BC$, using Thales theorem,

$$\frac{AD}{DB} = \frac{AP}{PC} \rightarrow (1)$$

Similarly, since $EQ \parallel AC$, $\frac{BE}{EA} = \frac{BQ}{QC} \rightarrow (2)$

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But, $\frac{AD}{DB} = \frac{BE}{BE+ED} = \frac{BE}{AD+ED} = \frac{BE}{AE} \rightarrow (3)$

From (1), (2) and (3), $\frac{AP}{PC} = \frac{BQ}{QC}$

$\Rightarrow PQ \parallel AB$ using Converse of Thales theorem.

4) Let the digit in ones place be y and that in tens place be x.

T	O
x	y

Original number $= 10x+y$

interchanged number $= 10y+x$

ATQ, $10x+y+10y+x = 110$

$11x+11y = 110$

$x+y = 10 \rightarrow (1)$

Also, $10x+y-10 = 5(x+y)+4$

$\Rightarrow 10x+y-10 = 5x+5y+4$

$\Rightarrow 5x-4y = 14 \rightarrow (2)$

$(1)\times 4 \Rightarrow 4x+4y = 40$

$(2) \Rightarrow 5x-4y = 14$

$(+)$, $9x = 54$

$x = 6$

$y = 4$

Hence, the first number is 64

5) $x+2y = 10$

$\Rightarrow x = 10-2y \rightarrow (1)$

$2x+y = 3$

$2(10-2y)+y = 3$

$20-4y+y = 3$

$20-3y = 3$

$-3y = -17$

$y = \frac{17}{3}$

From eq:(1), $x = 10 - \frac{34}{3}$

$= \frac{30-34}{3}$

$x = -\frac{4}{3}$

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6) Let $p(x)$ be of the form ax^2+bx+c; where $a=2, b=3, c=4$

$\alpha+\beta = -\frac{b}{a} = -\frac{3}{2}$

$\alpha\beta = \frac{c}{a} = \frac{4}{2} = 2$

$\therefore \alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha+\beta) = 2 \times -\frac{3}{2} = \underline{\underline{-3}}$

7) Let the zeroes be α and β.

$\alpha+\beta = -3$

$\alpha\beta = 4$

$\therefore$ The required quadratic polynomial is $k\{x^2-(\alpha+\beta)x+\alpha\beta\}$; where k is any non-zero real number

$= k\{x^2+3x+4\}$

$= x^2+3x+4$; where $k=1$

8) $104, 112, 120, \dots 496$ forms an AP with $a=104, d=8$, $a_n=496$

$a_n = a+(n-1)d = 496$

$\Rightarrow 104+(n-1)8 = 496$

$\Rightarrow (n-1)8 = 392$

$n-1 = 49$

$n = 50$

$S_n = \frac{n}{2}[a+a_n]$

$S_{50} = \frac{50}{2}[104+496] = 25 \times 600 = \underline{\underline{15000}}$

9) Let the given eqn be of the form $Ax^2+Bx+C=0$; where $A=a^2b^2$, $B=-(4b^4-3a^4)$, $C=-12a^2b^2$

$B^2-4AC = (4b^4-3a^4)^2 + 4a^2b^2 \times 12a^2b^2$

$= 16b^8+9a^8-24a^4b^4+48a^4b^4$

$= 16b^8+9a^8+24a^4b^4$

$= (4b^4+3a^4)^2$

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$$\therefore x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{(4b^4 - 3a^4) \pm (4b^4 + 3a^4)}{2a^2b^2}$$

$$x = \frac{4b^4 - \cancel{3a^4} + 4b^4 + \cancel{3a^4}}{2a^2b^2}$$

$$= \frac{\overset{4}{\cancel{8}} \overset{2}{\cancel{b^4}}}{\underset{1}{\cancel{2}} a^2 \underset{1}{\cancel{b^2}}} = \frac{4b^2}{a^2}$$

$$x = \frac{\cancel{4b^4} - 3a^4 - \cancel{4b^4} - 3a^4}{2a^2b^2}$$

$$= \frac{-6a^4}{2a^2b^2}$$

$$= \frac{-3a^2}{b^2}$$

10) Let the given eq:s be of the form $a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$

where $a_1 = 3k+1$, $b_1 = 3$, $c_1 = -2$
$a_2 = k^2+1$, $b_2 = k-2$, $c_2 = -5$

For ~~infinitely many~~ no solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \underset{\mathrm{I}}{\frac{3k+1}{k^2+1}} = \underset{\mathrm{II}}{\frac{3}{k-2}} \neq \underset{\mathrm{III}}{\frac{-2}{-5}}$$

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From I and II, $(3k+1)(k-2) = 3k^2+3$

$\cancel{3k^2} - 6k + k - 2 = \cancel{3k^2} + 3$

$-5k = 5$

$k = -1$

From II and III, $15 \neq 2k-4$

$2k \neq 19$

$k \neq \frac{19}{2}$

$\therefore$ Required value of k is -1

11) ~~Find the sp~~ Let the speed of stream be x km/hr.

Time = Distance/Speed

ATQ, $\frac{49}{17+x} + \frac{49}{17-x} = 9$

$49\left[\frac{17-\cancel{x}+17+\cancel{x}}{289-x^2}\right] = 9$

$\Rightarrow 49 \times 34 = 9(289-x^2)$

$\Rightarrow$

12) $a = -4, d = 3$

$a_n = x$

$S_n = 437$

$a_n = a+(n-1)d$

$x = -4+3(n-1)$

$\frac{x+4}{3} = n-1$

$n = \frac{x+4}{3}+1 = \frac{x+4+3}{3} = \frac{x+7}{3}$

$S_n = \frac{n}{2}[a+a_n]$

$437 = \left(\frac{x+7}{6}\right)[-4+x]$

$2622 = x^2+3x-28$

$x^2+3x-2650 = 0$

$x^2+53x-50x-2650 = 0$

$x(x+53)-50(x+53) = 0$

$(x-50)(x+53) = 0$

$x = 50, -53$

x cannot be -53

$\therefore$ required value of $x = 50$

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13) Let the four consecutive numbers be $a-3d, a-d, a+d$ and $a+3d$.

ATQ, $a-\cancel{3d}+a-\cancel{d}+a+\cancel{d}+a+\cancel{3d}=32$

$$4a=32$$

$$\boxed{a=8}$$

Also, $\dfrac{(a-3d)(a+3d)}{(a-d)(a+d)}=\dfrac{7}{15}$

$\Rightarrow \dfrac{a^2-9d^2}{a^2-d^2}=\dfrac{7}{15}$

$\Rightarrow 15(a^2-9d^2)=7(a^2-d^2)$

$\Rightarrow 15a^2-135d^2=7a^2-7d^2$

$\Rightarrow 8a^2=128d^2$

$\Rightarrow a^2=16d^2$

$\Rightarrow 64=16d^2$

$\Rightarrow d^2=4$

$\boxed{d=\pm 2}$

$\therefore$ The no.s are when $a=8, d=2$: $a-3d=8-6=2$

$a-d=8-2=6$

$a+d=8+2=10$

$a+3d=8+6=14$

When $a=8, d=-2$, the no.s are $14, 10, 6, 2$

14) $S_n=5n^2-3n$

$S_1=a_1=5-3=2$

$S_2=a_1+a_2=5\times4-6=14$

$a_2=S_2-S_1=14-2=12$

$d=a_2-a_1=12-2=10$

$\therefore$ The required AP is $2, 12, 22, 32, \dots$

$a_{16}=a+15d=2+15\times10=\underline{\underline{152}}$

15) $10, 20, 30, \dots 490$ forms an AP with

$a=10, d=10, a_n=490$

$$a_n = a + (n-1)d$$
$$490 = 10 + (n-1)10$$
$$48 = n-1$$
$$n = 49$$
$$S_n = \frac{n}{2}[a + a_n]$$
$$S_{49} = \frac{49}{2}[10 + 490]$$
$$= \frac{49}{2} \times 500$$
$$= 49 \times 250$$
$$= \underline{\underline{12250}}$$