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X H.W-9

1) Using quadratic formula solve for x : $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

2) If the equation $(1+m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, show that $c^2 = a^2(1+m^2)$

3) Seven years ago Varun's age was five times the square of Swati's age. Three years hence, Swati's age will be two-fifth of Varun's age. Find their present ages.

4) One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total no. of camels.

5) ₹9000 were divided equally among a certain no. of persons. Had there been 20 more persons, each would have got Rs 160 less. Find the original number of persons.

6) Two taps running together can fill a tank in $3\frac{1}{13}$ hrs. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank?

7) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

8) Find the middle term of the AP : 213, 205, 197, ... 37

9) If the m^{th} term of an AP is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, then show that its $(mn)^{th}$ term is 1.

10) Which term of the A.P : 3, 15, 27, 39, ... will be 120 more than its 21^{st} term?

11) Determine k so that k^2+4k+8, $2k^2+3k+6$, $3k^2+4k+4$ are three consecutive terms of an AP

12) The 10^{th} term of an AP is -4 and its 22^{nd} term is -16. Find its 38^{th} term.

13) If the sum of first n terms of an A.P is $S_n = 3n^2 - 4n$, find the n^{th} term.

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H.W-9 (Answers)

1) Let the given equation be of the form $Ax^2+Bx+C=0$; where

$A = p^2, B = p^2-q^2, C = -q^2$

$D = B^2-4AC = (p^2-q^2)^2+4p^2q^2 = p^4+q^4-2p^2q^2+4p^2q^2$

$= p^4+q^4+2p^2q^2$

$= (p^2+q^2)^2$

$\therefore x = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$

$= \frac{-(p^2-q^2) \pm (p^2+q^2)}{2p^2}$

$x = \frac{-(p^2-q^2)+(p^2+q^2)}{2p^2}$

$= \frac{-p^2+q^2+p^2+q^2}{2p^2}$

$= \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$

$x = \frac{-(p^2-q^2)-(p^2+q^2)}{2p^2}$

$= \frac{-p^2+q^2-p^2-q^2}{2p^2}$

$= \frac{-2p^2}{2p^2} = -1$

2) Let the given equation be of the form $Ax^2+Bx+C=0$;

Where $A = 1+m^2, B = 2mc, C = c^2-a^2$

For equal roots, $B^2-4AC = 0$

$\Rightarrow (2mc)^2-4(1+m^2)(c^2-a^2) = 0$

$\Rightarrow 4m^2c^2-4(c^2-a^2+m^2c^2-m^2a^2) = 0$

$\Rightarrow 4[m^2c^2-c^2+a^2-m^2c^2+m^2a^2] = 0$

$\Rightarrow a^2+m^2a^2 = c^2$

$\Rightarrow c^2 = a^2(1+m^2)$

3) Let Swati's age 7 years ago be x years. Then Varun's age was $5x^2$

Present ages of Varun and Swati are $5x^2+7$ and $x+7$

After three years, Varun's age is $5x^2+10$

Swati's age is $x+10$

ATQ, $x+10 = \frac{2}{5}(5x^2+10)$

$\Rightarrow x+10 = 2x^2+4$

$2x^2-x-6 = 0$

$2x^2-4x+3x-6 = 0$

$2x(x-2)+3(x-2) = 0$

$(2x+3)(x-2) = 0$

$x = -\frac{3}{2}, 2$

x cannot be negative, $\therefore$ required value of x is 2 years
Hence, present age of Varun is $5x^2+7 = 20+7 = 27$ years
and present age of Swati is $x+7 = 2+7 = 9$ years

4) Let the total no. of camels be x

ATQ $\frac{1}{4}x + 2\sqrt{x} + 15 = x$

$x + 8\sqrt{x} + 60 = 4x$

$3x - 8\sqrt{x} - 60 = 0$

Put $\sqrt{x} = y$

$\Rightarrow x = y^2$

$3y^2 - 8y - 60 = 0$

$\Rightarrow 3y^2 - 18y + 10y - 60 = 0$

$\Rightarrow 3y(y-6) + 10(y-6) = 0$

$\Rightarrow (3y+10)(y-6) = 0$

S	P
-8	-180
	-18, 10

$\therefore y = \frac{-10}{3}, 6$

When $y = \frac{-10}{3}$, $x = \frac{100}{3}$, which is not possible

When $y = 6$, $x = 36$

Hence, the total no. of camels = 36

5) Let the original number of persons be x

ATQ, $\frac{9000}{x} - \frac{9000}{x+20} = 160$

$9000\left[\frac{1}{x} - \frac{1}{x+20}\right] = 160$

$9000\left(\frac{x+20-x}{x^2+20x}\right) = 160$

$9000 \times 20 = 160(x^2+20x)$

$8x^2 + 160x - 9000 = 0$

$x^2 + 20x - 1125 = 0$

S	P
20	-1125
	45, 25

$(x+45)(x-25) = 0$

$x = -45, 25$

x cannot be -ve,

$\therefore$ required value of x is 25

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∴ the original number of persons = 25

6) Let the time taken by each tap to fill the tank separately be x hours and $(x+3)$ hours.

Part of the tank filled by larger pipe in 1 hour = $\frac{1}{x}$

Part of the tank filled by smaller pipe in 1 hour = $\frac{1}{x+3}$

Part of the tank filled by both the pipes together in 1 hour = $\frac{1}{40/13}$

ATQ, $\frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$

$\Rightarrow \frac{x+3+x}{x^2+3x} = \frac{13}{40}$

$\Rightarrow (2x+3)40 = 13(x^2+3x)$

$\Rightarrow 80x + 120 = 13x^2 + 39x$

$\Rightarrow 13x^2 - 41x - 120 = 0$

$\Rightarrow 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow 13x(x-5) + 24(x-5) = 0$

$\Rightarrow (13x+24)(x-5) = 0$

$\therefore x = -\frac{24}{13}, 5$

5	1560
3	312
2	104
2	52
2	26
	13

S = −41, P = −1560 = −65 × 24

x cannot be −ve, ∴ required value of x is 5

Hence, time taken by each tap to fill the tank are 5 hrs and 8 hours.

7) Let the no. of right answer be x and no. of wrong answer be y.

ATQ, $3x - 1y = 40 \longrightarrow (1)$

Also, $4x - 2y = 50 \xrightarrow{(\div 2)} 2x - y = 25 \longrightarrow (2)$

(1) − (2), $\boxed{x = 15}$

From eq: (1), $45 - y = 40$

$\boxed{y = 5}$

Hence, the total no. of questions in the test

= 15 + 5

= 20

8) $a = 213$

$d = 205 - 213 = -8$

$a_n = 37$

$a_n = a + (n-1)d$

$\Rightarrow 37 = 213 - 8(n-1)$

$\Rightarrow \frac{-176}{-8} = n-1$

$\Rightarrow n-1 = 22$

$\boxed{n = 23}$

$\therefore$ The middle most term is $\left(\frac{n+1}{2}\right)^{th}$ term

$= \left(\frac{23+1}{2}\right)^{th}$ term

$= 12^{th}$ term

$= a_{12}$

$= a + 11d$

$= 213 + 11 \times -8$

$= 213 - 88$

$= \underline{\underline{125}}$

9) Let a and d be the first term and common difference of the given AP.

ATQ, $a_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n} \rightarrow (1)$

$a_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m} \rightarrow (2)$

$(1) - (2),\ d(m-1-n+1) = \frac{1}{n} - \frac{1}{m}$

$d(m-n) = \frac{m-n}{mn}$

$\therefore \boxed{d = \frac{1}{mn}}$

From eq: (1), $a + \frac{m-1}{mn} = \frac{1}{n}$

$a = \frac{1 \times m}{n \times m} - \frac{m-1}{mn}$

$= \frac{m - m + 1}{mn}$

$\boxed{a = \frac{1}{mn}}$

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$\therefore a_{mn} = a+(mn-1)d = \frac{1}{mn} + \frac{mn-1}{mn}$

$= \frac{1+mn-1}{mn} = \frac{mn}{mn} = 1$

10) $a = 3$

$d = 15-3 = 12$

$a_{21} + 120 = a+20d+120 = 3+20\times12+120$

$= 3+240+120$

$= 363$

$a_n = 363$

$\Rightarrow a+(n-1)d = 363$

$\Rightarrow 3+(n-1)12 = 363$

$\Rightarrow n-1 = \frac{360}{12} = 30$

$n = 31$

Hence, the required term is the 31st term.

$a_{31} = a+30d = 3+30\times12 = 3+360 = 363$

11) Since the given terms are in A.P,

$a_2 - a_1 = a_3 - a_2$

$\Rightarrow 2k^2+3k+6-k^2-4k-8 = 3k^2+4k+4-2k^2-3k-6$

$\Rightarrow k^2-k-2 = k^2+k-2$

$2k = 0$

$k = 0$

12) $a_{10} = -4 \Rightarrow a+9d = -4$

$a_{22} = -16 \Rightarrow a+21d = -16$ $\quad (-)\ (-)\ (+)$

$-12d = 12$

$d = -1$

From eq: (2), $a-9 = -4$

$a = 5$

$\therefore a_{38} = a+37d = 5+37\times(-1) = 5-37 = -32$

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13) $S_n = 3n^2 - 4n$

When $n=1$; $S_1 = a_1 = 3-4 = -1$

When $n=2$, $S_2 = a_1 + a_2 = 3\times4 - 8 = 12-8 = 4$

$a_2 = S_2 - S_1 = 4+1 = 5$

$d = a_2 - a_1 = 5+1 = 6$

$\therefore a_n = a+(n-1)d = -1+(n-1)6$

$= -1+6n-6$

$= \underline{\underline{6n-7}}$