Date

IX Homework-3

1) If $x = 3+\sqrt{8}$, find the value of $x^2 + \frac{1}{x^2}$ 34

2) Simplify:- $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$ 11

3) If $x = 1-\sqrt{2}$, find the value of $\left(x-\frac{1}{x}\right)^3$ 8

4) Prove that: $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} +$

$$\frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}} = 2$$

5) Simplify: $\frac{\sqrt{5}-2}{\sqrt{5}+2} - \frac{\sqrt{5}+2}{\sqrt{5}-2}$ $-8\sqrt{5}$

6) Find the values of a and b if $\frac{5+\sqrt{3}}{7-4\sqrt{3}} = a+\sqrt{3}b$ 47, 27

7) If $\sqrt{5} = 2.236$, then find $\frac{2}{\sqrt{5}}$ upto three decimal places

8) Show that $\left(\frac{3^a}{3^b}\right)^{a+b} \cdot \left(\frac{3^b}{3^c}\right)^{b+c} \cdot \left(\frac{3^c}{3^a}\right)^{c+a} = 1$

9) Show that $\sqrt{x^{-1}y} \times \sqrt{y^{-1}z} \times \sqrt{z^{-1}x} = 1$

10) Simplify $\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left[\left(\frac{25}{9}\right)^{-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right]$

11) Simplify: $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$

12) Find the value of x if $5^{x-3} \times 3^{2x-8} = 225$

13) Prove that $\frac{a^{-1}}{a^{-1}+b^{-1}} + \frac{a^{-1}}{a^{-1}-b^{-1}} = \frac{2b^2}{b^2-a^2}$

14) Show that $\frac{(x^{a+b})^2 \cdot (x^{b+c})^2 \cdot (x^{c+a})^2}{(x^a \cdot x^b \cdot x^c)^4} = 1$

15) Simplify: $\frac{3^{30}+3^{29}+3^{28}}{3^{31}+3^{30}-3^{29}} + \frac{2^{30}+2^{29}+2^{28}}{2^{31}+2^{30}-2^{29}}$ $\frac{361}{330}$

No.
Date

IX Homework - 3 (Answers)

1) $x = 3+\sqrt{8}$

$x^2 = (3+\sqrt{8})^2 = (3)^2 + (\sqrt{8})^2 + 2\times 3\times\sqrt{8}$

$= 9+8+6\sqrt{8}$

$= 17+6\sqrt{8}$

$\frac{1}{x} = \frac{1}{3+\sqrt{8}} = \frac{3-\sqrt{8}}{(3)^2-(\sqrt{8})^2} = \frac{3-\sqrt{8}}{9-8} = 3-\sqrt{8}$

$\frac{1}{x^2} = (3-\sqrt{8})^2 = 9+(\sqrt{8})^2 - 2\times 3\times\sqrt{8}$

$= 9+8-6\sqrt{8}$

$= 17-6\sqrt{8}$

$\therefore x^2+\frac{1}{x^2} = 17+\cancel{6\sqrt{8}}+17-\cancel{6\sqrt{8}} = \underline{\underline{34}}$

2) $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$

$2\,|\,12$
$2\,|\,6$
3

$\frac{(3\sqrt{2}-2\sqrt{3})^2}{(3\sqrt{2})^2-(2\sqrt{3})^2} + \frac{2\sqrt{3}(\sqrt{3}+\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}$

$= \frac{(3\sqrt{2})^2+(2\sqrt{3})^2-2\times 3\sqrt{2}\times 2\sqrt{3}}{9\times 2-4\times 3} + \frac{2\times 3+2\sqrt{6}}{3-2}$

$= \frac{18+12-12\sqrt{6}}{18-12} + 6+2\sqrt{6}$

$= \frac{30-12\sqrt{6}}{6} + 6+2\sqrt{6}$

$= \frac{\cancel{6}^{1}(5-2\sqrt{6})}{\cancel{6}_{1}} + 6+2\sqrt{6}$

$= 5-\cancel{2\sqrt{6}}+6+\cancel{2\sqrt{6}}$

$= 5+6$

$= \underline{\underline{11}}$

No.
Date

3) $x = 1-\sqrt{2}$

$$\frac{1}{x} = \frac{1}{1-\sqrt{2}} = \frac{1+\sqrt{2}}{1^2-(\sqrt{2})^2} = \frac{1+\sqrt{2}}{1-2} = \frac{1+\sqrt{2}}{-1} = -(1+\sqrt{2})$$

$$\therefore x-\frac{1}{x} = (1-\sqrt{2})+(1+\sqrt{2}) = 1-\cancel{\sqrt{2}}+1+\cancel{\sqrt{2}} = 2$$

$$\therefore \left(x-\frac{1}{x}\right)^3 = 2^3 = \underline{\underline{8}}$$

4) LHS,

$$\frac{1\times(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})} + \frac{1\times(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})} + \frac{1\times(\sqrt{3}-\sqrt{4})}{(\sqrt{3}+\sqrt{4})(\sqrt{3}-\sqrt{4})} + \frac{1\times(\sqrt{4}-\sqrt{5})}{(\sqrt{4}+\sqrt{5})(\sqrt{4}-\sqrt{5})} + \frac{1\times(\sqrt{5}-\sqrt{6})}{(\sqrt{5}+\sqrt{6})(\sqrt{5}-\sqrt{6})}$$

$$+ \frac{1\times(\sqrt{6}-\sqrt{7})}{(\sqrt{6}+\sqrt{7})(\sqrt{6}-\sqrt{7})} + \frac{1\times(\sqrt{7}-\sqrt{8})}{(\sqrt{7}+\sqrt{8})(\sqrt{7}-\sqrt{8})} + \frac{1\times(\sqrt{8}-\sqrt{9})}{(\sqrt{8}+\sqrt{9})(\sqrt{8}-\sqrt{9})}.$$

$$= \frac{1-\sqrt{2}}{1-2} + \frac{\sqrt{2}-\sqrt{3}}{2-3} + \frac{\sqrt{3}-\sqrt{4}}{3-4} + \frac{\sqrt{4}-\sqrt{5}}{4-5} + \frac{\sqrt{5}-\sqrt{6}}{5-6} + \frac{\sqrt{6}-\sqrt{7}}{6-7} + \frac{\sqrt{7}-\sqrt{8}}{7-8} + \frac{\sqrt{8}-\sqrt{9}}{8-9}$$

$$= -(1-\sqrt{2})-(\sqrt{2}-\sqrt{3})-(\sqrt{3}-\sqrt{4})-(\sqrt{4}-\sqrt{5})-(\sqrt{5}-\sqrt{6})-(\sqrt{6}-\sqrt{7}) - (\sqrt{7}-\sqrt{8})-(\sqrt{8}-\sqrt{9})$$

$$= -1+\cancel{\sqrt{2}}-\cancel{\sqrt{2}}+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}+\cancel{\sqrt{4}}-\cancel{\sqrt{4}}+\cancel{\sqrt{5}}-\cancel{\sqrt{5}}+\cancel{\sqrt{6}}-\cancel{\sqrt{6}}+\cancel{\sqrt{7}}-\cancel{\sqrt{7}}+\cancel{\sqrt{8}} - \cancel{\sqrt{8}}+\sqrt{9}$$

$$= -1+\sqrt{9} = -1+3 = \underline{\underline{2}}$$, RHS

5)

$$\frac{(\sqrt{5}-2)(\sqrt{5}-2)}{(\sqrt{5})^2-(2)^2} - \frac{(\sqrt{5}+2)(\sqrt{5}+2)}{(\sqrt{5})^2-(2)^2}$$

$$= \frac{(\sqrt{5}-2)^2}{5-4} - \frac{(\sqrt{5}+2)^2}{5-4} = (5+4-4\sqrt{5})-(5+4+4\sqrt{5})$$

$$= (9-4\sqrt{5})-(9+4\sqrt{5})$$

$$= \cancel{9}-4\sqrt{5}-\cancel{9}-4\sqrt{5}$$

$$= \underline{\underline{-8\sqrt{5}}}$$

No.

Date

6) $\dfrac{(5+\sqrt{3})(7+4\sqrt{3})}{(7-4\sqrt{3})(7+4\sqrt{3})} = \dfrac{35+20\sqrt{3}+7\sqrt{3}+12}{49-48}$

$= 47+27\sqrt{3}$

On comparing with $a+\sqrt{3}\,b$, $a = 47$
$b = 27$

7) $\dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5} = \dfrac{2 \times 2.236}{5} = \dfrac{4.472}{5} = 0.8944$
$\approx \underline{\underline{0.894}}$

8) LHS; $\left(3^{a-b}\right)^{a+b} \cdot \left(3^{b-c}\right)^{b+c} \cdot \left(3^{c-a}\right)^{c+a}$

$= 3^{a^2-b^2} \cdot 3^{b^2-c^2} \cdot 3^{c^2-a^2}$

$= 3^{\cancel{a^2}-\cancel{b^2}+\cancel{b^2}-\cancel{c^2}+\cancel{c^2}-\cancel{a^2}}$

$= 3^0 = \underline{\underline{1}}$, RHS

9) LHS; $\sqrt{\dfrac{y}{x}} \times \sqrt{\dfrac{z}{y}} \times \sqrt{\dfrac{x}{z}}$

$= \sqrt{\dfrac{\cancel{y}^{1}}{\cancel{x}_{1}} \times \dfrac{\cancel{z}^{1}}{\cancel{y}_{1}} \times \dfrac{\cancel{x}^{1}}{\cancel{z}_{1}}} = \sqrt{1} = \underline{\underline{1}}$, RHS

10) $\dfrac{3^{\cancel{4}^{1}x-\frac{3}{\cancel{4}_{1}}}}{2^{\cancel{4}^{1}x-\frac{3}{\cancel{4}_{1}}}} \times \left[\dfrac{5^{\cancel{2}^{1}x-\frac{3}{\cancel{2}_{1}}}}{3^{\cancel{2}^{1}x-\frac{3}{\cancel{2}_{1}}}} \div \dfrac{5^{-3}}{2^{-3}}\right]$

$= \dfrac{\cancel{3^{-3}}^{1}}{\cancel{2^{-3}}_{1}} \times \dfrac{\cancel{5^{-3}}^{1}}{\cancel{3^{-3}}_{1}} \times \dfrac{\cancel{2^{-3}}^{1}}{\cancel{5^{-3}}_{1}}$

$= \underline{\underline{1}}$

11) $\frac{4}{6^{3x-\frac{2}{3}}} + \frac{1}{4^{4x-\frac{3}{4}}} + \frac{2}{3^{5x-\frac{1}{5}}}$

$= \frac{4}{6^{-2}} + \frac{1}{4^{-3}} + \frac{2}{3^{-1}}$

$= 4 \times 6^2 + 4^3 + 2 \times 3$

$= 144 + 64 + 6 = \underline{\underline{214}}$

12) $\frac{5^{x-3}}{\cancel{5}^3} \times \frac{3^{2x-8}}{\cancel{3}^8} = 5^2 \times 3^2$

$$\begin{array}{r|l} 5 & 225 \\ 5 & 45 \\ 3 & 9 \\ & 3 \end{array}$$

$\therefore x - 3 = 2$ or $2x - 8 = 2$

$x = \underline{\underline{5}}$ $2x = 10$

$x = \underline{\underline{5}}$

13) LHS, $\frac{\frac{1}{a}}{\frac{1}{a}+\frac{1}{b}} + \frac{\frac{1}{a}}{\frac{1}{a}-\frac{1}{b}} = \frac{\frac{1}{a}}{\frac{b+a}{ab}} + \frac{\frac{1}{a}}{\frac{b-a}{ab}}$

$= \frac{1}{\cancel{a}} \times \frac{\cancel{a}b}{a+b} + \frac{1}{\cancel{a}} \times \frac{\cancel{a}b}{b-a}$

$= \frac{b}{a+b} + \frac{b}{b-a}$

$= \frac{b(b-a) + b(b+a)}{b^2 - a^2}$

$= \frac{b^2 \cancel{-ab} + b^2 \cancel{+ab}}{b^2 - a^2}$

$= \underline{\underline{\frac{2b^2}{b^2 - a^2}}}$, RHS

No.
Date

14) LHS, $\dfrac{x^{2(a+b)} \cdot x^{2(b+c)} \cdot x^{2(c+a)}}{x^{4a} \cdot x^{4b} \cdot x^{4c}}$

$= \dfrac{x^{2a+2b} \cdot x^{2b+2c} \cdot x^{2c+2a}}{x^{4a} \cdot x^{4b} \cdot x^{4c}}$

$= \dfrac{x^{2a+2b+2b+2c+2c+2a}}{x^{4a+4b+4c}}$

$= \dfrac{x^{4a+4b+4c}}{x^{4a+4b+4c}}$

$= 1$, RHS

15) $\dfrac{\cancel{3^{28}}(3^2+3^1+1)}{\cancel{3^{28}}(3^3+3^2-3^1)} + \dfrac{\cancel{2^{28}}(2^2+2^1+1)}{\cancel{2^{28}}(2^3+2^2-2^1)}$

$= \dfrac{9+3+1}{27+9-3} + \dfrac{4+2+1}{8+4-2}$

$= \dfrac{13 \times 10}{33 \times 10} + \dfrac{7 \times 33}{10 \times 33}$

$= \dfrac{130}{330} + \dfrac{231}{330}$

$= \dfrac{361}{330}$