Homework-5

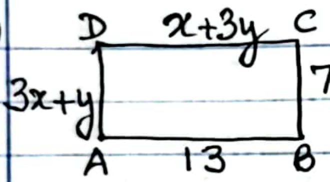
1) Solve graphically, the pair of equations $2x+y=6$ and $2x-y+2=0$. Find the ratio of the areas of the two triangles formed by the lines representing these equations with x-axis and the lines with y-axis.

2) Solve for x and y: (i) $ax+by=1$, $bx+ay=\frac{2ab}{a^2+b^2}$ (ii) $\frac{x}{a}+\frac{y}{b}=a+b$, $\frac{x}{a^2}+\frac{y}{b^2}=2$

3) For what value(s) of p, does the pair of linear equations $px+y=p^2$ and $x+py=1$ have
(i) no solution? (ii) infinitely many solutions? (iii) a unique solution?

4) Find a, if the line $3x+ay=8$ passes through the intersection of lines represented by equations $3x-2y=10$ and $5x+y=8$

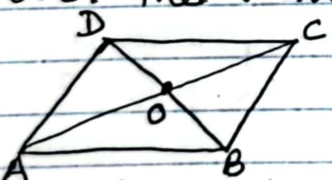
5) Father's age is 3 times the sum of ages of his two children. After 5 years, his age will be twice the sum of ages of the two children. Find the age of father.

6) 

Find the values of x and y in the given rectangle.

7) Determine the values of a and b for which the following pairs of linear equations has infinitely many solutions
$3x-(a+1)y=2b-1$; $5x+(1-2a)y=3b$

8) Find the value of $x+y$, if $3x-2y=5$ and $3y-2x=3$

9) 

ABCD is a parallelogram.
OA = 9cm, OB = (x−y) cm, OC = (x+y) cm,
OD = 5cm

10) For what value of p, will the following system of linear equations represent parallel lines?
$-x+py=1$; $px-y=1$

11) If α and β are the zeroes of the polynomial $p(x)=2x^2+5x+k$ satisfying the relation $\alpha^2+\beta^2+\alpha\beta=\frac{21}{4}$, then find the value of k.

12) If one of the zeroes of $p(x)=3x^2-8x+2k+1$ is seven times the other, find the value of k.

13) If the zeroes of the polynomial x^2+px+q are double in value to the zeroes of $2x^2-5x-3$, find the values of p and q.

14) If one of the zeroes of $f(x)=14x^2-42k^2x-9$ is negative of the other, find the value of k.

15) If one zero of polynomial $(a^2+9)x^2+13x+6a$ is reciprocal of the other, find the value of a.

MCQs

1) The pair of linear equations $2kx+5y=7$; $6x-5y=11$ has a unique solution, if
(a) $k \neq -3$ (b) $k \neq \frac{2}{3}$ (c) $k \neq 5$ (d) $k \neq \frac{2}{9}$

2) A pair of linear equations which has a unique solution $x=2$ and $y=-3$ is
(a) $x+y=1$; $2x-3y=-5$ (b) $2x+5y=-11$; $4x+10y=-22$
(c) $2x-y=1$; $3x+2y=0$ (d) $x-4y-14=0$; $5x-y-13=0$

3) If the lines given by $3x+2ky=2$; $2x+5y=1$ are parallel, then the value of k is (a) $-5/4$ (b) $2/5$ (c) $15/4$ (d) $3/2$

4) The value of k for which the pair of linear equations $3x+5y=8$ and $kx+15y=24$ has infinitely many solutions is
(a) 3 (b) 9 (c) 5 (d) 15

5) Which of the following pair of equations are inconsistent ?
(a) $3x-y=9$; $x-\frac{y}{3}=3$ (b) $4x+3y=24$; $-2x+3y=6$
(c) $5x-y=10$; $10x-2y=20$ (d) $-2x+y=3$; $-4x+2y=10$

6) The value of k for which the pair of linear equations $x+y-4=0$ and $2x+ky-3=0$ have no solution is (a) 0 (b) 2 (c) 6 (d) 8

7) If a pair of linear equations is consistent, then the lines will be
(a) parallel (b) always coincident (c) intersecting or coincident (d) always intersecting

8) The pair of equations $x+2y+5=0$ and $-3x-6y+1=0$ has
(a) a unique solution (b) exactly two solutions
(c) infinitely many solutions (d) no solution.

9) The pair of linear equation $y=0$ and $y=-5$ has
(a) one solution (b) two solutions (c) infinitely many solutions (d) no solution

10) If α and β are zeroes and the quadratic polynomial $p(s)=3s^2-6s+4$, then the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta$ is
(a) 7 (b) 6 (c) 8 (d) 10

11) If the square of difference of the zeroes of $p(x)=x^2+px+45$ is equal to 144, then the value of p is
(a) ± 9 (b) ± 12 (c) ± 15 (d) ± 18

12) If one zero of $p(x)=(k-1)x^2+kx+1$ is -3, then the value of k is
(a) $4/3$ (b) $-4/3$ (c) $2/3$ (d) $-2/3$

13) A quadratic polynomial having sum and product of its zeroes are 5 and 0 respectively is (a) x^2+5x (b) $2x(x-5)$ (c) $5x^2-1$ (d) x^2-5x+5

14) If the sum of the zeroes of $p(x)=kx^2+2x+3k$ is equal to their product then k = (a) $1/3$ (b) $-1/3$ (c) $2/3$ (d) $-2/3$

15) If α, β are zeroes of $p(x)=x^2-(k+6)x+2(2k-1)$, then k = —, if $\alpha+\beta=\frac{1}{2}\alpha\beta$ (a) -7 (b) 7 (c) -3 (d) 3

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X Homework-5 (Answers)

1) $2x+y=6$

$y=6-2x$

x	0	3	1
y	6	0	4

$2x-y+2=0$

$y=2x+2$

x	0	-1	1
y	2	0	4

graph

2) (i) $ax+by=1 \rightarrow (1) \times a \Rightarrow a^2x+aby=a$

$bx+ay=\frac{2ab}{a^2+b^2} \rightarrow (2)\times b \Rightarrow \overset{(-)}{b^2x} \overset{(-)}{+} aby \overset{(-)}{=} \frac{2ab^2}{a^2+b^2}$

$x(a^2-b^2)= a-\frac{2ab^2}{a^2+b^2}$

$\Rightarrow x(a^2-b^2)=\frac{a^3+ab^2-2ab^2}{a^2+b^2}$

$\Rightarrow x(a^2-b^2)=\frac{a^3-ab^2}{a^2+b^2}$

$\Rightarrow x(a^2-b^2)=\frac{a(a^2-b^2)}{a^2+b^2}$

$\therefore x=\frac{a}{a^2+b^2}$

From eq:(1), $a\times\frac{a}{a^2+b^2}+by=1$

$\Rightarrow by=1-\frac{a^2}{a^2+b^2}$

$\Rightarrow by=\frac{a^2+b^2-a^2}{a^2+b^2}$

$\Rightarrow by=\frac{b^2}{a^2+b^2}$

$y=\frac{b}{a^2+b^2}$

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(ii) $\frac{x}{a} + \frac{y}{b} = a+b \Rightarrow bx + ay = ab(a+b) \rightarrow (1)$

$\frac{x}{a^2} + \frac{y}{b^2} = 2 \Rightarrow b^2x + a^2y = 2a^2b^2 \longrightarrow (2)$

$(1) \times a \Rightarrow abx + a^2y = a^2b(a+b)$
$(2) \Rightarrow \overset{(-)}{} b^2x \overset{(-)}{+} a^2y = \overset{(-)}{} 2a^2b^2$

$(-)$, $x(ab - b^2) = a^3b + a^2b^2 - 2a^2b^2$

$\Rightarrow bx(a-b) = a^3b - a^2b^2$

$bx(a-b) = a^2b(a-b)$

$\boxed{x = a^2}$

From eq: (2), $a^2b^2 + a^2y = 2a^2b^2$

$a^2y = 2a^2b^2 - a^2b^2$

$a^2y = a^2b^2$

$\boxed{y = b^2}$

3) $a_1 = p, b_1 = 1, c_1 = -p^2$
$a_2 = 1, b_2 = p, c_2 = -1$

(i) For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{p}{1} = \frac{1}{p} \neq \frac{-p^2}{-1}$
I II III

From I and II, $p^2 = 1$
$p = \pm 1$

From II and III, $p^3 \neq 1$
$p \neq 1$

$\therefore$ Required value of $p = -1$

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(ii) For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\Rightarrow \frac{p}{1} = \frac{1}{p} = \frac{-p^2}{-1}$$
I II III

From I and II, $p^2 = 1$
$p = \pm 1$.
From II and III, $p^3 = 1$
$p = 1$
$\therefore$ the required value of p is 1.

(iii) For a unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\Rightarrow \frac{p}{1} \neq \frac{1}{p}$$

$$\Rightarrow p^2 \neq 1$$
$$p \neq \pm 1$$

Thus, p can take any real values except ± 1.

4) $3x - 2y = 10 \rightarrow (1)$
$5x + y = 8 \rightarrow (2) \times 2$

$(1) \Rightarrow 3x - 2y = 10$
$(2) \times 2 \Rightarrow 10x + 2y = 16$
$(+)$, $13x = 26$
$\boxed{x = 2}$

From eq:(2), $10 + y = 8$
$\boxed{y = -2}$

Since $3x + ay = 8$ passes through the point of intersection of given two lines,
when $x = 2$ and $y = -2$
$6 - 2a = 8$
$-2a = 2$
$\boxed{a = -1}$

5) Let the father's age be x years and sum of ages of his two children be y years.

	Father	two children
	x	y
After 5yrs,	$x+5$	$y+10$

ATQ, $x = 3y \rightarrow (1)$
Also, $x + 5 = 2(y + 10)$
$\Rightarrow x + 5 = 2y + 20$
$\Rightarrow x - 2y = 15 \rightarrow (2)$

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On Substituting eq:(1) in eq:(2),

$x - 2y = 15$

$3y - 2y = 15$

$\boxed{y = 15}$

From eq:(1) $\boxed{x = 45}$

Hence, the present age of father is 45 years

6) We know that opposite sides of a rectangle are equal,

$x + 3y = 13 \rightarrow (1)$

$3x + y = 7 \rightarrow (2) \times 3$

$(1) \Rightarrow x + 3y = 13$

$(2)\times 3 \Rightarrow 9x + 3y = 21$

$(-),\quad -8x = -8$

$\boxed{x = 1}$

From eq:(1), $1 + 3y = 13$

$3y = 12$

$\boxed{y = 4}$

7) Let the given equations be of the form

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$; where

$a_1 = 3$, $b_1 = -(a+1)$, $c_1 = -(2b-1)$

$a_2 = 5$, $b_2 = +(1-2a)$, $c_2 = -3b$

For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{5} = \frac{-(a+1)}{(1-2a)} = \frac{-(2b-1)}{-3b}$

I II III

From I and II, $3 - 6a = -5a - 5$

$-a = -8$

$\boxed{a = 8}$

From II and III,

$\frac{-9}{1-16} = \frac{2b-1}{3b}$

$\frac{3}{5} = \frac{2b-1}{3b}$

$9b = 10b - 5$

$\boxed{b = 5}$

8) $3x - 2y = 5 \rightarrow (1)$
$3y - 2x = 3 \rightarrow (2)$

$(1) \Rightarrow 3x - 2y = 5$
$(2) \Rightarrow -2x + 3y = 3$
$(+), \quad x + y = 8$

9) We know that diagonals of a parallelogram bisect eachother,
$OA = OC \Rightarrow x + y = 9 \rightarrow (1)$
$OB = OD \Rightarrow x - y = 5 \rightarrow (2)$
$(1) + (2), \quad 2x = 14$
$\boxed{x = 7}$
$\boxed{y = 2}$

10) The given eq: be of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
$a_1 = -1, b_1 = p, c_1 = -1$
$a_2 = p, b_2 = -1, c_2 = -1$
For parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\Rightarrow \underset{\text{I}}{\frac{-1}{p}} = \underset{\text{II}}{\frac{p}{-1}} \neq \underset{\text{III}}{\frac{\not -1}{\not -1}}$$

From eq: I and II, $p^2 = 1$, $p = \pm 1$ | From II and III, $p \neq -1$

$\therefore$ Required value of p is 1

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11) Let $p(x) = 2x^2 + 5x + k$ be of the form ax^2+bx+c; where $a = 2$, $b = 5$, $c = k$

$\alpha + \beta = -\frac{b}{a} = -\frac{5}{2}$

$\alpha\beta = \frac{c}{a} = \frac{k}{2}$

Given, $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$

$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta + \alpha\beta = \frac{21}{4}$

$\Rightarrow (\alpha+\beta)^2 - \alpha\beta = \frac{21}{4}$

$\Rightarrow \frac{25}{4} - \frac{k}{2} = \frac{21}{4}$

$-\frac{k}{2} = \frac{21}{4} - \frac{25}{4}$

$\frac{k}{2} = \frac{4}{4}$

$\therefore$ $\boxed{k = 2}$

12) Let the given ~~equatio~~ polynomial be of the form ax^2+bx+c; where $a = 3$, $b = -8$, $c = 2k+1$ and let α and 7α be the zeroes.

Sum of zeroes, $\alpha + 7\alpha = -\frac{b}{a}$

$\Rightarrow 8\alpha = \frac{8}{3}$

$\therefore \alpha = \frac{1}{3}$

Product of zeroes, $\alpha \times 7\alpha = \frac{c}{a}$

$\Rightarrow 7\alpha^2 = \frac{2k+1}{3}$

$\Rightarrow \frac{7}{9} = \frac{2k+1}{3}$

$\Rightarrow 6k+3 = 7$

$\Rightarrow 6k = 4$

$\boxed{k = \frac{2}{3}}$

13) Let the given polynomial $p(x) = 2x^2 - 5x - 3$;
Where $a = 2, b = -5, c = -3$ and the zeroes be α and β

$$\alpha + \beta = -\frac{b}{a} = \frac{5}{2}$$

$$\alpha\beta = \frac{c}{a} = \frac{-3}{2}$$

Let the zeroes of $f(x) = x^2 + px + q$ be 2α and 2β.
Then, sum of zeroes, $2\alpha + 2\beta = -p$

$$\Rightarrow 2(\alpha + \beta) = -p$$

$$\Rightarrow 2 \times \frac{5}{2} = -p$$

$$\therefore \boxed{p = -5}$$

Product of zeroes, $2\alpha \times 2\beta = q$

$$\Rightarrow 4\alpha\beta = q$$

$$\Rightarrow 4 \times \frac{-3}{2} = q$$

$$\therefore \boxed{q = -6}$$

14) Let $f(x) = 14x^2 - 42k^2x - 9$ be of the form $ax^2 + bx + c$; where
$a = 14$, $b = -42k^2$, $c = -9$ and the zeroes be α and $-\alpha$.

Then, sum of zeroes $= \alpha + (-\alpha) = -\frac{b}{a}$

$$\Rightarrow 0 = \frac{42k^2}{14}$$

$$\Rightarrow 42k^2 = 0$$

$$\Rightarrow k^2 = 0$$

$$\therefore \boxed{k = 0}$$

15) Let $p(x) = (a^2 + 9)x^2 + 13x + 6a$ be of the form $Ax^2 + Bx + C$;
where $A = a^2 + 9$, $B = 13$, $C = 6a$ and let α and $\frac{1}{\alpha}$ be the zeroes.
Then, product of zeroes $= \alpha \times \frac{1}{\alpha} = \frac{C}{A}$

$$\Rightarrow 1 = \frac{6a}{a^2 + 9}$$

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$\Rightarrow a^2+9=6a$

$\Rightarrow a^2-6a+9=0$

$\Rightarrow (a-3)^2=0 \Rightarrow (a-3)=0$

$\therefore \boxed{a=3}$

MCQs

1) $a_1 = 2k, b_1 = 5, c_1 = -7$

$a_2 = 6, b_2 = -5, c_2 = -11$

For unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{2k}{6} \neq \frac{5}{-5}$ -1

$\Rightarrow 2k \neq -6$

$k \neq -3$ (a)

2) $x-4y-14=0$; $5x-y-13=0$ (d)

3) $a_1 = 3, b_1 = 2k, c_1 = -2$

$a_2 = 2, b_2 = 5, c_2 = -1$

For parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{-1}$

$\therefore 4k = 15$

$k = \frac{15}{4}$ (c)

4) $a_1 = 3, b_1 = 5, c_1 = -8$

$a_2 = k, b_2 = 15, c_2 = -24$

for infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{k} = \frac{5}{15} = \frac{-8}{-24}$

I II

From I and II, $k=9$ (b)

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5) $-2x+y=3$; $-4x+2y=10$ (d)

6) $a_1=1, b_1=1, c_1=-4$

$a_2=2, b_2=k, c_2=-3$

For no solution, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$$\Rightarrow \frac{1}{2}=\frac{1}{k}\neq\frac{-4}{-3}$$

$\therefore k=2$ (b)

7) intersecting or coincident (c)

8) $a_1=1, b_1=2, c_1=5$

$a_2=-3, b_2=-6, c_2=1$

$\frac{a_1}{a_2}=\frac{-1}{3}$

$\frac{b_1}{b_2}=\frac{-2}{6}=\frac{-1}{3}$

$\frac{c_1}{c_2}=\frac{5}{1}$

$\therefore \frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

no solution (d)

9) $y=0$ is the equation of x-axis

$y=-5$ is the line parallel to x-axis.

So no solution (d)

10) $a=3, b=-6, c=4$

$\alpha+\beta=\frac{-b}{a}=\frac{6}{3}=2$

$\alpha\beta=\frac{c}{a}=\frac{4}{3}$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=\frac{\alpha^2+\beta^2}{\alpha\beta}+2\left(\frac{\beta+\alpha}{\alpha\beta}\right)+3\alpha\beta$$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}+2\left(\frac{\alpha+\beta}{\alpha\beta}\right)+3\alpha\beta$$

$$=\frac{4-\frac{8}{3}}{\frac{4}{3}}+2\times\frac{2}{4}\times 3+3\times\frac{4}{3}$$

$$=\frac{4}{3}\times\frac{3}{4}+3+4=1+7=\underline{8}\text{ (c)}$$

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11) $a=1, b=p, c=45$; $\alpha+\beta=-\frac{b}{a}=-p$; $\alpha\beta=\frac{c}{a}=45$

$(\alpha-\beta)^2=144$

$\Rightarrow (\alpha+\beta)^2-4\alpha\beta=144$

$\Rightarrow \quad p^2-4\times45=144$

$\therefore p^2=324$

$p=\pm18$ (d)

12) $p(-3)=0$

$\Rightarrow 9(k-1)-3k+1=0$

$\Rightarrow 9k-9-3k+1=0$

$\Rightarrow 6k-8=0$

$k=\frac{8}{6}=\frac{4}{3}$ (a)

13) $\alpha+\beta=5$

$\alpha\beta=0$

$x^2-(\alpha+\beta)x+\alpha\beta=x^2-5x$

$=x(x-5)$

$=2\times x(x-5)$ (b)

14) $a=k, b=2, c=3k$

$\alpha+\beta=-\frac{b}{a}=\frac{-2}{k}$

$\alpha\beta=\frac{c}{a}=\frac{3k}{k}=3$

ATQ, $\alpha+\beta=\alpha\beta$

$\Rightarrow -\frac{2}{k}=3$

$\therefore k=-\frac{2}{3}$ (d)

15) $a=1, b=-(k+6), c=2(2k-1)$

$\alpha+\beta=-\frac{b}{a}=k+6$

$\alpha\beta=\frac{c}{a}=2(2k-1)$

ATQ, $\alpha+\beta=\frac{1}{2}\alpha\beta$

$\Rightarrow k+6=\frac{1}{2}\times2(2k-1)$

$\Rightarrow k-2k=-1-6$

$\Rightarrow -k=-7$

$k=7$ (b)