Homework-3

1) Find a quadratic polynomial whose sum and product respectively of the zeroes are given as (i) $-2\sqrt{3}, -9$ (ii) $-\frac{3}{2\sqrt{5}}, -\frac{1}{2}$. Also, find the zeroes.

2) Find the zeroes of the following polynomials by factorisation method and verify the relations between the zeroes and the coefficients of the polynomials
(i) $2x^2 + \frac{7}{2}x + \frac{3}{4}$ (ii) $4x^2 + 5\sqrt{2}x - 3$ (iii) $2s^2 - (1+2\sqrt{2})s + \sqrt{2}$

3) If the zeroes of a quadratic polynomial ax^2+bx+c are both positive, then (a) a, b and c all have the same sign
(b) a and c are positive and b is negative
(c) a and b are positive and c is negative
(d) none of these

4) If the ~~graph~~ graph of a polynomial intersects the x-axis at only one point, it cannot be a quadratic polynomial. True or false? Justify your answer.

5) Prove that $\sqrt{p}+\sqrt{q}$ is irrational, where p, q are primes.

6) During a sale, colour pencils were being sold in packets of 24 each and crayons in packets of 32 each. If you want full packs of both and the same number of pencils and crayons, how many of each would you need to buy?

7) α, β are zeroes of polynomial x^2-6x+a. Find the value of a, if $3\alpha+2\beta=20$.

8) If $\overset{\alpha}{2}$ and $\overset{\beta}{-3}$ are the zeroes of the quadratic polynomial $x^2+(a+1)x+b$, then find the values of a and b.

9) If α and β are the zeroes of $f(t) = t^2 - p(t+1) - c$, show that $(\alpha+1)(\beta+1) = 1-c$.

10) If α, β are the zeroes of $f(x) = 2x^2+5x+k$ such that $\alpha^2+\beta^2+\alpha\beta = \frac{21}{4}$, find the value of k.

11) If α and β are the zeroes of the quadratic polynomial $f(x) = 2x^2-5x+7$, find a polynomial whose zeroes are $2\alpha+3\beta$ and $3\alpha+2\beta$.

Answers (HW-3)

1) (i) Let the zeroes be α and β.
Then, $\alpha+\beta = -2\sqrt{3}$ and $\alpha\beta = -9$.
$\therefore$ The required polynomial is
$k[x^2-(\alpha+\beta)x+\alpha\beta]$; where k is any non-zero real number.
$= k[x^2+2\sqrt{3}x-9]$
$= \underline{\underline{x^2+2\sqrt{3}x-9}}$; where $k=1$

To find the zeroes :-

S	P
$2\sqrt{3}$	-9
	$3\sqrt{3}, -1\sqrt{3}$

$x^2+2\sqrt{3}x-9$
$= x^2+3\sqrt{3}x-\sqrt{3}x-9$
$= x(x+3\sqrt{3})-\sqrt{3}(x+3\sqrt{3})$
$= (x-\sqrt{3})(x+3\sqrt{3})$
$\therefore$ The zeroes are $\sqrt{3}$ and $-3\sqrt{3}$ //

(ii) Let α and β be the zeroes.

Then, $\alpha+\beta = -\frac{3}{2\sqrt{5}}$ and $\alpha\beta = -\frac{1}{2}$

$\therefore$ The required polynomial is

$k[x^2 - (\alpha+\beta)x + \alpha\beta]$; where k is any non-zero real number.

$= k\left[x^2 + \frac{3x}{2\sqrt{5}} - \frac{1}{2}\right]$

$= \frac{k}{2\sqrt{5}}\left[2\sqrt{5}x^2 + 3x - \sqrt{5}\right]$

$= \underline{\underline{2\sqrt{5}x^2 + 3x - \sqrt{5}}}$; where $k = 2\sqrt{5}$

To find the zeroes:-

S	P
3	−10
	−2, 5

$2\sqrt{5}x^2 + 3x - \sqrt{5}$

$= 2\sqrt{5}x^2 + 5x - 2x - \sqrt{5}$

$= \sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5})$

$= (\sqrt{5}x - 1)(2x + \sqrt{5})$

$\therefore$ The zeroes are $\frac{1}{\sqrt{5}}$ and $-\frac{\sqrt{5}}{2}$ //

2) (i) $2x^2 + \frac{7}{2}x + \frac{3}{4} = \frac{8x^2+14x+3}{4}$

Let the given polynomial $p(x) = 8x^2+14x+3$ be of the form ax^2+bx+c; where $a=8$, $b=14$ and $c=3$ and let α, β be the zeroes.

$p(x) = 8x^2+14x+3$ S   P

$= 8x^2+12x+2x+3$ 14 $24 < \begin{smallmatrix} 12 \\ 2 \end{smallmatrix}$

$= 4x(2x+3)+1(2x+3)$

$= (4x+1)(2x+3)$

$\therefore$ The zeroes are $-\frac{1}{4}$ and $-\frac{3}{2}$

For verification:-

let $\alpha = -\frac{1}{4}$ and $\beta = -\frac{3}{2}$

Sum of zeroes $= \alpha+\beta = -\frac{1}{4} - \frac{3}{2} = -\frac{7}{4} = \frac{-14}{8}$

$= -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of zeroes $= \alpha\beta = -\frac{1}{4} \times -\frac{3}{2} = \frac{3}{8} = \frac{c}{a}$

$= \frac{\text{constant term}}{\text{coefficient of } x^2}$

Hence verified

(ii) $4x^2+5\sqrt{2}x-3$

Let the given polynomial $p(x)=4x^2+5\sqrt{2}x-3$ be of the form ax^2+bx+c; where $a=4$, $b=5\sqrt{2}$ and $c=-3$ and α, β be the zeroes.

$$p(x)=4x^2+5\sqrt{2}x-3$$

$$=4x^2+6\sqrt{2}x-\sqrt{2}x-3$$

S P

$5\sqrt{2}$ $\quad -12 < \begin{matrix} 6\sqrt{2} \\ -1\sqrt{2} \end{matrix}$

$$=2\sqrt{2}x(\sqrt{2}x+3)-1(\sqrt{2}x+3)$$

$$=(2\sqrt{2}x-1)(\sqrt{2}x+3)$$

$\therefore$ The zeroes are $\frac{1}{2\sqrt{2}}$ and $-\frac{3}{\sqrt{2}}$

For verification:-

let $\alpha=\frac{1}{2\sqrt{2}}$ and $\beta=\frac{-3}{\sqrt{2}}$

Sum of zeroes $=\alpha+\beta=\frac{1}{2\sqrt{2}}-\frac{3\times 2}{\sqrt{2}\times 2}=\frac{-5\times\sqrt{2}}{2\sqrt{2}\times\sqrt{2}}=\frac{-5\sqrt{2}}{4}$

$$=-\frac{b}{a}=-\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Product of zeroes $=\alpha\beta=\frac{1}{2\sqrt{2}}\times\frac{-3}{\sqrt{2}}=-\frac{3}{4}=\frac{c}{a}$

$$=\frac{\text{constant term}}{\text{coefficient of } x^2}$$

Hence verified

(iii) Let $p(s) = 2s^2 - (1+2\sqrt{2})s + \sqrt{2}$ be of the form $ax^2 + bx + c$; where $a = 2$,

$b = -(1+2\sqrt{2})$

$c = \sqrt{2}$

and α, β be the zeroes.

$$p(s) = 2s^2 - s - 2\sqrt{2}s + \sqrt{2}$$
$$= s(2s-1) - \sqrt{2}(2s-1)$$
$$= (s-\sqrt{2})(2s-1)$$

$\therefore$ the zeroes are $\sqrt{2}$ and $\frac{1}{2}$.

For verification:-

let $\alpha = \sqrt{2}$ and $\beta = \frac{1}{2}$

Sum of zeroes $= \alpha + \beta = \sqrt{2} + \frac{1}{2} = \frac{2\sqrt{2}+1}{2} = \frac{-b}{a}$

$= -\dfrac{\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of zeroes $= \alpha \times \beta = \sqrt{2} \times \frac{1}{2} = \frac{\sqrt{2}}{2} = \frac{c}{a}$

$= \dfrac{\text{constant term}}{\text{coefficient of } x^2}$

Hence verified

3) a and c are positive and b is negative (b)

4) False, the polynomial has two equal zeroes and hence it can be a quadratic polynomial.

5) Let $\sqrt{p}+\sqrt{q}$ is a rational number.

Then, $\sqrt{p}+\sqrt{q} = \frac{a}{b}$; where a and b are co-prime integers.

Squaring on both sides,

$$(\sqrt{p}+\sqrt{q})^2 = \frac{a^2}{b^2}$$

$$\Rightarrow p+q+2\sqrt{pq} = \frac{a^2}{b^2}$$

$$\Rightarrow 2\sqrt{pq} = \frac{a^2}{b^2} - p - q$$

$$\Rightarrow \sqrt{pq} = \frac{a^2 - pb^2 - qb^2}{2b^2}$$

Since a and b are integers, $\frac{a^2-pb^2-qb^2}{2b^2}$ is rational. Thus $\sqrt{pq}$ is also rational. But this contradicts the fact that $\sqrt{pq}$ is irrational since p and q are prime numbers. This contradiction arises due to our wrong assumption that $\sqrt{p}+\sqrt{q}$ is rational.

Hence $\sqrt{p}+\sqrt{q}$ is an irrational number.

6) no. of pencils contained in 1 packet = 24
no. of crayons contained in 1 packet = 32

$24 = 2^3 \times 3$
$32 = 2^5$
$LCM(24, 32) = 2^5 \times 3 = 96$ pencils and 96 crayons need to buy.

$\therefore$ No. of packets for pencils $= \frac{96}{24} = 4$
No. of packets for crayons $= \frac{96}{32} = 3$

7) Let $p(x) = x^2 - 6x + a$ be of the form $Ax^2 + Bx + C$; where $A = 1$, $B = -6$, $C = a$.

$\alpha + \beta = \frac{-B}{A} = 6$
$\alpha\beta = \frac{C}{A} = a$
Given, $3\alpha + 2\beta = 20$
$\Rightarrow \alpha + 2\alpha + 2\beta = 20$
$\Rightarrow \alpha + 2(\alpha + \beta) = 20$
$\Rightarrow \alpha + 2 \times 6 = 20$
$\alpha = 20 - 12 = 8$
$\beta = 6 - 8 = -2$

$\therefore a = \alpha\beta = \underline{\underline{-16}}$

8) Let $p(x) = x^2 + (a+1)x + b$ be of the form $Ax^2 + Bx + C$; where $A = 1$, $B = a+1$ and $C = b$ and α, β be the zeroes.

$\alpha + \beta = -\frac{B}{A}$

$\Rightarrow 2 - 3 = -(a+1)$

$\Rightarrow -1 = -(a+1)$

$\Rightarrow 1 = a + 1$

$\therefore a = 0$

$\alpha\beta = \frac{C}{A}$

$\Rightarrow -6 = b$

$\therefore b = -6$

9) Let $p(t) = t^2 - pt - p - c$ be of the form $Ax^2 + Bx + C$; where $A = 1$, $B = -p$, $C = -p - c$

$\alpha + \beta = -\frac{B}{A} = p$

$\alpha\beta = \frac{C}{A} = -p - c$

LHS, $(\alpha+1)(\beta+1) = \alpha\beta + (\alpha+\beta) + 1$

$= -\cancel{p} - c + \cancel{p} + 1$

$= \underline{\underline{1 - c}}$, RHS

10) $a = 2, b = 5, c = k$

$\alpha + \beta = -\dfrac{b}{a} = -\dfrac{5}{2}$

$\alpha\beta = \dfrac{c}{a} = \dfrac{k}{2}$

Given, $\alpha^2 + \beta^2 + \alpha\beta = \dfrac{21}{4}$

$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta + \alpha\beta = \dfrac{21}{4}$

$\Rightarrow (\alpha+\beta)^2 - \alpha\beta = \dfrac{21}{4}$

$\Rightarrow \dfrac{25}{4} - \dfrac{k}{2} = \dfrac{21}{4}$

$\Rightarrow \dfrac{k}{2} = \dfrac{25}{4} - \dfrac{21}{4}$

$\Rightarrow \dfrac{k}{2} = \dfrac{\cancel{4}}{\cancel{4}}$

$\Rightarrow \boxed{k = 2}$

11) $a = 2, b = -5, c = 7$

$\alpha + \beta = -\dfrac{b}{a} = \dfrac{5}{2}$

$\alpha\beta = \dfrac{c}{a} = \dfrac{7}{2}$

For the new polynomial :-

Sum of zeroes $= 2\alpha + 3\beta + 3\alpha + 2\beta = 5\alpha + 5\beta = 5(\alpha+\beta)$

$= 5 \times \dfrac{5}{2} = \dfrac{25}{2}$

Product of zeroes $= (2\alpha + 3\beta)(3\alpha + 2\beta)$

$= 6\alpha^2 + 4\alpha\beta + 9\alpha\beta + 6\beta^2$

$= 6(\alpha^2 + \beta^2) + 13\alpha\beta$

$= 6[(\alpha+\beta)^2 - 2\alpha\beta] + 13\alpha\beta$

$= 6\left[\dfrac{25}{4} - \cancel{2} \times \dfrac{7}{\cancel{2}}\right] + 13 \times \dfrac{7}{2}$

$$= 6\left[\frac{25}{4} - 7\right] + \frac{91}{2} = \overset{3}{\cancel{6}}\left[-\frac{3}{\underset{2}{\cancel{4}}}\right] + \frac{91}{2}$$

$$= \frac{-9+91}{2} = \frac{82}{2} = 41$$

$\therefore$ The required polynomial is

$k[x^2 - (\text{sum of zeroes})x + \text{product of zeroes}]$;

where k is any non-zero real number.

$$= k\left[x^2 - \frac{25}{2}x + 41\right]$$

$$= \frac{k}{2}\left[2x^2 - 25x + 82\right]$$

$$= \underline{\underline{2x^2 - 25x + 82}}\text{; where } k = 2$$