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TEST - 3 :-

1. Let us assume that $\sqrt{5}$ is rational.

$\sqrt{5} = \frac{a}{b}$; where a and b are co-prime integers $b \neq 0$

Squaring on both sides, $5 = \frac{a^2}{b^2}$

$\Rightarrow 5b^2 = a^2 \longrightarrow (1)$

$\Rightarrow$ 5 divides a^2

$\Rightarrow$ 5 divides a

Let $a = 5c$; where c is any integer

From eq : (1), $5b^2 = (5c)^2$

$\Rightarrow \cancel{5}b^2 = \cancel{25}c^2$

$\Rightarrow b^2 = 5c^2$

$\Rightarrow$ 5 divides b^2

$\Rightarrow$ 5 divides b

Thus, 5 is a factor of both a and b.

But this contradicts the fact that a and b are co-prime (i.e, HCF = 1).

This contradiction arises due to our wrong assumption that $\sqrt{5}$ is rational.

Hence, $\sqrt{5}$ is irrational.

Now, let us assume that $7 + 3\sqrt{5}$ is rational.

$7 + 3\sqrt{5} = \frac{p}{q}$, where p and q are co-prime integers $q \neq 0$

$\Rightarrow 3\sqrt{5} = \frac{p - 7q}{q}$

$\Rightarrow \sqrt{5} = \frac{p - 7q}{3q}$

p, q are integers)

Since $\frac{p-7q}{3q}$ is a rational number, $\sqrt{5}$ is also rational.

This contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction arises due to our wrong assumption that $7+3\sqrt{5}$ is rational.

Hence, $7+3\sqrt{5}$ is irrational.