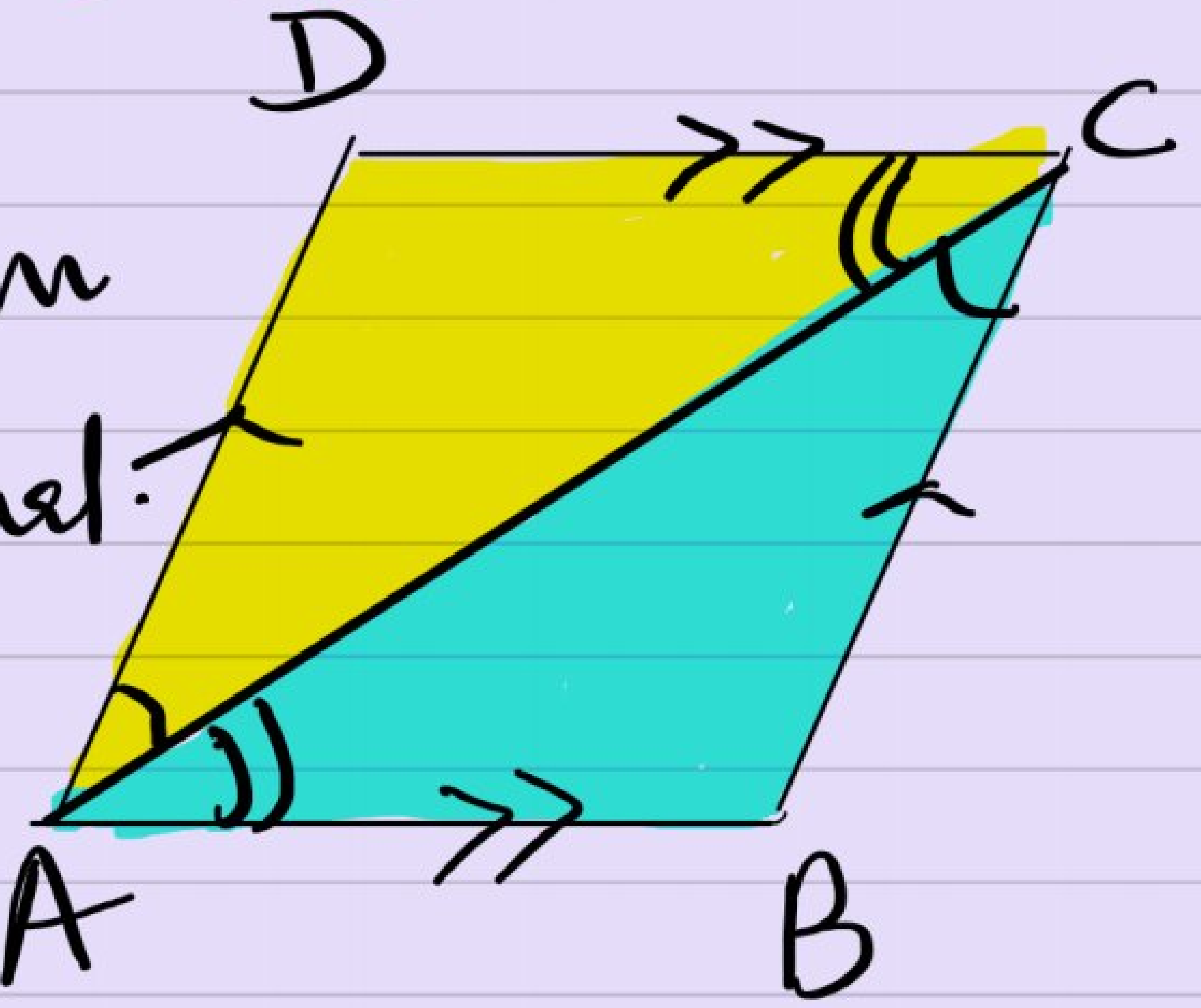
8 Quadrilaterals

answers

1) Given: in parallelogram ABCD, AC is a diagonal.



To prove:

$\triangle ADC \cong \triangle CBA$

Proof:- In $\triangle ADC$ and $\triangle CBA$,

$\angle DAC = \angle BCA$ [alternate interior angles; $AD \parallel BC$]

$AC = AC$ (common side)

$\angle DCA = \angle BAC$ [alternate interior angles; $AB \parallel DC$]

$\therefore \triangle ADC \cong \triangle CBA$ [ASA congruency]

Hence, a diagonal of a parallelogram divides it into two congruent $\triangle$s.

2)

Given:- in rectangle ABCD, $\angle A = 90°$

To prove:-

$\angle B = \angle C = \angle D = 90°$

Proof:- we know that rectangle is a parallelogram with one angle measures 90°.

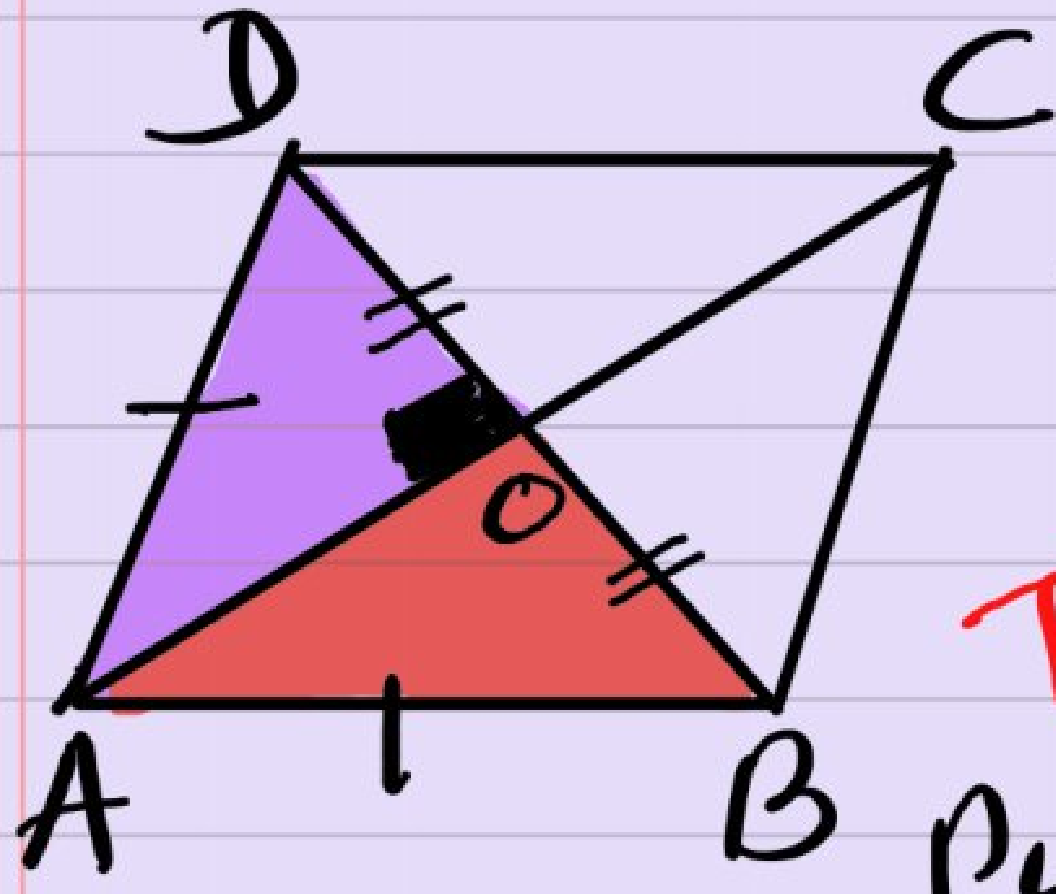
Since $AB \parallel DC$ and AD is the transversal,

$\angle A + \angle D = 180°$ (co-interior angles)

$\Rightarrow \angle D = 180^\circ - \angle A$
$= 180^\circ - 90^\circ$
$\angle D = 90^\circ$

Thus, $\angle A = \angle C = 90^\circ$ (opposite angles are equal)
Also, $\angle B = \angle D = 90^\circ$
$\therefore \angle B = \angle C = \angle D = 90^\circ$ - Hence proved

3)

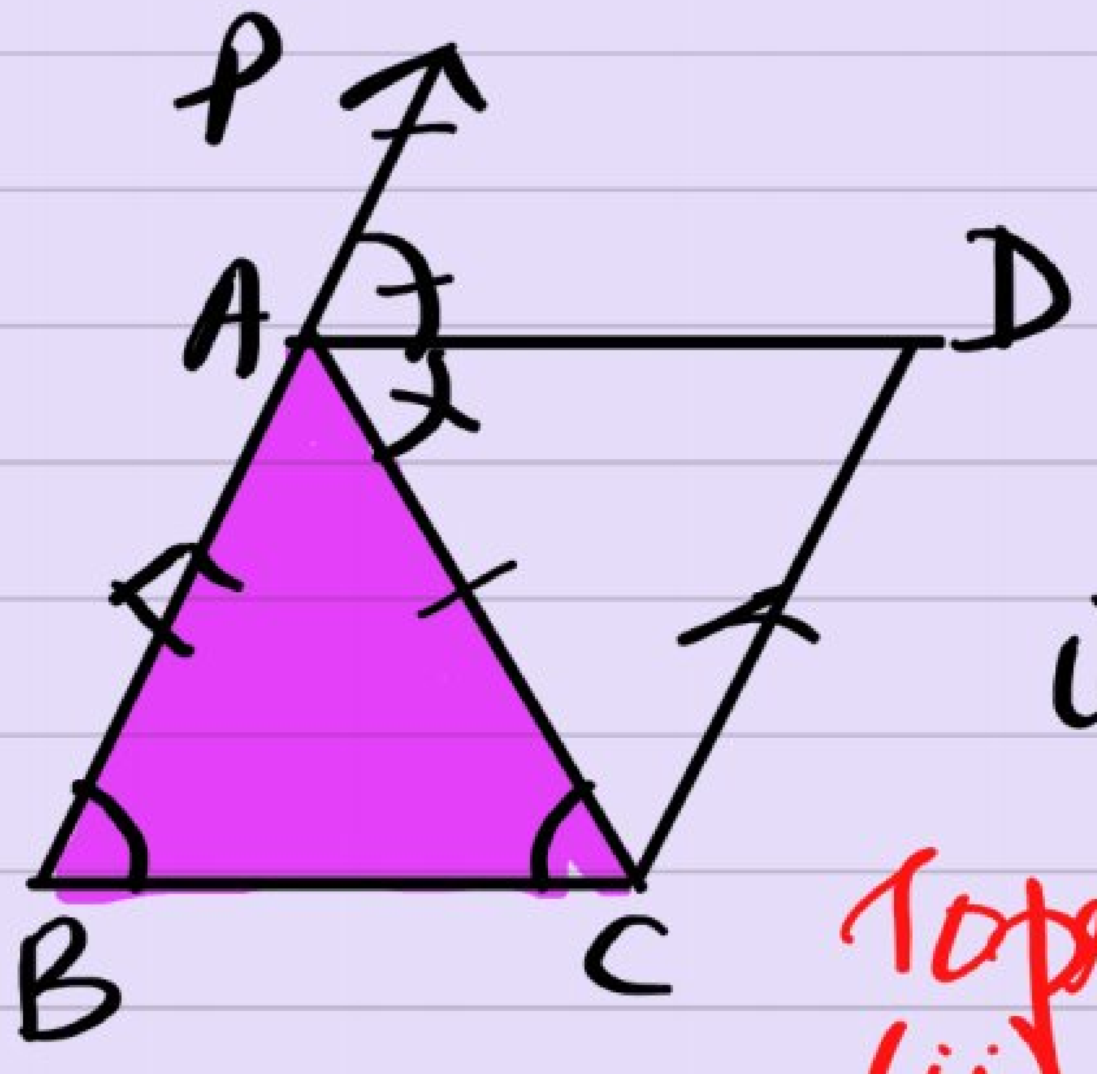


Given :- in rhombus ABCD, AC and BD are the diagonals.
To prove :- $BD \perp AC$

Proof :-
In $\triangle AOD$ and $\triangle AOB$,
$AD = AB$ (sides of a rhombus)
$OD = OB$ (diagonals bisect each other)
$OA = OA$ (common side)
$\therefore \triangle AOD \cong \triangle AOB$ (SSS congruency)

Thus, $\angle DOA = \angle BOA$ (by cpct)
These angles form a linear pair and are equal as well.
$\therefore \angle DOA = \angle BOA = 90^\circ$
$\Rightarrow BD \perp AC$
Hence, the diagonals of a rhombus are perpendicular to each other.

4)



Given :- in isosceles $\triangle ABC$, $AB = AC$.
AD bisects $\angle PAC$
ie, $\angle PAD = \angle DAC \rightarrow (1)$
$CD \parallel AB$

To prove :- (i) $DAC = \angle BCA$
(ii) ABCD is a parallelogram

Proof :- Since $AB = AC$,
$\angle ABC = \angle BCA \rightarrow (2)$

(i) Using exterior angle property in $\triangle ABC$,

$\angle ABC + \angle ACB = \angle PAC$

$\Rightarrow \angle ABC + \angle BCA = \angle PAD + \angle DAC$

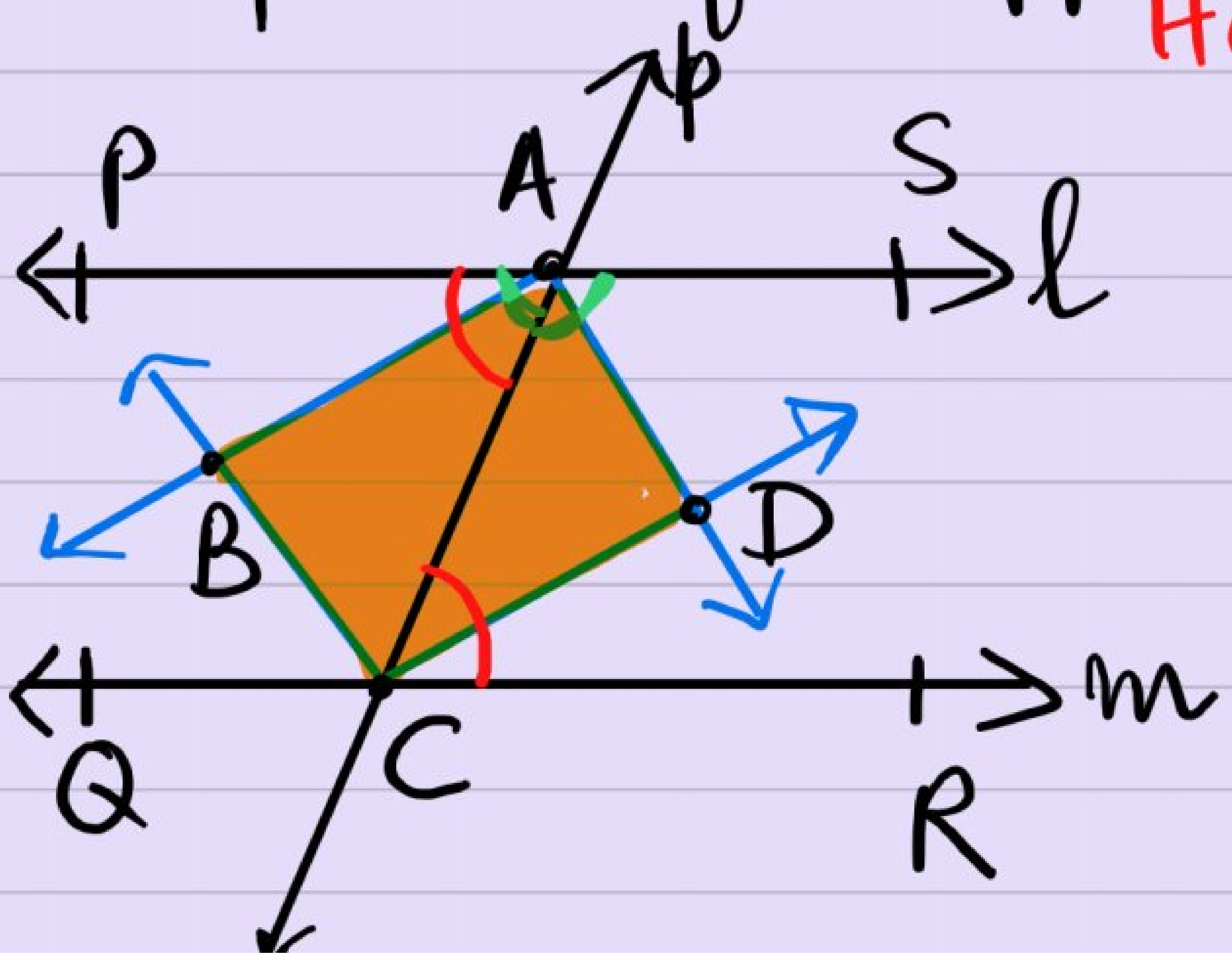
$\Rightarrow \cancel{2}\angle BCA = \cancel{2}\angle DAC$ [from eq:s (1) & (2)]

$\therefore \boxed{\angle BCA = \angle DAC}$

(ii) These angles form a pair of alternate interior angles only when $BC \parallel AD$
Also, $AB \parallel CD$ (given)

Thus, ABCD is a parallelogram with both pairs of opposite sides parallel.
Hence proved

5)



Given :- $l \parallel m$ and p is the transversal.
AB bisects $\angle PAC$
BC bisects $\angle ACQ$
CD bisects $\angle ACR$
AD bisects $\angle SAC$
To prove :- ABCD is a rectangle.
Proof :-
since $l \parallel m$ and p is the transversal,
$\angle PAC = \angle ACR$ [alternate interior angles]

$\Rightarrow \frac{1}{2} \angle PAC = \frac{1}{2} \angle ACR$

$\Rightarrow \angle BAC = \angle ACD$

These angles form a pair of alternate interior angles only when $AB \parallel CD$.
Similarly, we can prove that $AD \parallel BC$.

Thus, quadrilateral ABCD is a parallelogram with both pairs of opposite sides parallel.

Also, $\angle PAC + \angle SAC = 180^\circ$ (linear pair)

$$\frac{1}{2} \angle PAC + \frac{1}{2} \angle SAC = \frac{1}{2} \times 180^\circ$$

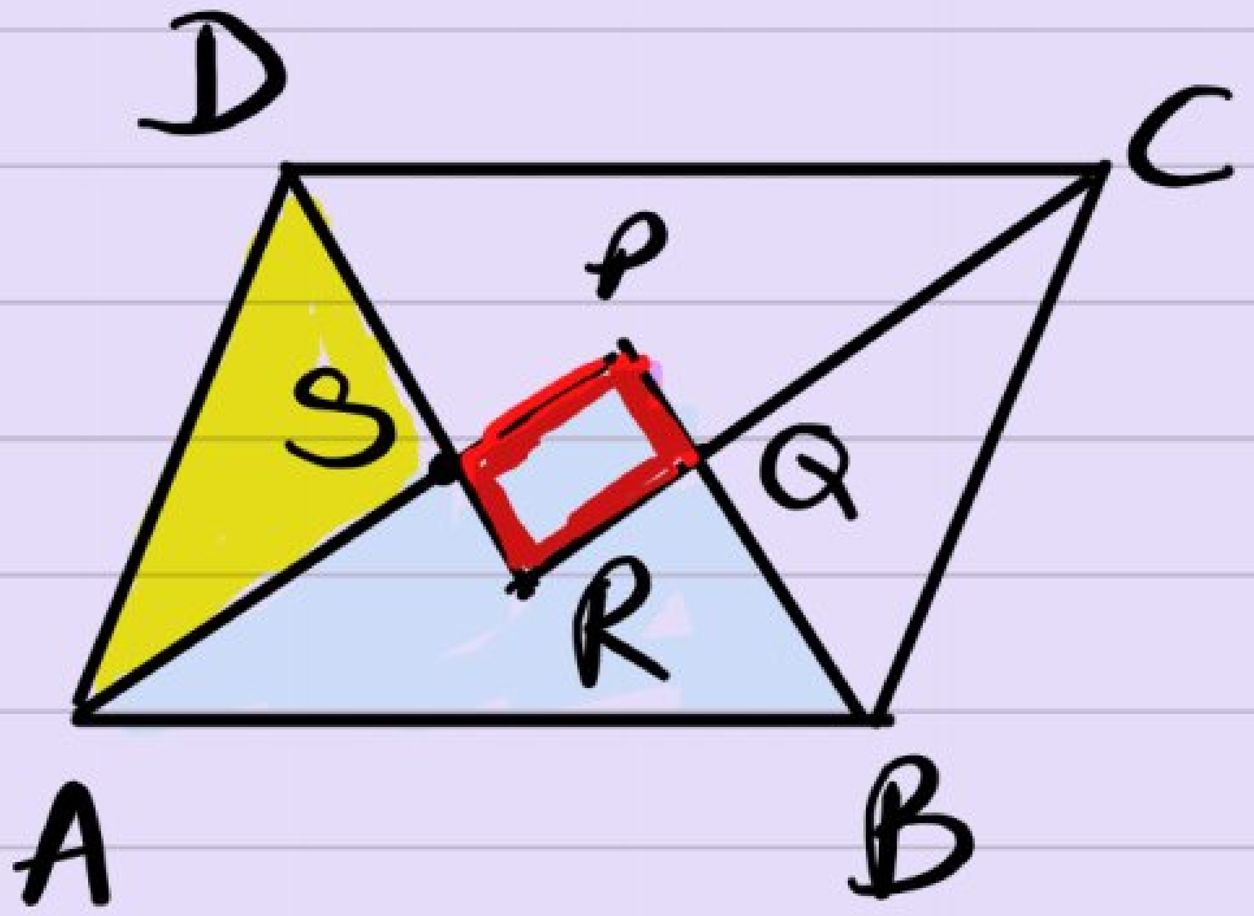
$$\Rightarrow \angle BAC + \angle DAC = 90^\circ$$

$$\Rightarrow \angle BAD = 90^\circ$$

Thus, parallelogram ABCD is a rectangle with each angle measures 90°.

Hence Proved

6)



Given:- in parallelogram ABCD,

AP bisects $\angle A$
BP bisects $\angle B$
CR bisects $\angle C$
DR bisects $\angle D$

To prove:- PQRS is a rectangle

Proof:-

$\angle A + \angle D = 180^\circ$ (adjacent angles of a ||gm)

$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle D = \frac{1}{2} \times 180^\circ$

$\Rightarrow \angle SAD + \angle SDA = 90^\circ \rightarrow (1)$

Using angle sum property in ΔDSA,

$(\angle SDA + \angle SAD) + \angle DSA = 180^\circ$

$90^\circ + \angle DSA = 180^\circ$ [from eq:(1)]

$\therefore \angle DSA = 90^\circ$

Now, $\angle DSA = \angle PSR = 90^\circ$ (VOA)

Similarly, we can prove $\angle PQR = 90^\circ$

Also, $\angle A + \angle B = 180^\circ$ (adjacent angles of a ||gm)

$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B = \frac{1}{2} \times 180^\circ$

$\Rightarrow \angle PAB + \angle PBA = 90^\circ \longrightarrow (2)$

Using angle sum property in $\triangle APB$,

$(\angle PAB + \angle PBA) + \angle APB = 180^\circ$

$\Rightarrow 90^\circ + \angle APB = 180^\circ$

$\therefore \angle APB = 90^\circ \Rightarrow \angle SPQ = 90^\circ$

Similarly, we can prove $\angle DRC = 90^\circ$

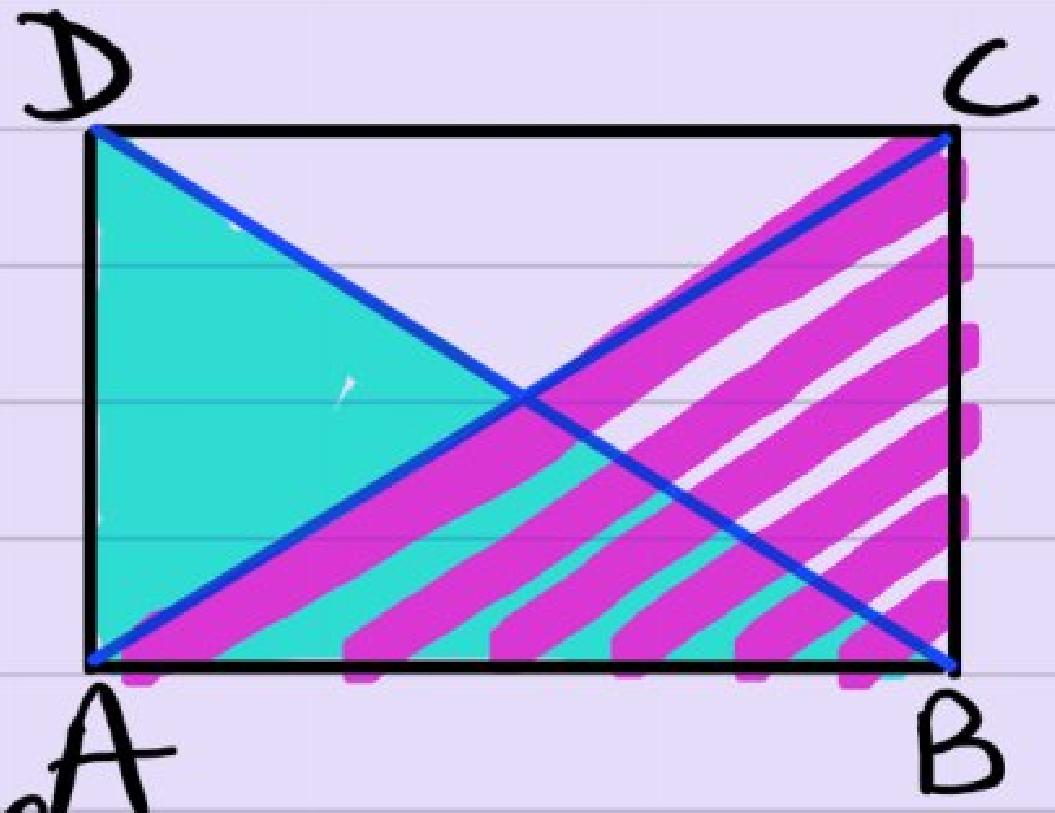
$\Rightarrow \angle SRQ = 90^\circ$

Thus, in quadrilateral PQRS,

$\angle PSR = \angle SRQ = \angle RQP = \angle SPQ = 90^\circ$

$\Rightarrow$ PQRS is a rectangle with each angle measures 90°.

Hence proved

7) 

Given:- in parallelogram ABCD, diagonal AC = diagonal BD.

To prove: ABCD is a rectangle

Proof:-

In $\triangle DAB$ and $\triangle CBA$,

$AD = BC$ (opposite sides of a parallelogram)

$AB = AB$ (common side)

$BD = AC$ (given)

$\therefore \triangle DAB \cong \triangle CBA$ (SSS congruency)

Thus $\angle DAB = \angle CBA$ (by cpct) $\longrightarrow (1)$

But, $\angle DAB + \angle CBA = 180^\circ$ (co-interior angles) $\rightarrow (2)$

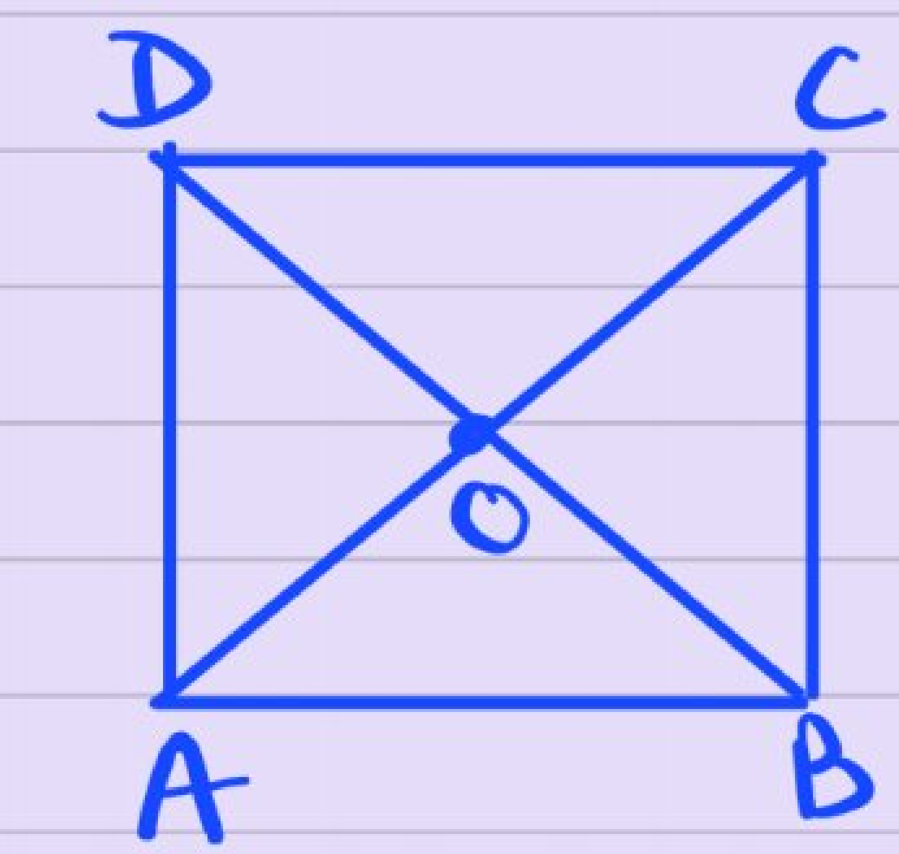
From eq: (1) and (2),

Thus, $\angle DAB = \angle CBA = 90° \Rightarrow \angle A = \angle B = 90°$

$\Rightarrow$ ||gm ABCD is a rectangle with each angle measures 90°

Hence Proved

8)



Given:- in square ABCD, AC and BD are the diagonals and intersect at O.

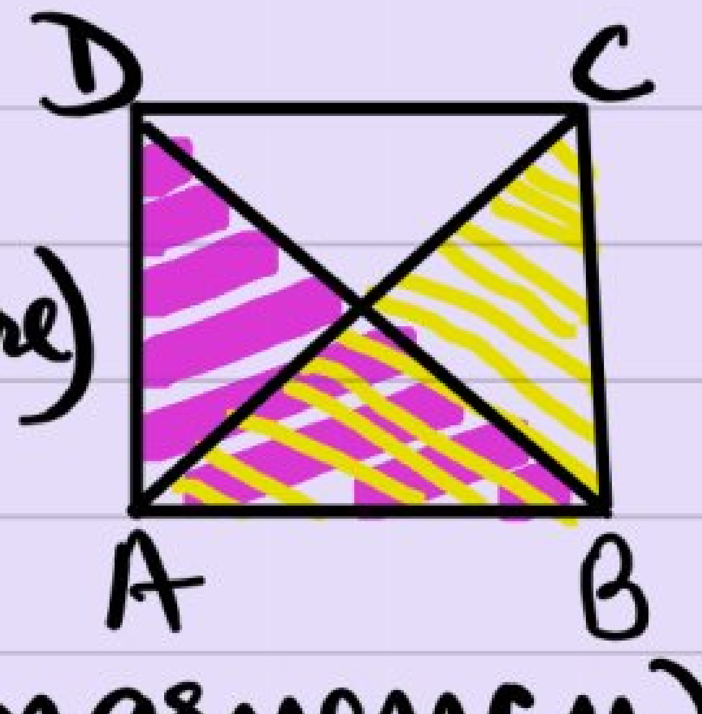
To prove:- $AC = BD$

$OA = OC$

$OB = OD$

$\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$

Proof:-



In $\triangle DAB$ and $\triangle CBA$,

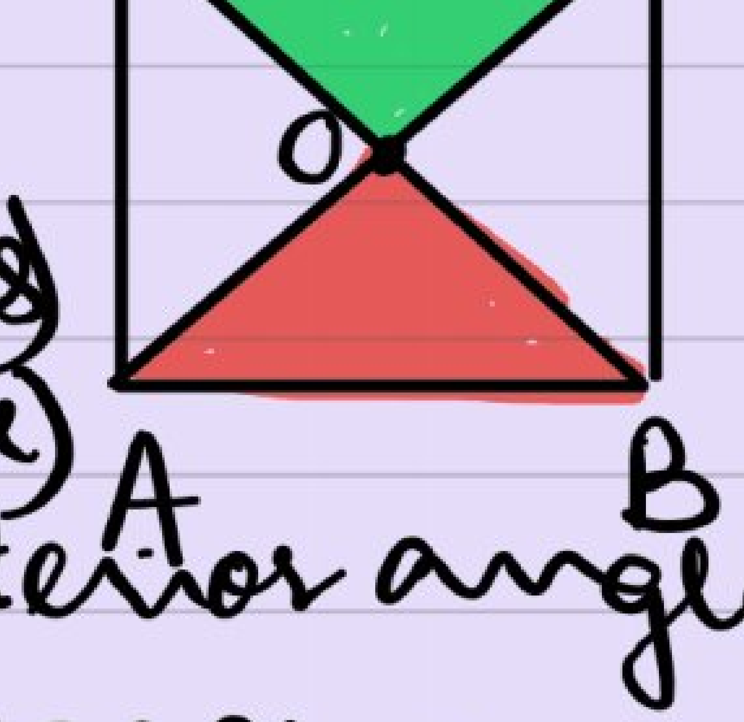
$AD = BC$ (sides of a square)

$\angle DAB = \angle CBA$ (each 90°)

$AB = AB$ (common side)

$\therefore \triangle DAB \cong \triangle CBA$ (SAS congruency)

Thus, $BD = AC$ (by cpct)



In $\triangle AOB$ and $\triangle COD$,

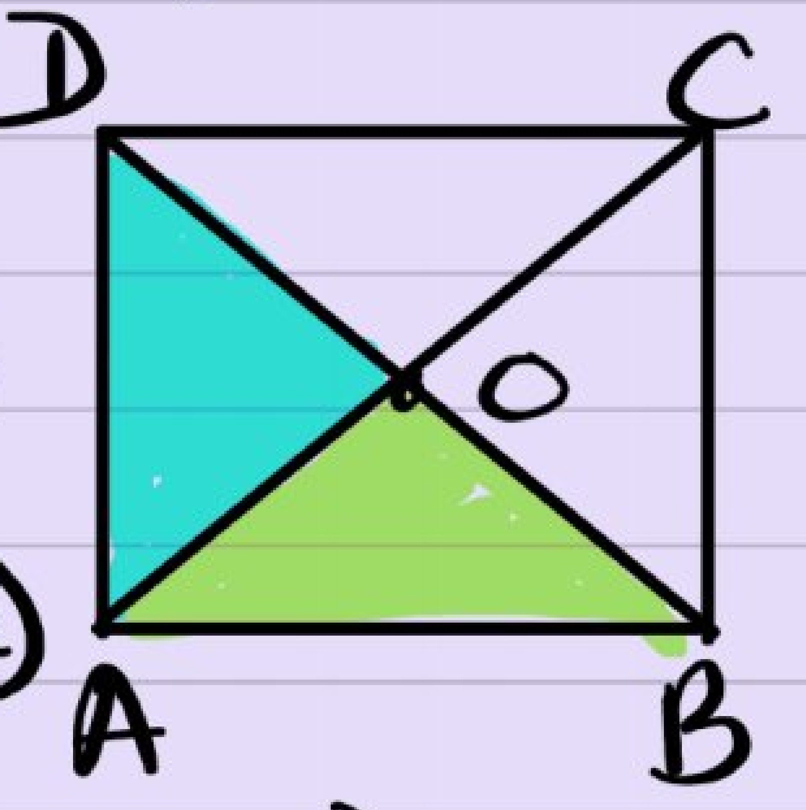
$\angle OAB = \angle OCD$ (alternate interior angles)

$AB = CD$ (sides of a square)

$\angle OBA = \angle OCD$ (alternate interior angles)

$\therefore \triangle AOB \cong \triangle COD$ (ASA congruency)

Thus, $\left.\begin{array}{l} OA = OC \\ OB = OD \end{array}\right\}$ (by cpct)



In $\triangle DOA$ and $\triangle BOA$,

$DO = BO$ (proved above)

$OA = OA$ (common side)

$DA = AB$ (sides of a square)

$\therefore \triangle DOA \cong \triangle BOA$

(SSS congruency)

Thus, $\angle DOA = \angle BOA$ (by cpct)

But these angles form a linear pair and are equal.

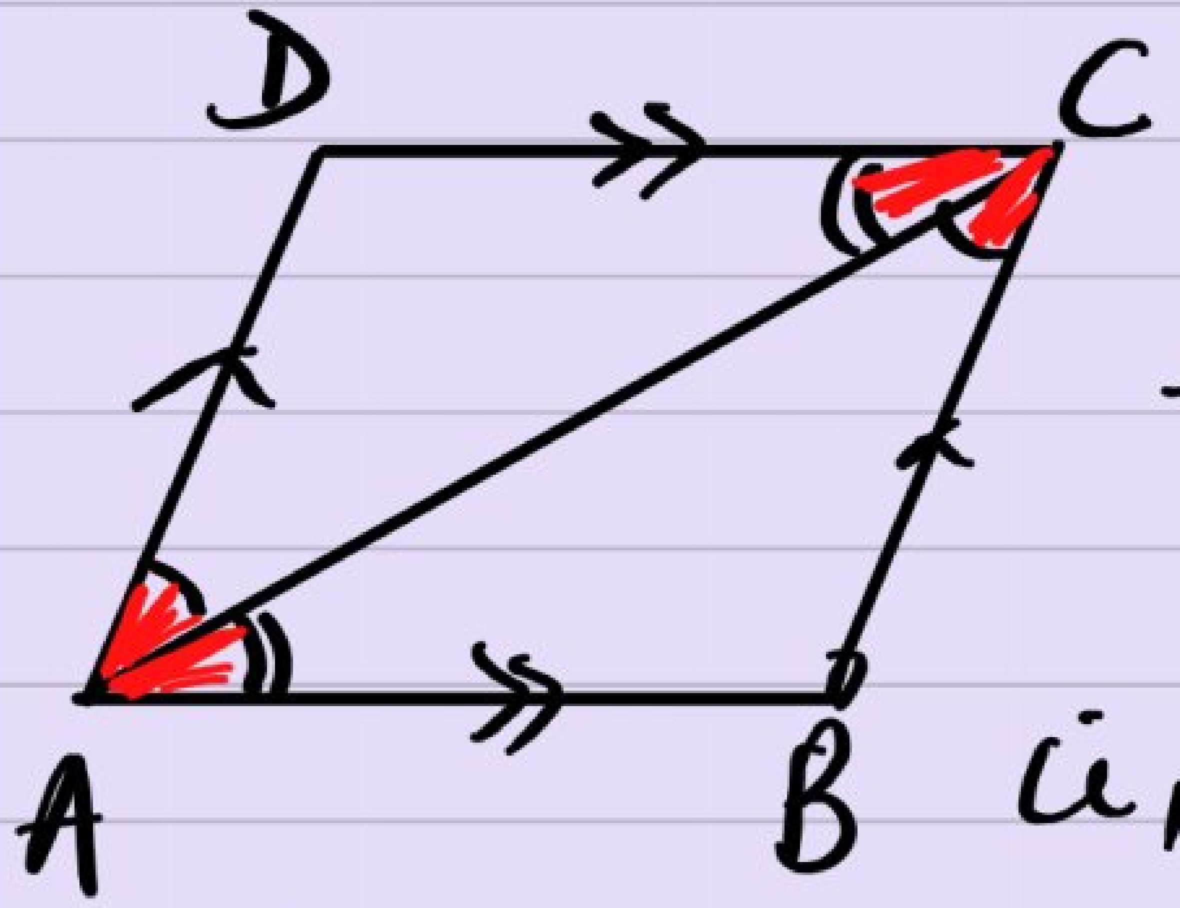
$\therefore \angle DOA = \angle BOA = 180°/2 = 90°$

Similarly, $\angle BOC = \angle COD = 90°$

Thus, $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$

Hence Proved

9)



Given :- in parallelogram diagonal AC bisects $\angle A$

i.e, $\angle DAC = \angle BAC \longrightarrow (1)$

To prove :- (i) AC bisects $\angle C$

(ii) ABCD is a rhombus

Proof :- (i) $\angle DAC = \angle BCA$ (alternate interior angles, $AD \parallel BC$) $\rightarrow (2)$

$\angle BAC = \angle DCA$ (alternate interior angles, $AB \parallel DC$) $\rightarrow (3)$

But $\angle DAC = \angle BAC$ (from eq : (1))

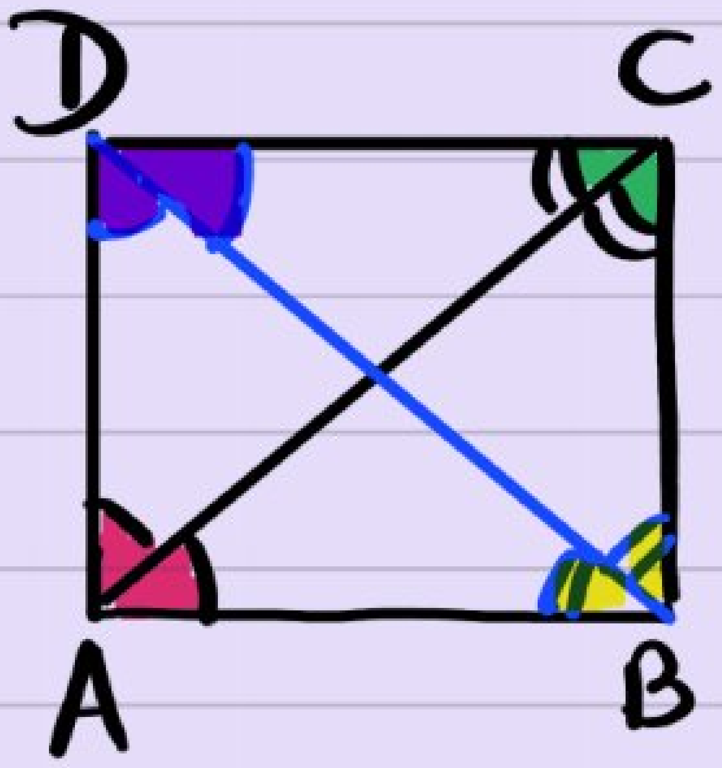
Thus, $\angle BCA = \angle DCA \longrightarrow (4)$

$\Rightarrow$ AC bisects $\angle C$

(ii) In $\triangle DAC$ and $\triangle BAC$,

$\angle DAC = \angle BAC$ (given)
$AC = AC$ (common side)
$\angle DCA = \angle BCA$ (from eq :(ii))
$\therefore \triangle DAC \cong \triangle BAC$ (ASA congruency)
Thus, $AD = AB$ (by cpct)
$\therefore$ parallelogram ABCD is a rhombus with adjacent sides equal.
Hence proved

10)



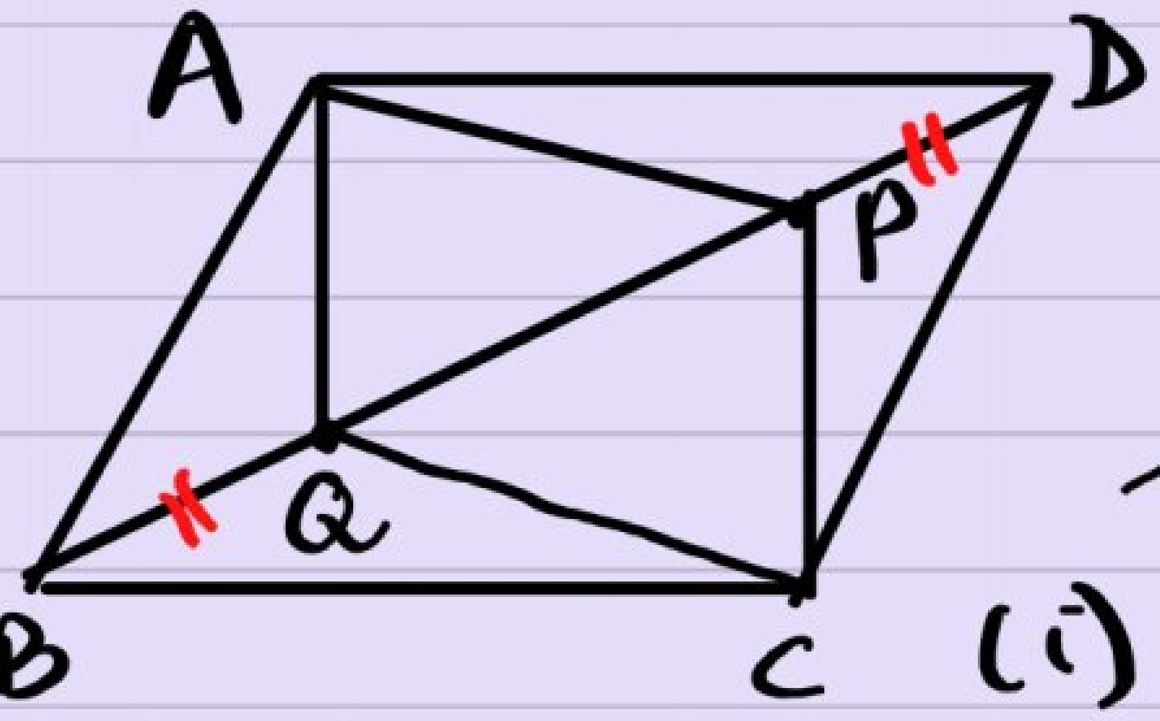
Given: in rectangle ABCD, diagonal AC bisects $\angle A$
ie, $\angle DAC = \angle BAC \rightarrow (1)$
diagonal AC bisects $\angle C$
ie, $\angle DCA = \angle BCA \rightarrow (2)$

To prove:- (i) ABCD is a square
(ii) diagonal BD bisects $\angle B$ as well as $\angle D$.

Proof: (i) In $\triangle DAC$ and $\triangle BAC$,
$\angle DAC = \angle BAC$ (given)
$AC = AC$ (common side)
$\angle DCA = \angle BCA$ (given)
$\therefore \triangle DAC \cong \triangle BAC$ (ASA congruency)
Thus, $AB = AD$ (by cpct)
$\therefore$ rectangle ABCD is a square with adjacent sides equal.

(ii) In $\triangle DAB$ and $\triangle DCB$,
$DA = DC$ (sides of a square)
$\angle DAB = \angle DCB$ (each 90°)
$AB = BC$ (sides of a square)
$\therefore \triangle DAB \cong \triangle DCB$ (SAS congruency)
Thus, $\angle ADB = \angle CDB$ and $\angle ABD = \angle CBD$ } by cpct
$\therefore$ diagonal BD bisects $\angle B$ and $\angle D$. Hence Proved

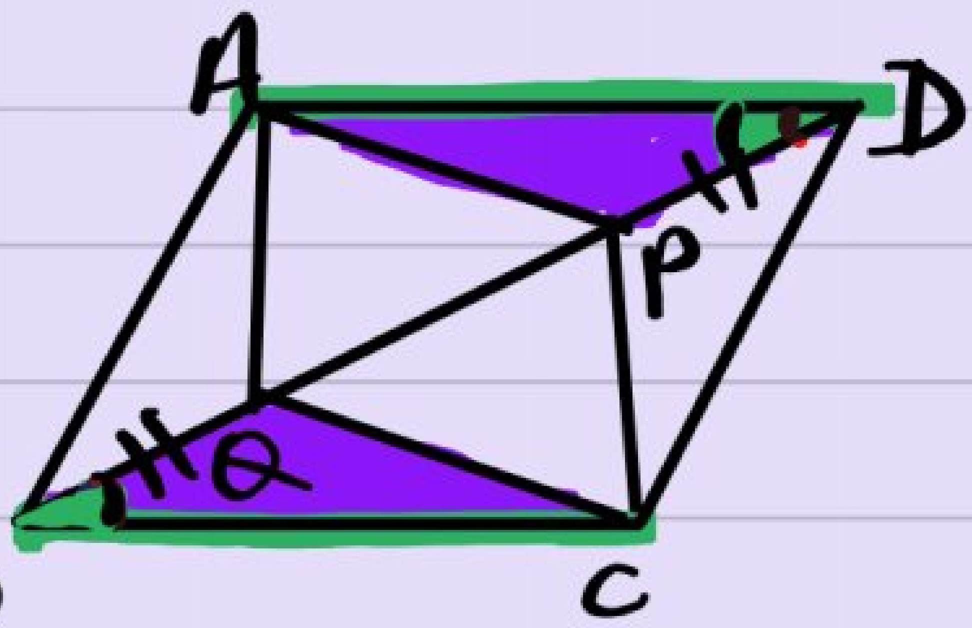
11)



Given:- in parallelogram ABCD,
$DP = BQ$ → (1)

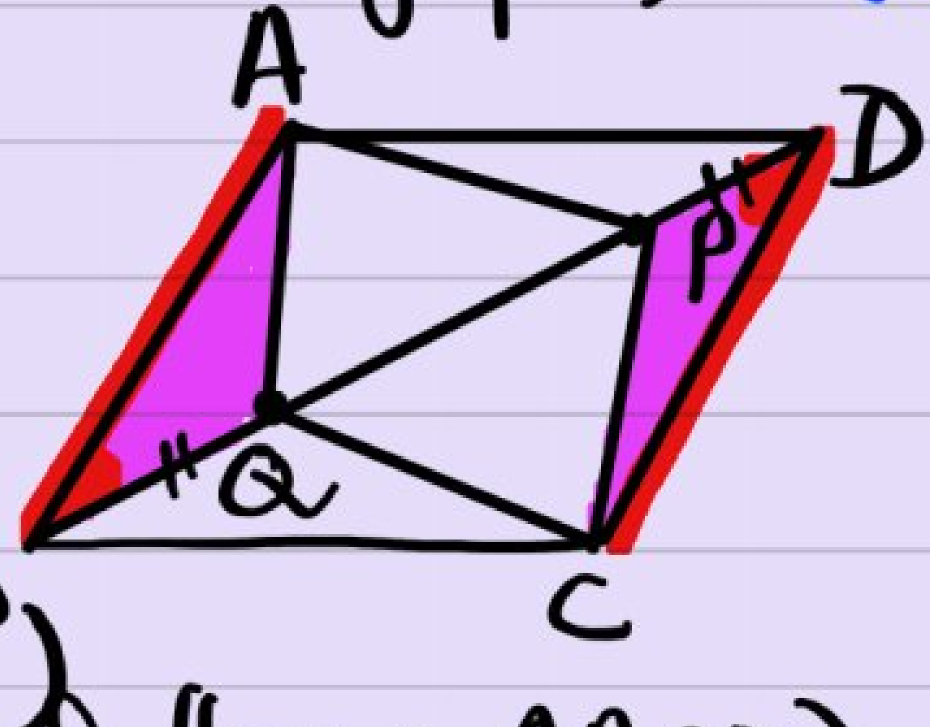
To prove:-
(i) $\triangle APD \cong \triangle CQB$
(ii) $AP = CQ$
(iii) $\triangle AQB \cong \triangle CPD$
(iv) $AQ = CP$
(v) APCQ is a parallelogram.

Proof :-



(i) In $\triangle APD$ and $\triangle CQB$,
$PD = QB$ (given)
$\angle PDA = \angle QBC$ (alternate interior angles, $AD \parallel BC$)
$AD = BC$ (opposite sides of ||gm ABCD)
$\therefore \triangle APD \cong \triangle CQB$ (SAS congruency)

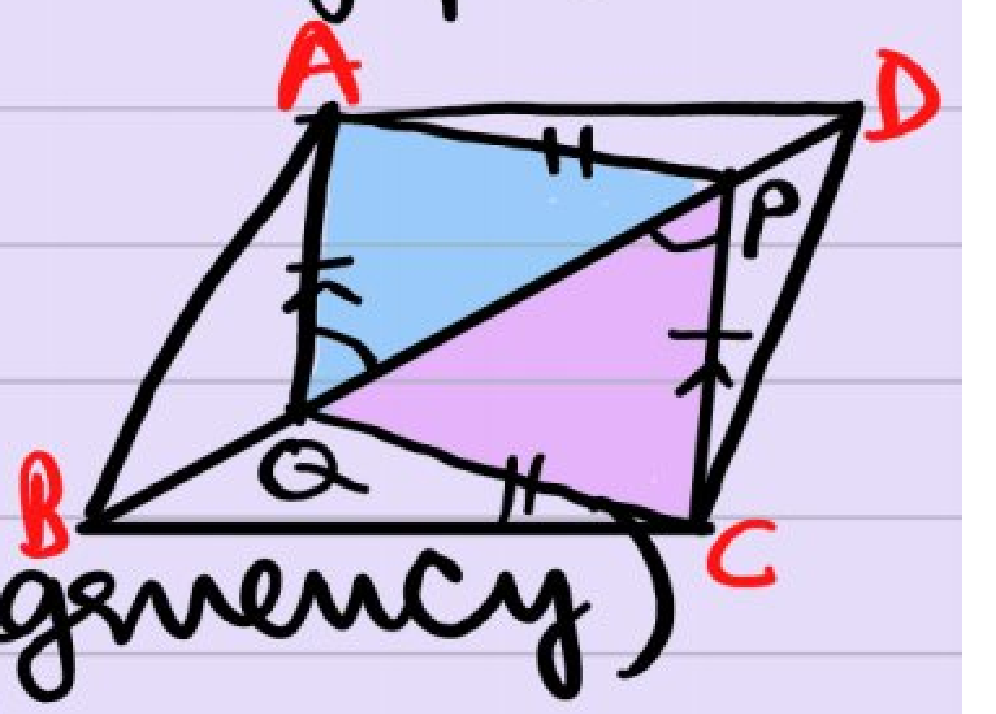
(ii) Since $\triangle APD \cong \triangle CQB$, $AP = CQ$ (by cpct) → (2)



(iii) In $\triangle AQB$ and $\triangle CPD$,
$BQ = PD$ (given)
$\angle ABQ = \angle CDP$ (alternate interior angles, $AB \parallel CD$)
$AB = CD$ (opposite sides of ||gm ABCD)
$\therefore \triangle AQB \cong \triangle CPD$ (SAS congruency)

(iv) Since $\triangle AQB \cong \triangle CPD$, $AQ = CP$ (by cpct) → (3)

(v) In $\triangle AQP$ and $\triangle CPQ$,

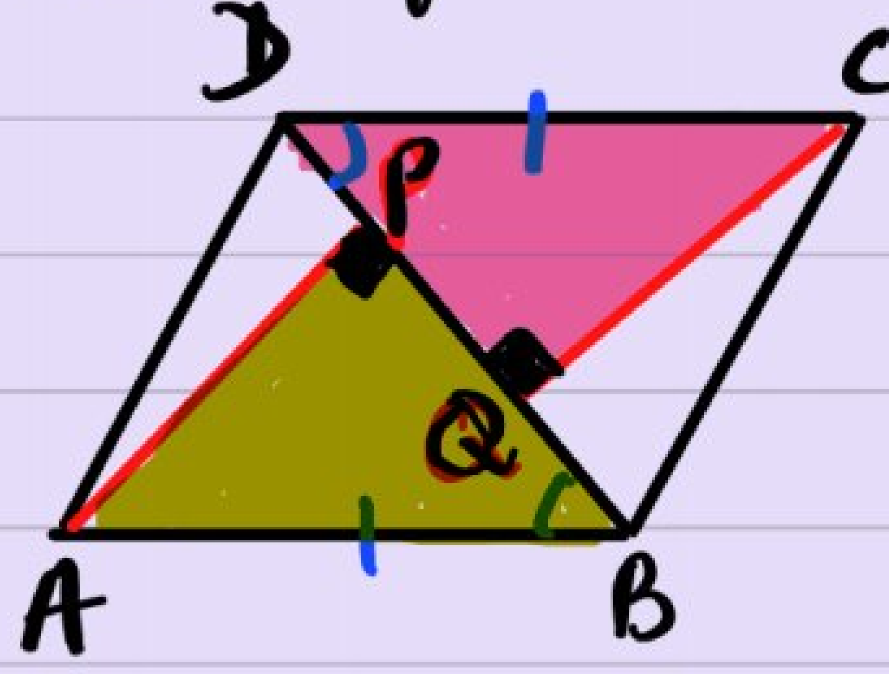


$AQ = CP$ (from eq:(3))
$AP = CQ$ (from eq:(2))
$QP = QP$ (common side)
$\therefore \triangle AQP \cong \triangle CPQ$ (SSS congruency)
Thus $\angle AQP = \angle CPQ$ (by cpct)

These angles form a pair of alternate interior angles only when $AQ \parallel CP$.
$\therefore$ APCQ is a parallelogram with one pair of opposite sides equal and parallel.

Hence Proved

12)



Given:- in parallelogram ABCD
$AP \perp BD$
$CQ \perp BD$

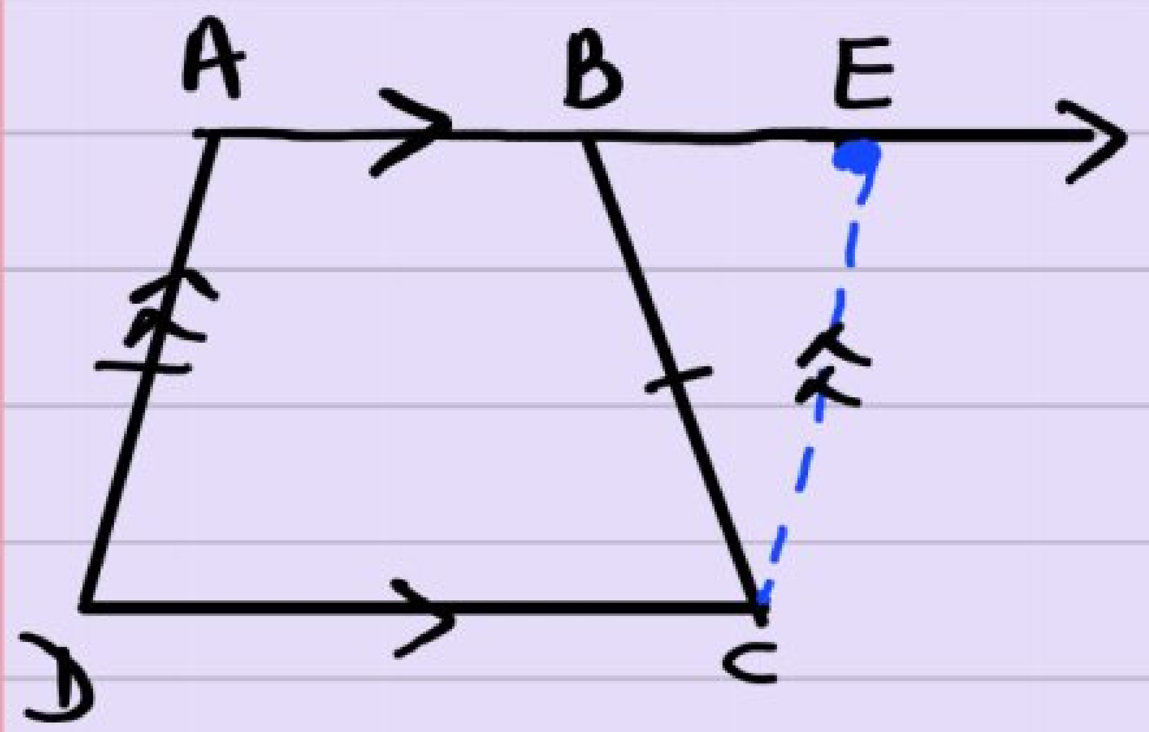
To prove:- (i) $\triangle APB \cong \triangle CQD$
(ii) $AP = CQ$

Proof:- (i) In $\triangle APB$ and $\triangle CQD$,
$\angle APB = \angle CQD$ (each $90°$)
$\angle ABP = \angle CDQ$ (alternate interior angles; $AB \parallel CD$)
$AB = CD$ (opposite sides of ||gm ABCD)
$\therefore \triangle APB \cong \triangle CQD$ (AAS congruency)

(ii) Since $\triangle APB \cong \triangle CQD$, then $AP = CQ$ (by cpct).

Hence Proved

13)



Given:- in trapezium ABCD,
$AB \parallel CD$, $AD = BC$

To prove:- (i) $\angle A = \angle B$
(ii) $\angle C = \angle D$
(iii) $\triangle ABC \cong \triangle BAD$
(iv) diagonal AC = diagonal BD

Construction:- draw $CE \parallel AD$ to meet AB produced at E.

Proof:- (i) since both pairs of opposite sides $AB \parallel DC$ and $AD \parallel EC$, then ADCE is a parallelogram.

Thus, $AD = EC$ (opposite sides of ||gm ADCE) $\rightarrow$ (1)
Given, $AD = BC$

$\therefore EC = BC$

$\Rightarrow \angle CBE = \angle CEB$ [angles opposite to equal sides] $\rightarrow (2)$

Also, $\angle DAE + \angle CEA = 180°$ (adjacent angles of $\parallel$gm ADCE)

$\Rightarrow \angle DAE + \angle CBE = 180°$ (from eq: (2)) $\rightarrow (3)$

And, $\angle ABC + \angle CBE = 180°$ (linear pair) $\rightarrow (4)$

From (3) and (4), $\angle DAE + \cancel{\angle CBE} = \angle ABC + \cancel{\angle CBE}$

$\Rightarrow \angle DAE = \angle ABC$

$\therefore \angle A = \angle B$ //

(ii) Since $AB \parallel DC$, $\angle A + \angle D = 180° \rightarrow (5)$

Also, $\angle B + \angle C = 180° \rightarrow (6)$

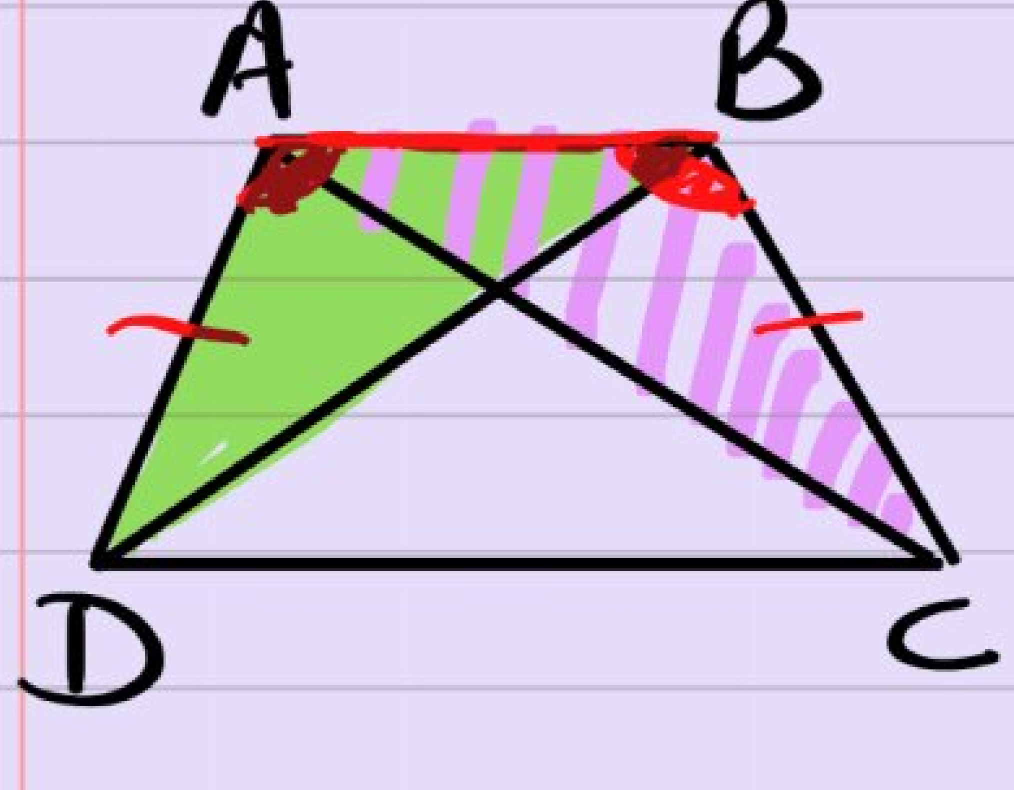
From (5) and (6),

$\cancel{\angle A} + \angle D = \cancel{\angle B} + \angle C$

$[\because \angle A = \angle B]$

$\therefore \angle D = \angle C$ //

(iii)



In $\triangle ABC$ and $\triangle BAD$,

$AB = AB$ (common side)

$\angle ABC = \angle BAD$ (proved above)

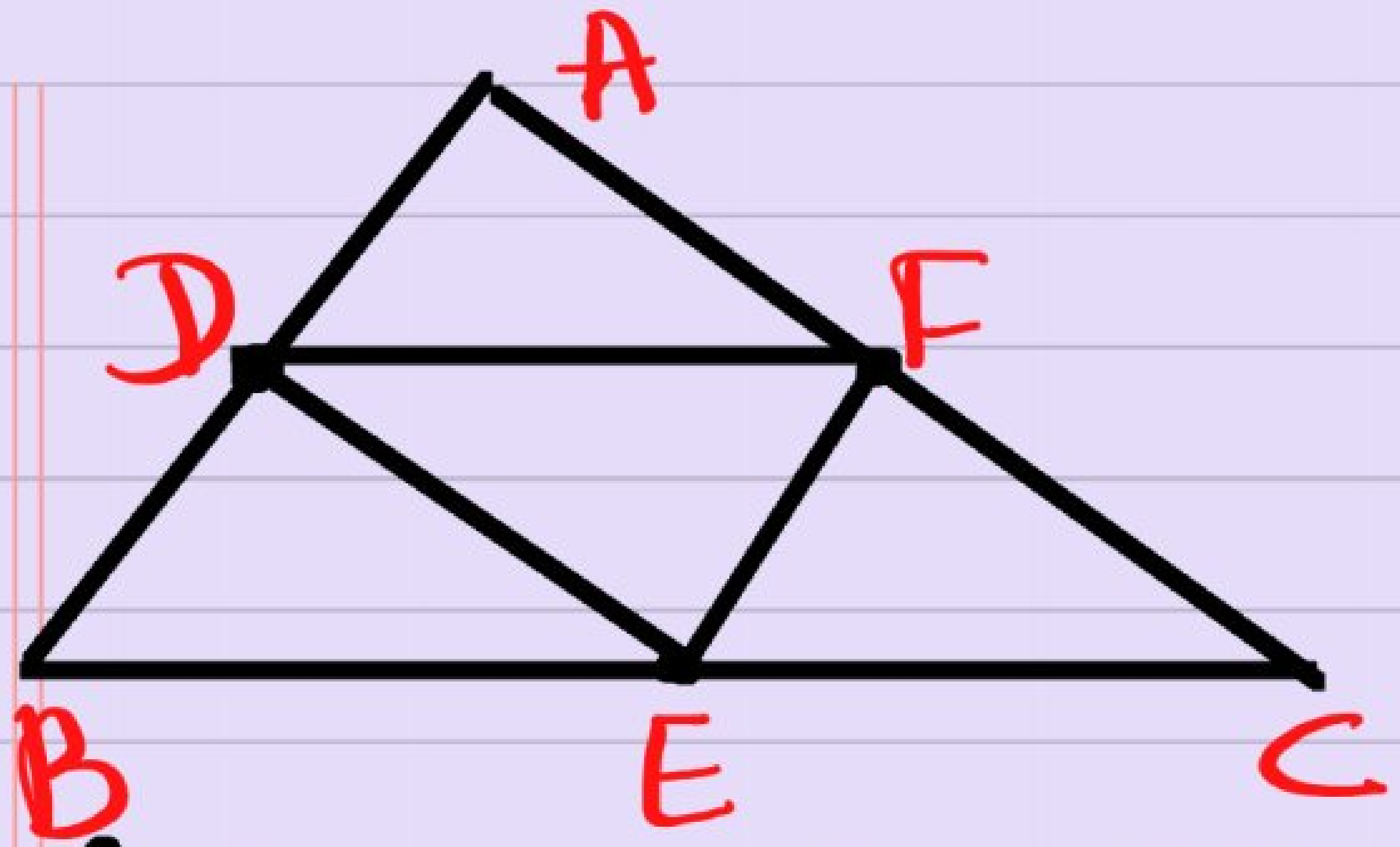
$BC = AD$ (given)

$\therefore \triangle ABC \cong \triangle BAD$ (SAS congruency)

(iv) Since $\triangle ABC \cong \triangle BAD$, $AC = BD$ (by cpct)

Hence Proved

4)



Given :- in $\triangle ABC$,

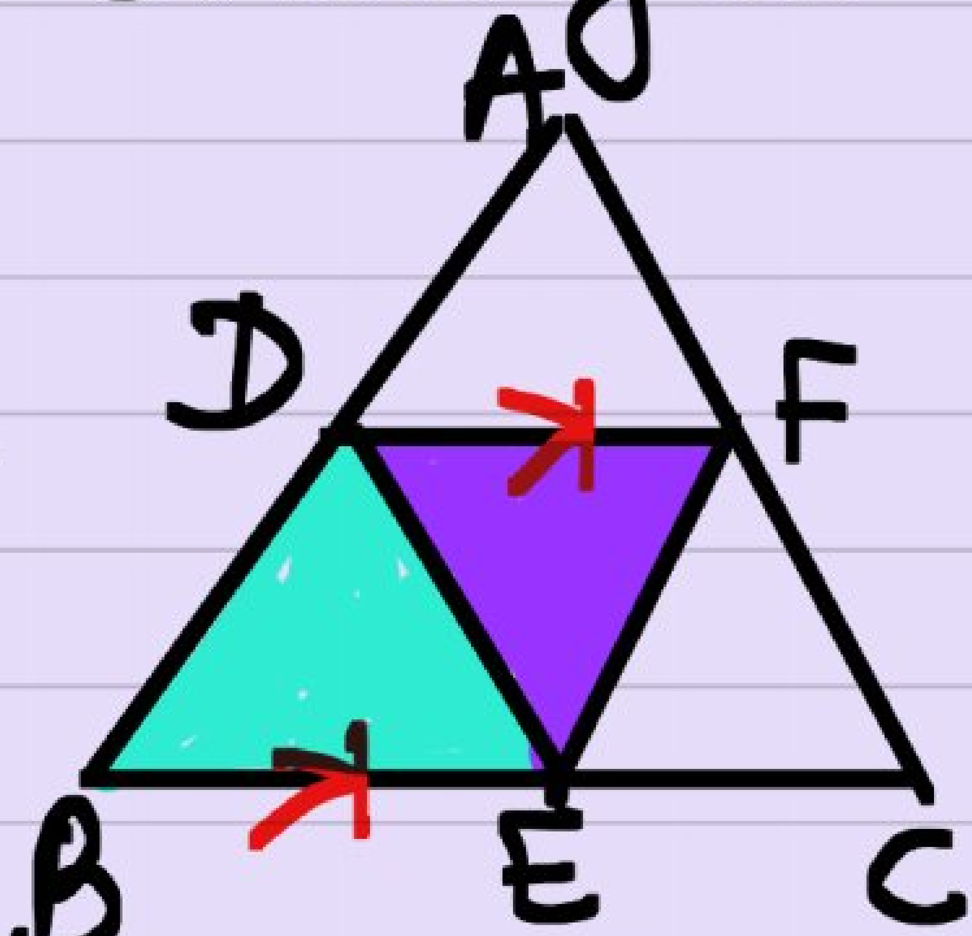
D is the mid-pt of AB

E is the mid-pt of BC

F is the mid-pt of AC

To prove :- $\triangle ABC$ is divided into four congruent triangles.

Proof :-



Using mid-point theorem, since D and F are the mid-points of sides AB and AC,

$DF \parallel BC \Rightarrow DF \parallel BE$

and $DF = \frac{1}{2}BC \Rightarrow DF = BE$

Thus, DFEB is a parallelogram with one pair of opposite sides equal and parallel.

Similarly, we can prove DECF and ADEF are parallelograms

We know that diagonal of a parallelogram divides it into two congruent triangles.

Thus, in ||gm DFEB,

$\triangle DEF \cong \triangle EBD$

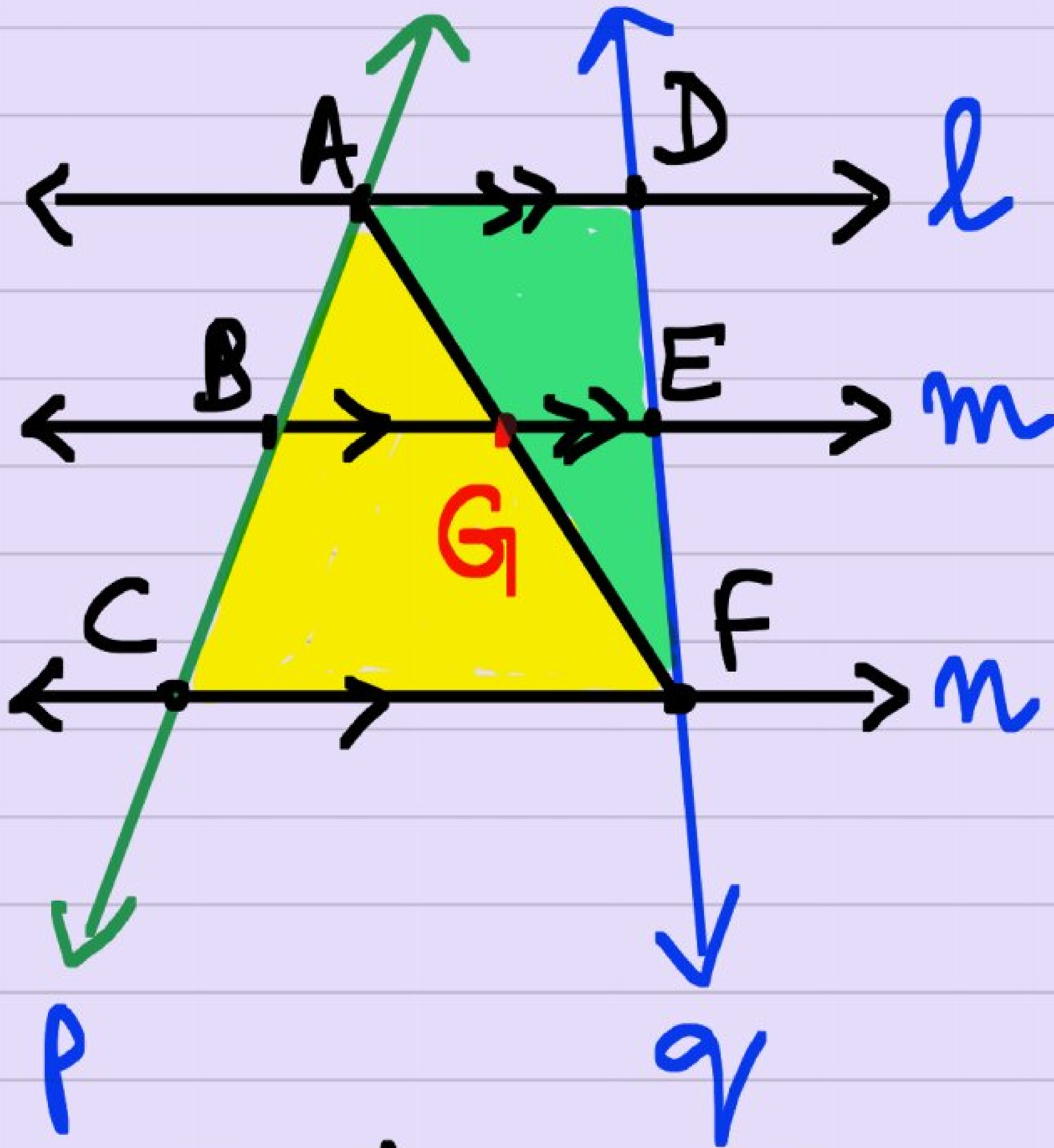
Similarly, $\triangle DEF \cong \triangle CFE$

and $\triangle DEF \cong \triangle FAD$

$\therefore \triangle DEF \cong \triangle EBD \cong \triangle CFE \cong \triangle FAD$

Hence Proved

15)



Given:-

$l \parallel m \parallel n$

$AB = BC$

To prove:-

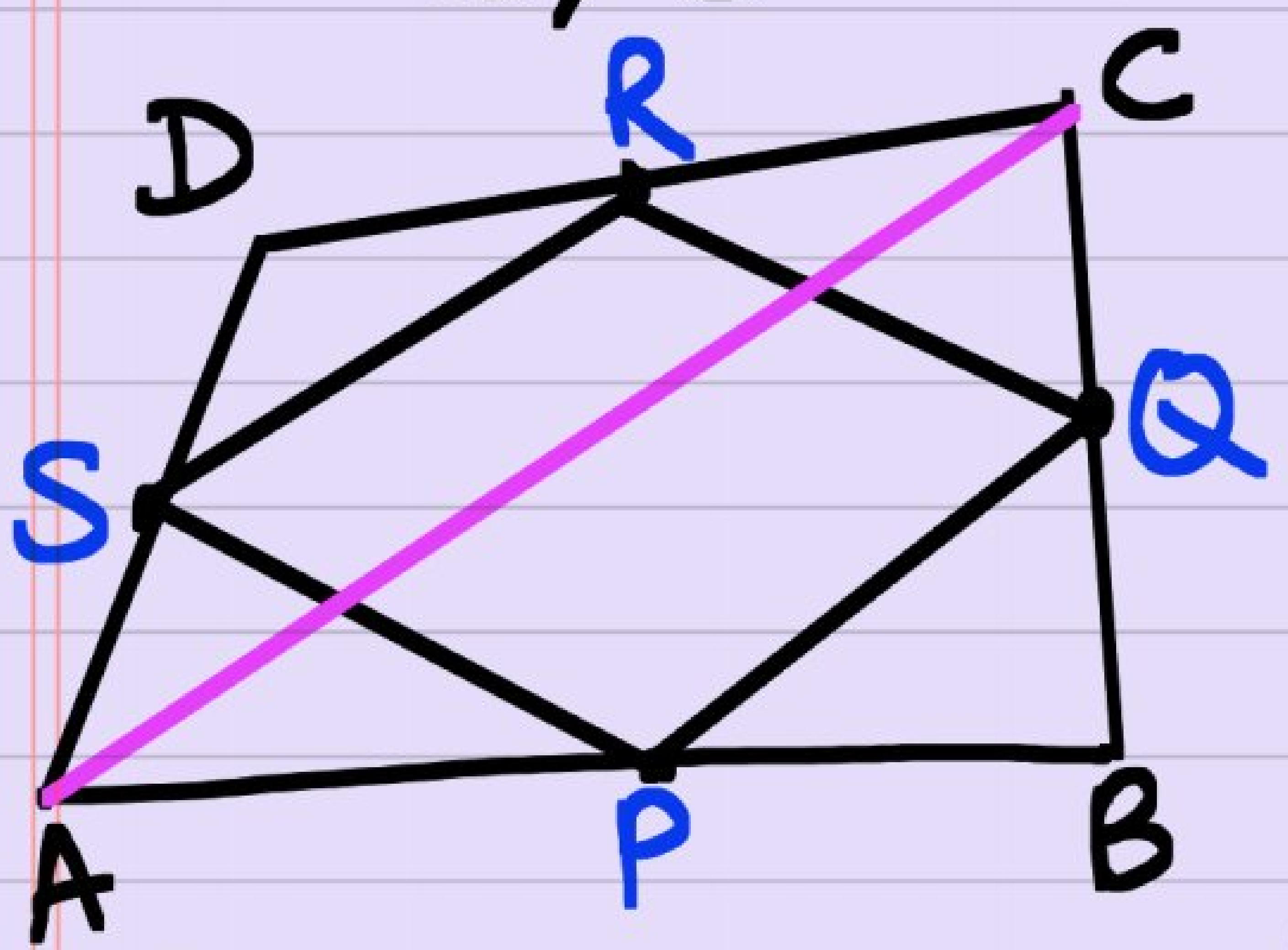
$DE = EF$

Proof:- In $\triangle ACF$, using converse of

mid-point theorem, since B is the mid-point of AC and BG || CF, G is also the mid-point of AF.
Thus, AG = GF → (1)

Similarly, in △AFD, since G is the mid-point of AF and GE || AD, E is also the mid-point of DF.
Thus, DE = EF. Hence Proved

16)



Given :- in quad ABCD P, Q, R and S are the mid-points of sides AB, BC, CD and AD respectively AC is a diagonal

To prove :- (i) SR || AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram

Proof :- (i) Using mid-point theorem in △ADC, since S and R are the mid-points of sides AD and DC respt, SR || AC and SR = $\frac{1}{2}$ AC → (1)

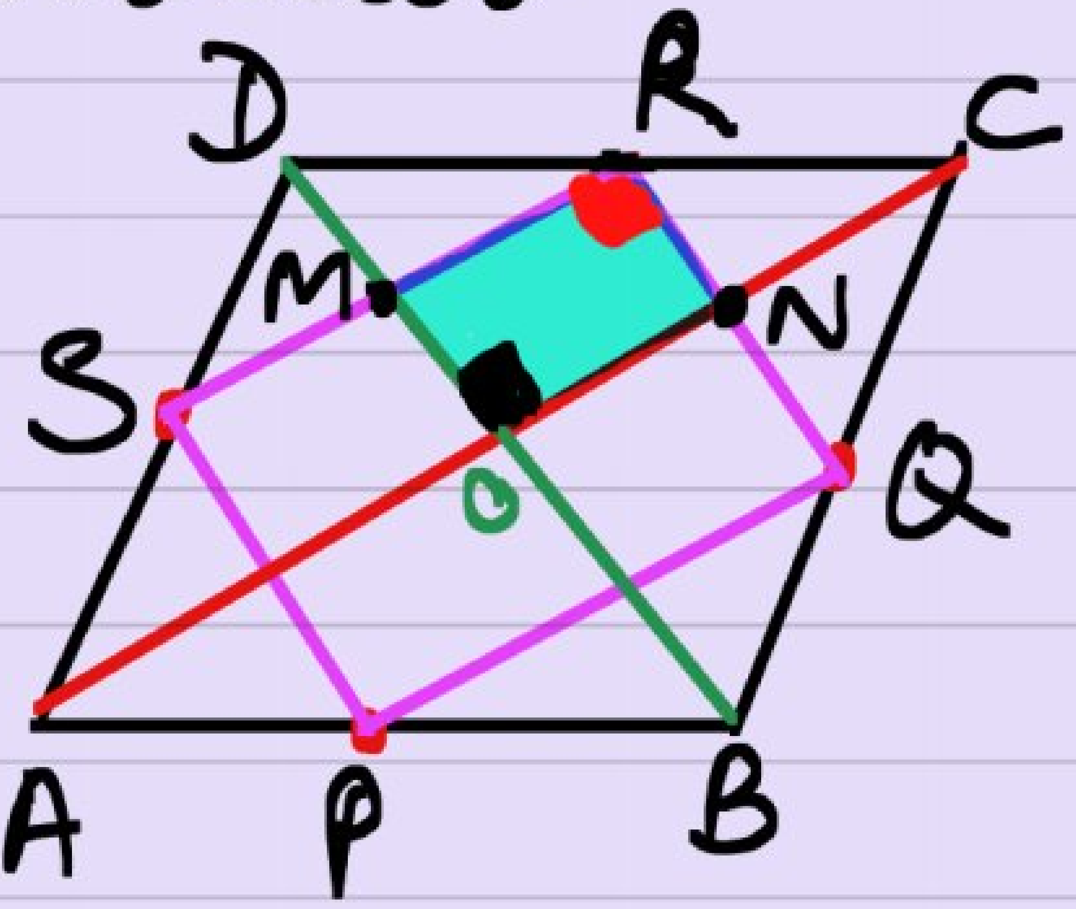
(ii) Similarly, in △ABC, since P and Q are the mid-points of sides AB and BC respectively, then

$PQ \parallel AC$ and $PQ = \frac{1}{2}AC \rightarrow (2)$

From (1) and (2), $PQ = SR$

(iii) From (1) and (2), $SR \parallel PQ$ and $SR = PQ$
Thus PQRS is a parallelogram with one pair of opposite side equal and parallel.

Hence Proved

17) 

Given:- in rhombus ABCD, P, Q, R and S are the mid-points of sides AB, BC, CD and AD respectively.

To prove:- quad. PQRS is a rectangle.

Proof:- Using mid-point theorem,
in $\triangle ADC$, since S and R are the mid-points of sides AD and DC respt.,
$SR \parallel AC$ and $SR = \frac{1}{2}AC \rightarrow (1)$

Similarly, in $\triangle ABC$, since P and Q are the mid-points of AB and BC respectively,

$PQ \parallel AC$ and $PQ = \frac{1}{2}AC \rightarrow (2)$

From (1) and (2), $SR \parallel PQ$ and $SR = PQ$.
Thus quadrilateral PQRS is a $\parallel$gm with one pair of opposite sides

equal and parallel.

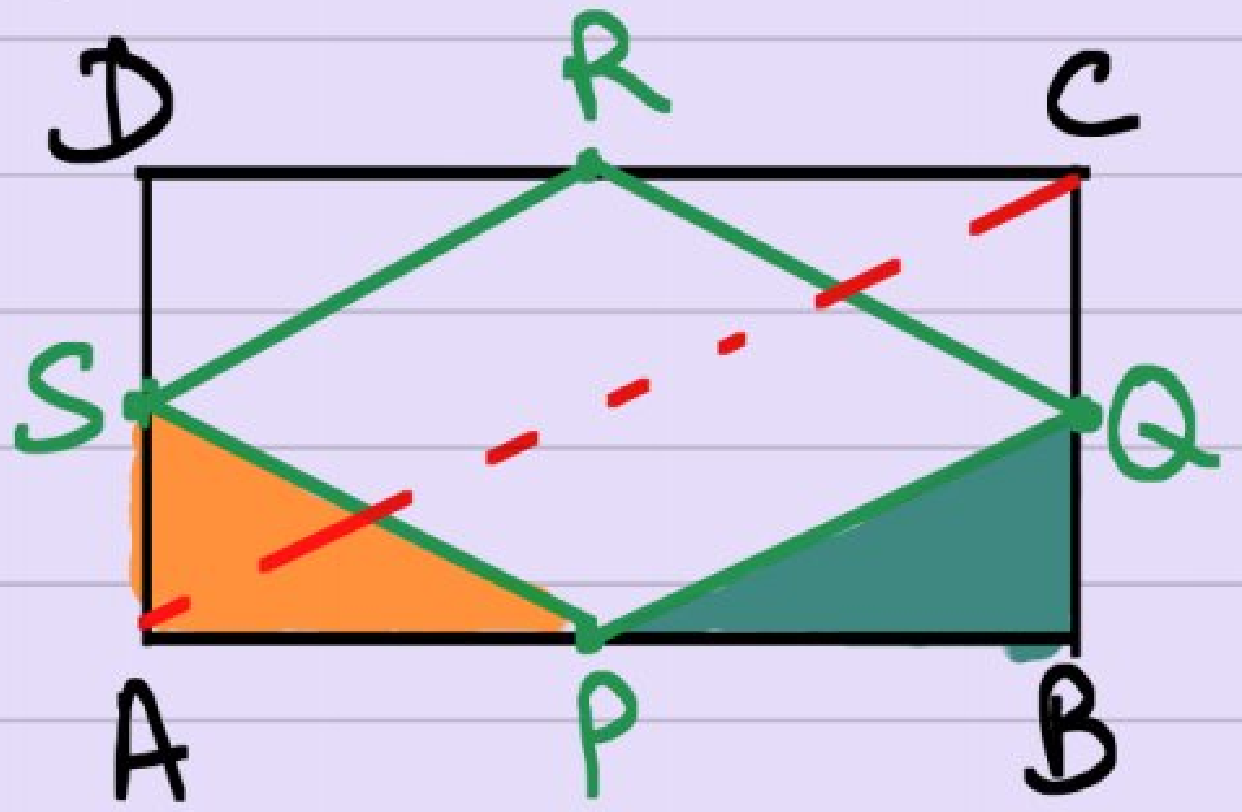
Now, using mid-point theorem,
since $SR \parallel AC \Rightarrow MR \parallel ON$
and $RQ \parallel DB \Rightarrow RN \parallel MO$
Thus OMRN is a parallelogram with both pairs of opposite sides parallel.
We know that diagonals of a rhombus bisect each other at 90°.

$\therefore \angle DOC = 90° \Rightarrow \angle MON = 90°$

Thus, $\angle MON = \angle MRN = 90°$ [opposite angles of ||gm OMRN]

$\Rightarrow \angle R = 90°$
Thus, parallelogram PQRS is a rectangle with each angle measures 90°.

Hence Proved

18) 

Given:- in rectangle ABCD, P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively.
To prove:- quad. PQRS is a rhombus
Proof:- Using mid-point theorem,

Since S and R are the mid-points of sides AD and DC respectively,

$SR \parallel AC$ and $SR = \frac{1}{2}AC \longrightarrow (1)$

Similarly, in ΔABC since P and Q are the mid-points of sides AB and BC respectively,

$PQ \parallel AC$ and $PQ = \frac{1}{2}AC \longrightarrow (2)$

From (1) and (2), $SR \parallel PQ$ and $SR = PQ$

Thus, quadrilateral PQRS is a parallelogram with one pair of opposite sides equal and parallel.

In ΔSAP and ΔQBP,

$SA = BQ$ $[\frac{1}{2}DA = \frac{1}{2}BC]$

$\angle SAP = \angle QBP$ (each $90°$)

$AP = BP$ [$\because$ P is the mid-point of AB]

$\therefore \Delta SAP \cong \Delta QBP$ (SAS congruency)

Thus, $SP = QP$ (by cpct)

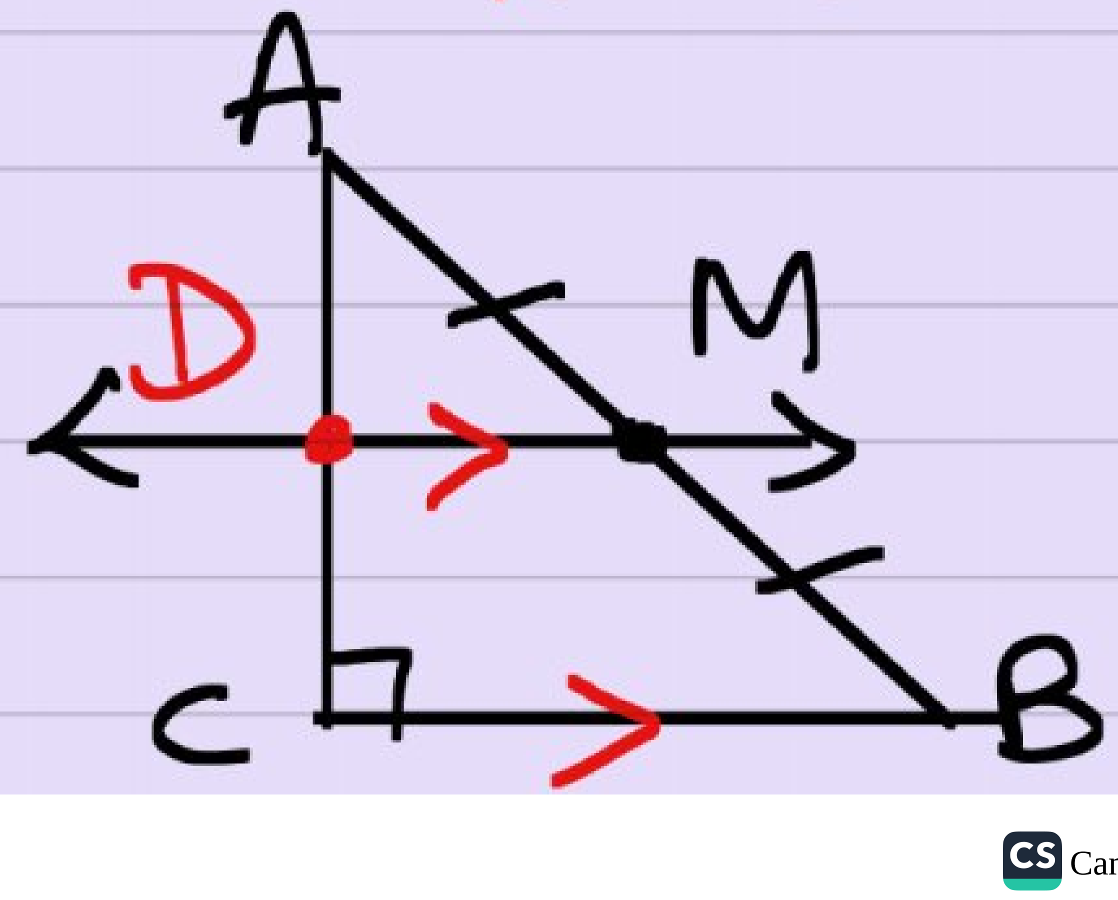
Thus, ||gm PQRS is a rhombus with adjacent sides equal. Hence Proved

19) Given:- in ΔABC,

$\angle C = 90°$

$AM = BM$

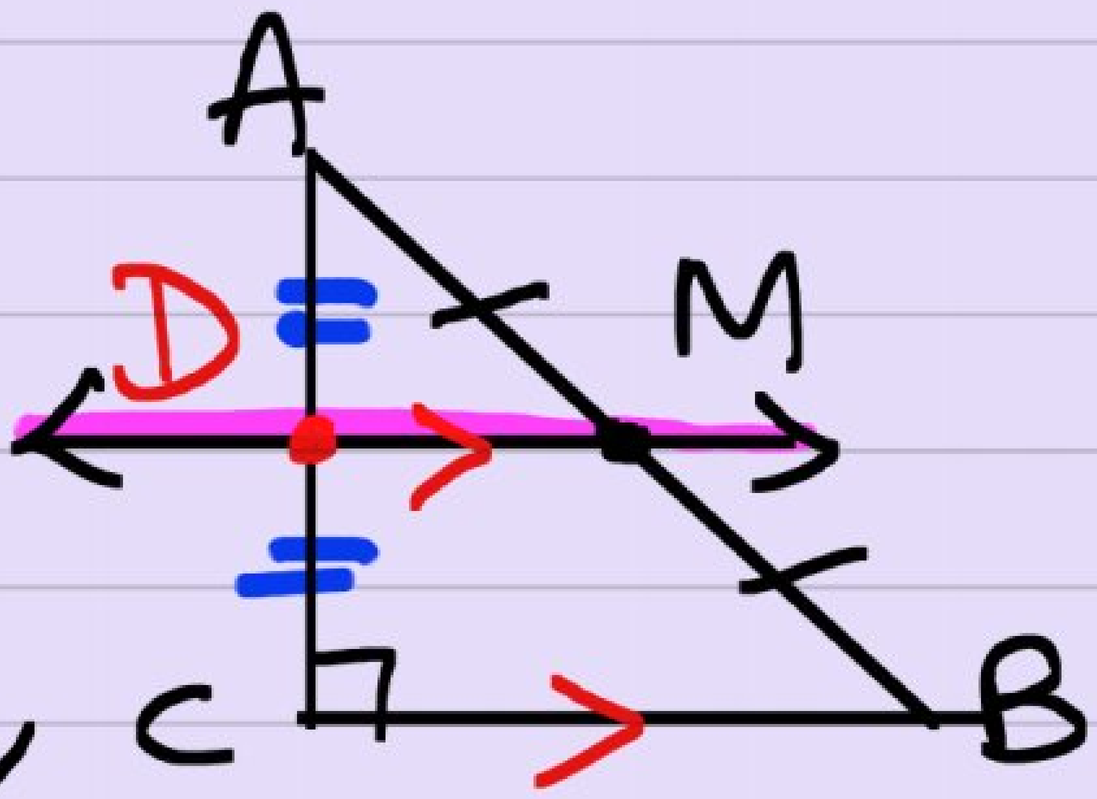
$DM \parallel BC$



To prove:- (i) D is the mid_point of AC
(ii) $MD \perp AC$
(iii) $CM = MA = \frac{1}{2} AB$

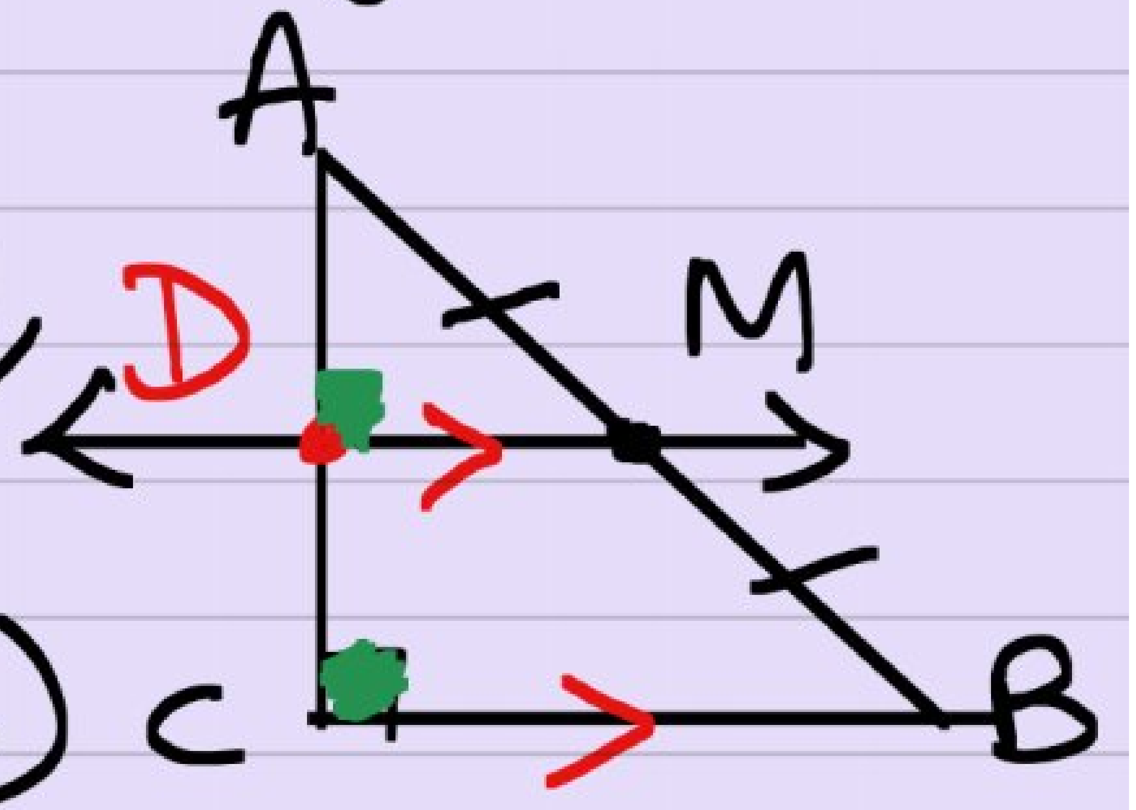
proof:-

(i) Since M is the mid-point of AB and $DM \parallel BC$, using mid-point theorem,



D is also the mid-point of AC.

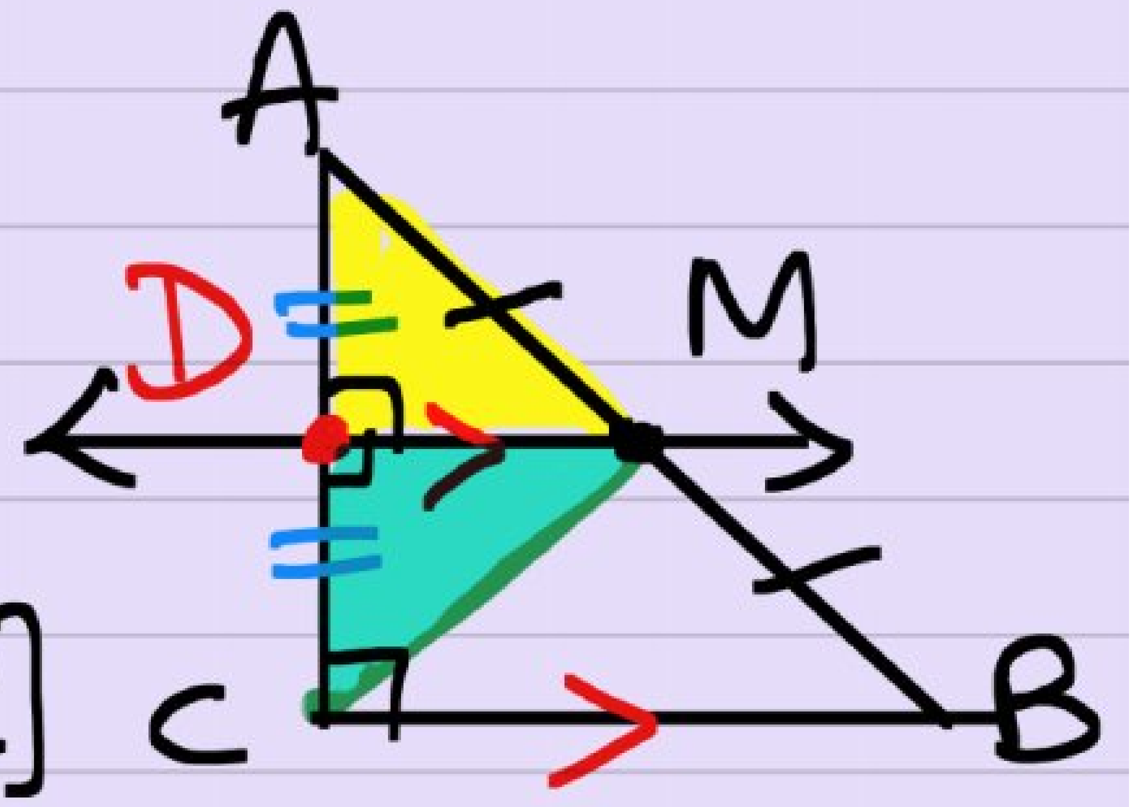
(ii) Since $DM \parallel BC$ and AC is the transversal,



$\angle ADM = \angle DCB = 90^\circ$
(corresponding angles)

$\therefore MD \perp AC$

(iii) In $\triangle ADM$ and $\triangle CDM$,



$AD = DC$ [$\because$ D is the mid-pt of AC]

$\angle ADM = \angle CDM$ (each 90°)
$DM = DM$ (common side)
$\therefore \triangle ADM \cong \triangle CDM$ (SAS congruency)

Thus, $MA = CM$ (by cpct)
But $MA = \frac{1}{2} AB$

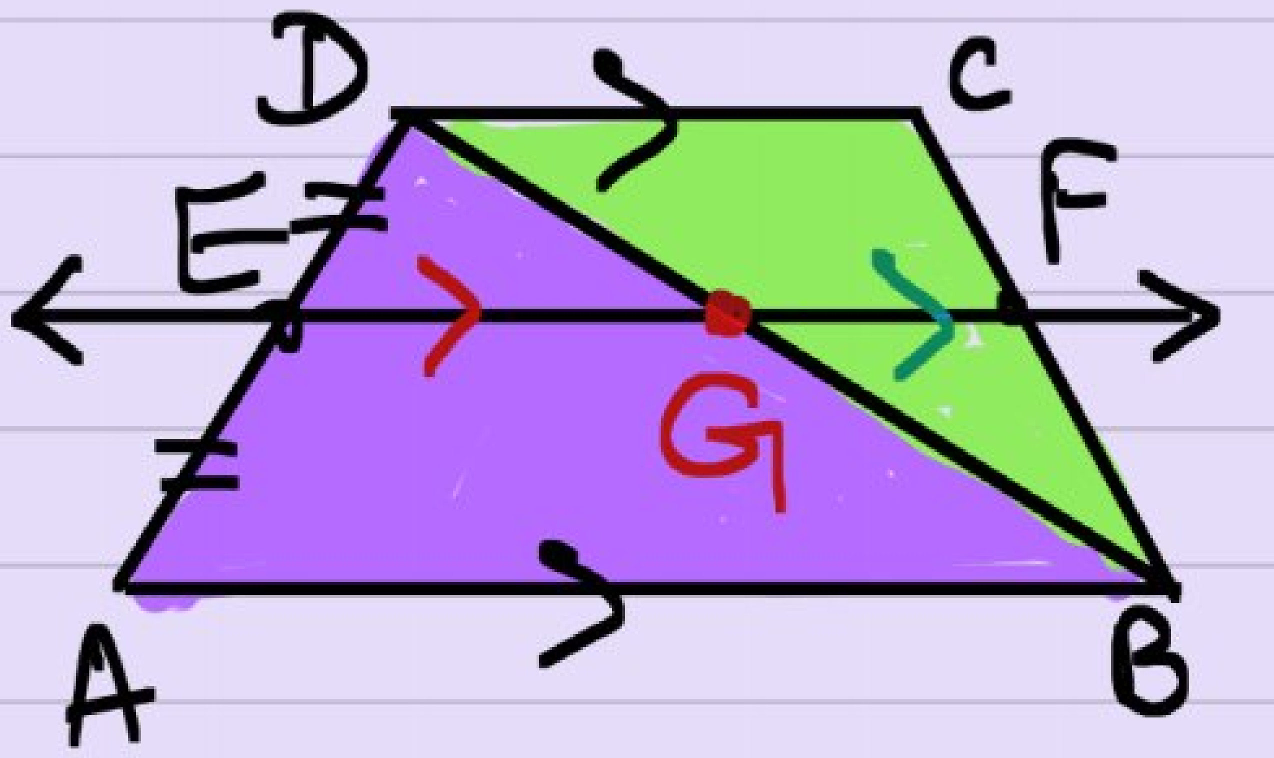
$\therefore CM = MA = \frac{1}{2} AB$. Hence Proved

20) Given:-

in trapezium ABCD,

AB || DC; EF || AB

DE = EA



To prove:- CF = FB

Proof:- Let diagonal BD intersect EF at G.

Since AB || DC and EF || AB, then AB || EF || DC

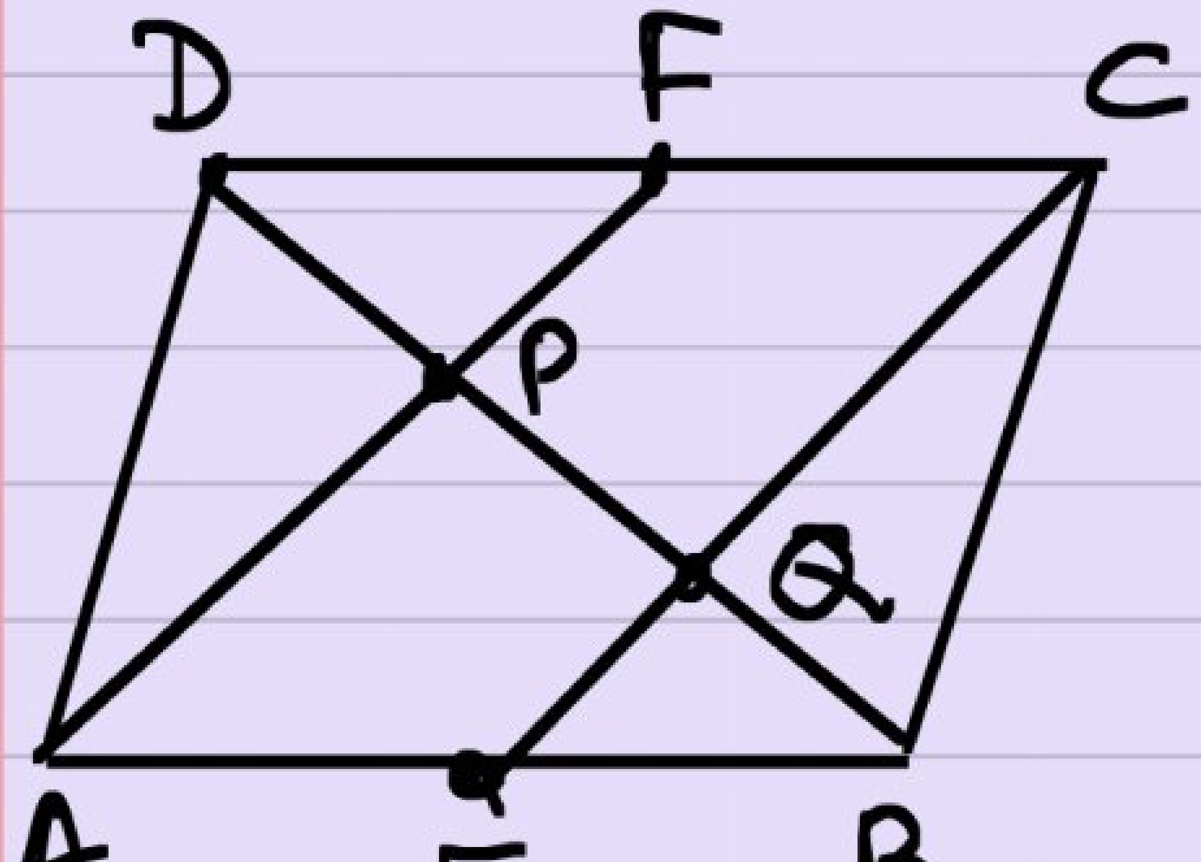
In △DAB, since E is the mid-point of AD and EG || AB, G is also the mid-point of BD (using converse of mid-point theorem)

Similarly, in △DBC, since G is the mid-point of DB and GF || DC, then F is also the mid-point of BC.

i.e, CF = FB

Hence Proved

21)



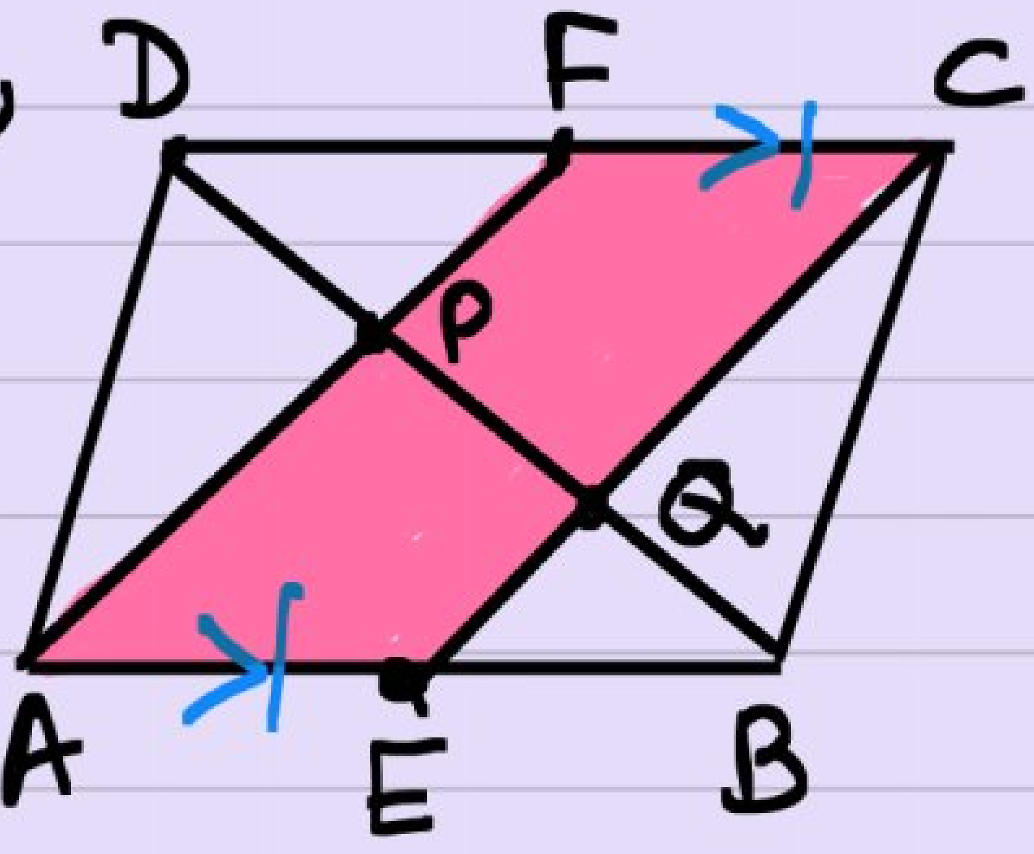
Given:- in ||gm ABCD,

E is the mid-point of AB.

F is the mid-point of DC.

To prove:- AF and CE trisect BD
i.e, DP = PQ = QB

Proof:* Since AB || CD,
then AE || FC.
Also, since
AB = DC (opposite
sides of ||gm ABCD)

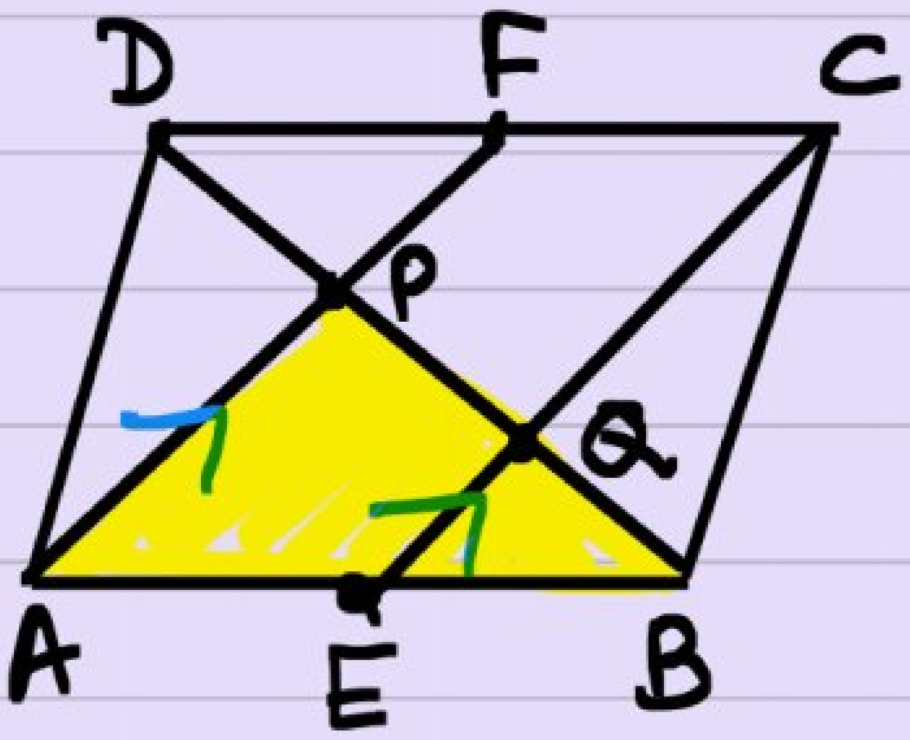


$\Rightarrow \frac{1}{2}AB = \frac{1}{2}DC$

$\Rightarrow AE = FC$

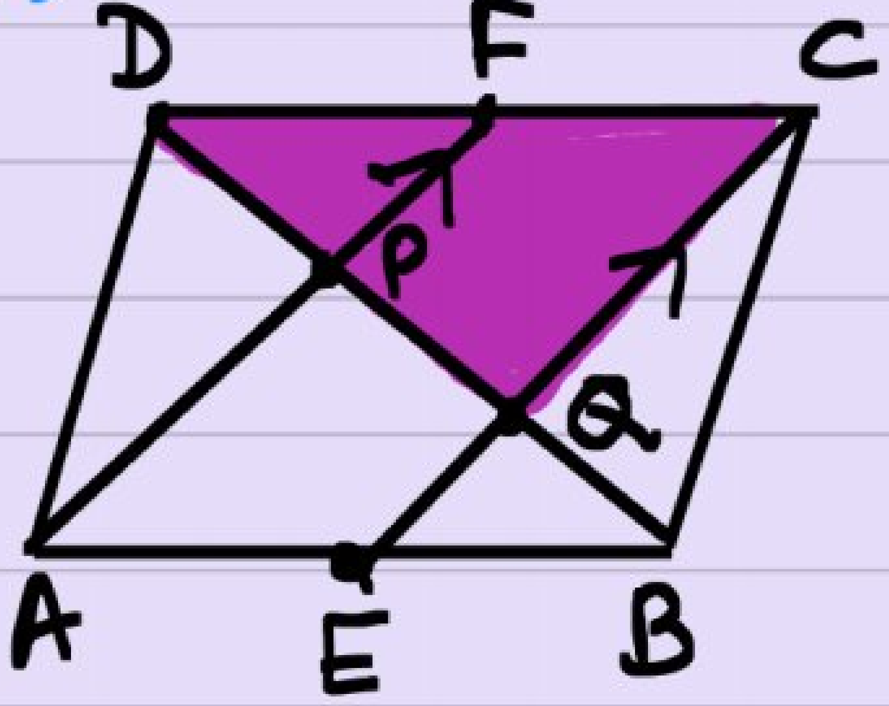
Thus, AECF is a ||gm with parallelogram with one pair of opposite sides equal and parallel.

* In $\triangle ABP$, since
AP || EQ and E is
the mid-point of
AB, using converse
of mid-point theorem, Q is also the mid-point of BP.



i.e, PQ = BQ $\longrightarrow$ (1)

* Similarly, in $\triangle DQC$,
since PF || QC and
F is the mid-point

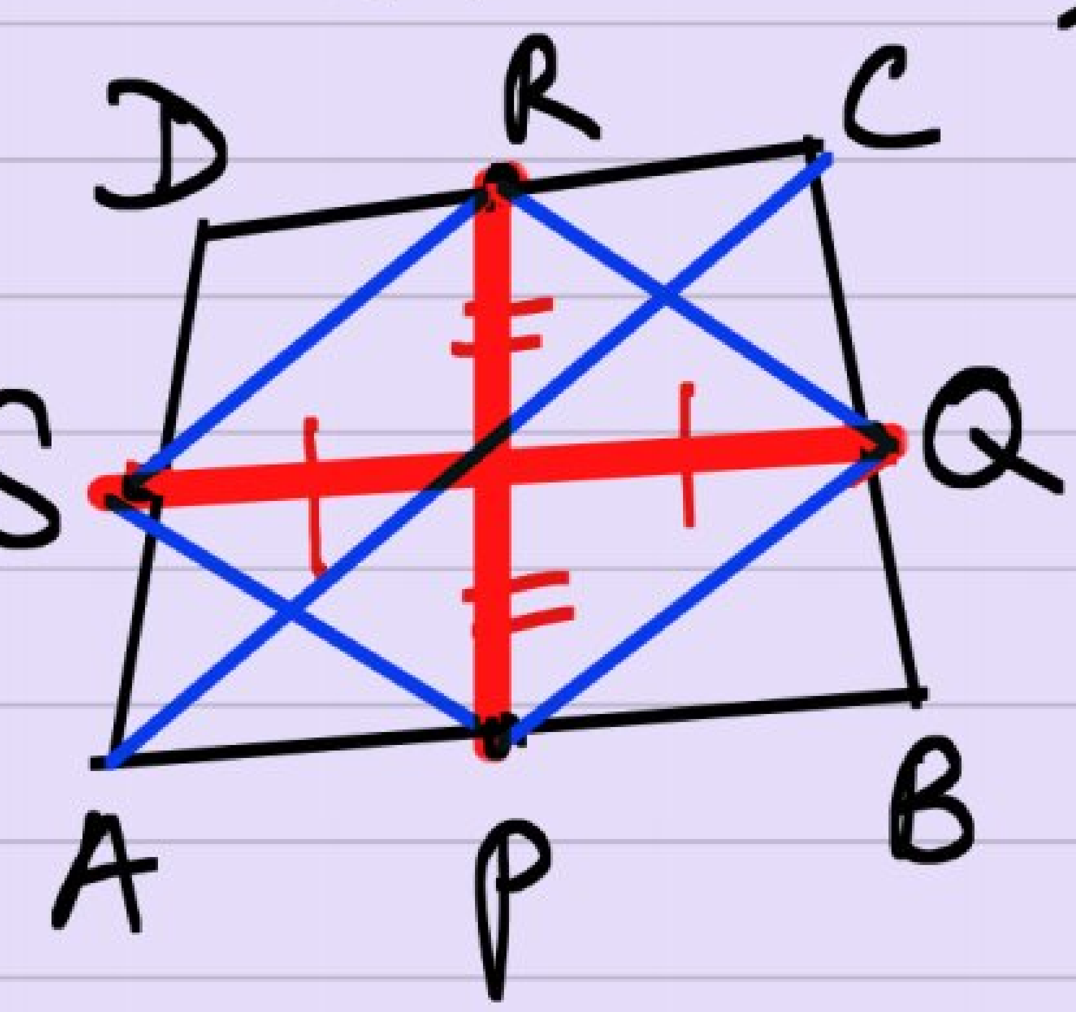


of CD, then P is also the mid-point of DQ.

i.e., DP = PQ → (2)

from (1) and (2), DP = PQ = QB

Hence proved.

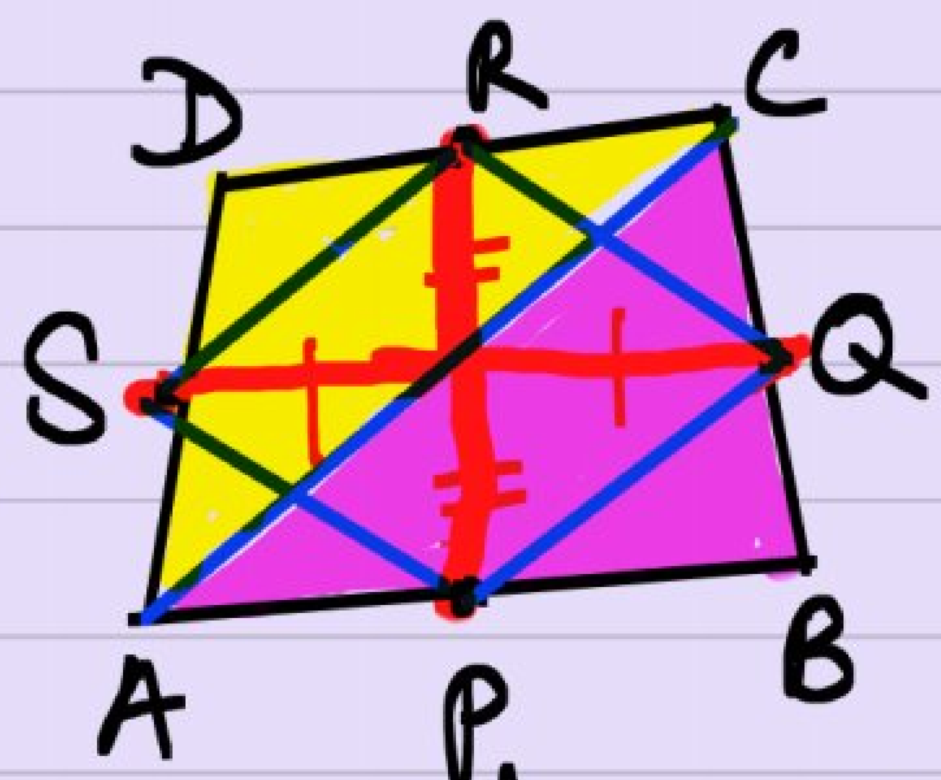
22) 

Given:- in quad. ABCD, P, Q, R and S are the mid-points of AB, BC, CD and AD respectively.

To prove:- PR and SQ bisect each other.

Construction:- Join AC, PQ, QR, RS and PS.

Proof:- In △ADC,



since S and R are the mid-points of sides AD and DC respectively, using mid-point theorem, SR || AC and $SR = \frac{1}{2}AC$ → (1)

Similarly, in △ABC, since P and Q are the mid-points of sides AB and BC respectively, then

$PQ \parallel AC$ and $PQ = \frac{1}{2}AC \rightarrow (2)$

From (1) and (2),

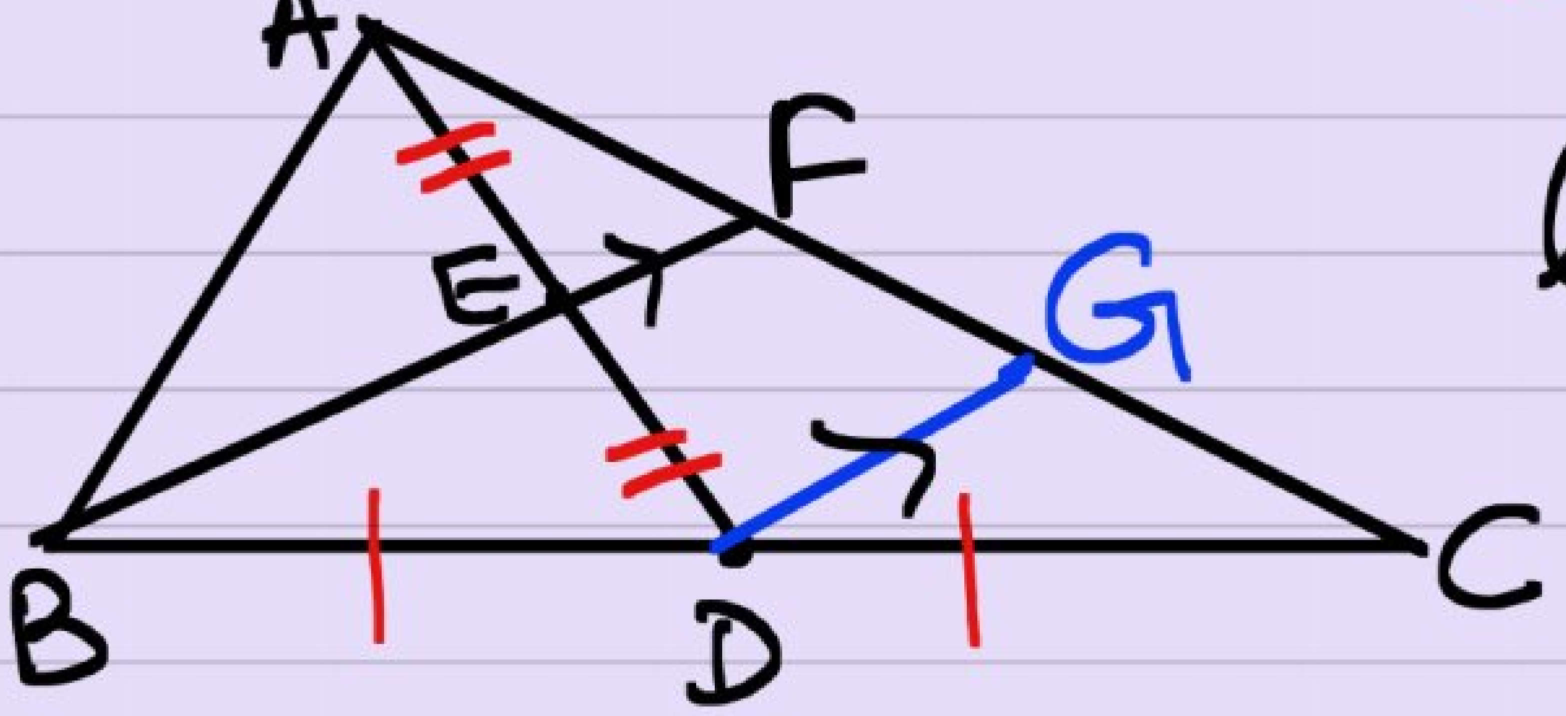
$SR \parallel PQ$ and $SR = PQ$.

Thus, PQRS is a parallelogram with one pair of ~~opposite~~ sides equal and parallel.

Since, PQRS is a parallelogram, the diagonals PR and SQ bisect each other.

Hence proved

23)

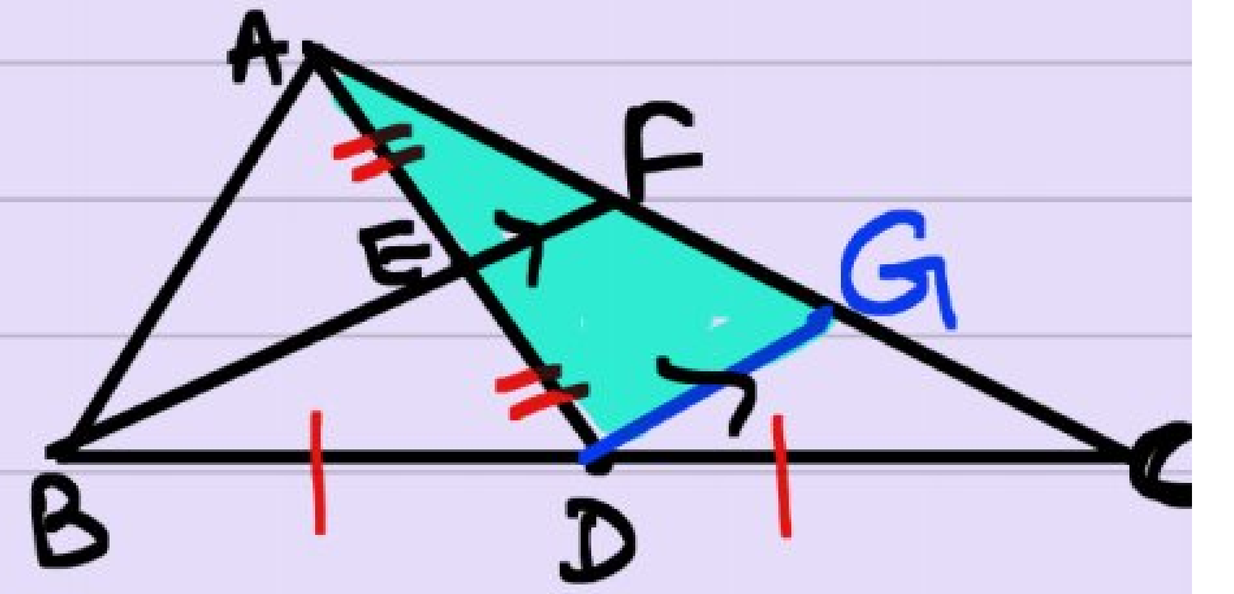


Given:- in ΔABC, $BD = DC$ and $AE = ED$

To prove:- $AF = \frac{1}{3}AC$

Construction:- draw $DG \parallel BF$ to meet AC at G

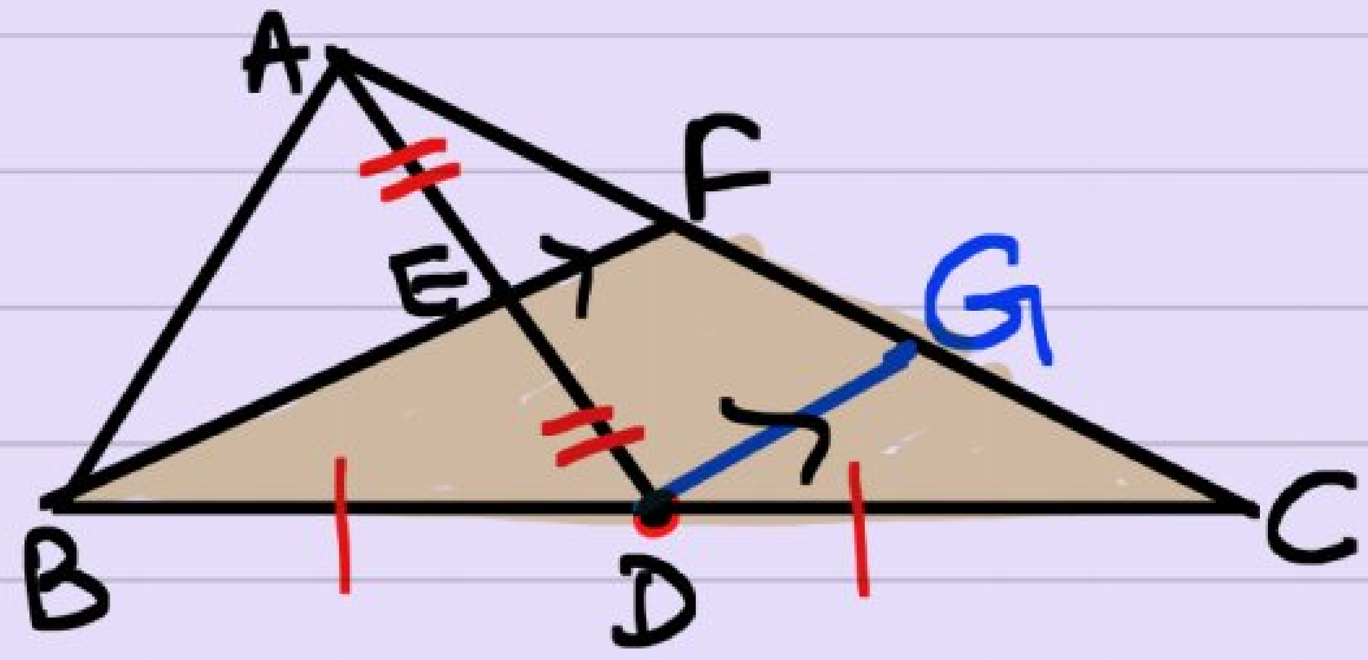
Proof:- in ΔADG,



since E is the mid-point of AD and $EF \parallel DG$, then using converse of mid-point theorem F is also the mid-point of AG.

i.e, $AF = FG \rightarrow (1)$

Similarly, in $\triangle BFC$, since D is the mid-pt of BC and



$DG \parallel BF$, then G is also the mid-point of FC.

i.e, $FG = GC \rightarrow (2)$

From (1) and (2), $AF = FG = GC \rightarrow (3)$

But, $AC = AF + FG + GC$

$\Rightarrow AC = 3AF$ [from eq: (3)]

$\therefore AF = \frac{1}{3} AC$

Hence Proved