X REVISION

1) If the sum of the roots of quadratic equation $ax^2+bx+c=0$ is equal to the sum of squares of their reciprocals, then prove that $2a^2c = c^2b + b^2a$.

2) The sum of the four consecutive terms in an AP is 32 and the ratio of the product of the first and last terms to the product of the middle terms is 7:15. Find the numbers.

3) A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/hr from the usual speed. Find its usual speed.

4) Find the values of k so that the equation $(k+4)x^2+(k+1)x+1=0$ has equal roots.

5) Find the sum of all the 11 terms of an AP whose middle most term is 30.

6) If $(k+9)$, $(2k-1)$ and $(2k+7)$ are the consecutive terms of an AP, then find the value of k.

7) Solve for x: $\dfrac{1}{(x-2)(x-1)} + \dfrac{1}{(x-2)(x-3)} = \dfrac{2}{3}$; $x \neq 1, 2, 3$

8) RCB Machine started making road roller 10 years ago. The company increased its production uniformly by a fixed number every year. The company produces 800 rollers in the 6th year and 1130 roller in the 9th year.

Case-Study

(i) What was the company's production in the second year?
(a) 290 (b) 360 (c) 250 (d) 200

(ii) What was the increase in the company's production in 4 years (when compared to the first year)
(a) 160 (b) 130 (c) 330 (d) 110

(iii) What was the company's production in the 10th year?
(a) 880 (b) 820 (c) 760 (d) 1240

(iv) In which year the company's production will be 1460 rollers?
(a) 11th (b) 6th (c) 5th (d) 12th

9)* Solve for x: $\sqrt{3}x^2 - 2x - 8\sqrt{3} = 0$

10) One root of $2x^2+kx-6=0$ is $\dfrac{-3}{2}$, then find the value of k.

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11) Solve for x : $9x^2 - 9(a+b)x + (2a^2+5ab+2b^2) = 0$

12) A two-digit number is four times the sum of the digits. It is also equal to 3 times the product of digits. Find the number.

13) If $4m$, $m+20$ and $3m-2$ are in A.P, then find the value of m.

14) If the sum of first 13 terms of an AP is 182 and that of 19 terms is 380, find the sum of first n terms.

15) The sum of first six terms of a AP is 42. The ratio of its 10^{th} term to its 30^{th} term is 1 : 3. Calculate the first and the thirteenth term of the AP.

16) An AP is given as $\frac{1}{3}, \frac{1-3b}{3}, \frac{1-6b}{3}, \ldots$

(i) The c.d of the given AP is (a) b (b) $-b$ (c) $1/b$ (d) $1/3b$

(ii) What is the sixth term of the AP?
(a) $\frac{1+9b}{3}$ (b) $\frac{1-9b}{3}$ (c) $\frac{1+15b}{3}$ (d) $\frac{1-15b}{3}$

(iii) The — term of the AP will be $\frac{1-21b}{3}$
(a) seventh (b) eighth (c) nineth (d) 10^{th}

(iv) The sum of third and fifth term is
(a) $\frac{2-18b}{3}$ (b) $\frac{2+18b}{3}$ (c) $\frac{2+15b}{3}$ (d) $\frac{2-15b}{3}$

(v) The difference of second and sixth term is
(a) $-b$ (b) $-2b$ (c) $-3b$ (d) $-4b$

Case-Study 17) Given below are taxi fares in two cities comprise of a fixed charge together with the charge for the distance covered.

Name of the City	Distance travelled (Km)	Amount paid (₹)
Delhi	10	100
	15	145
Mumbai	8	88
	14	148

Situation 1: In Delhi, for a journey of 10 km, the charge paid is ₹100 and for a journey of 15 km, the charge paid is ₹145.

Situation 2: In Mumbai, for a journey of 8 km, the charge paid is ₹88 and for a journey of 14 km, the charge paid is ₹148.

(a) In Delhi, if the fixed charge be ₹x and the running charges be ₹y km/hr, the pair of LE representing the situation is

(i) $x+10y=145$; $x+15y=100$
(ii) $x+10y=100$; $x+15y=145$
(iii) $10x+y=100$; $15xy=145$
(iv) $10x+y=145$; $15x+y=100$

(b) What is the fixed charge in Mumbai ?
(i) ₹8 (ii) ₹10 (iii) ₹12 (iv) ₹15

(c) What are the charges per km in Delhi?
(i) ₹8 (ii) ₹9 (iii) ₹10 (iv) ₹12

(d) A person travels 40 km in Delhi. The amount he must pay is (i) ₹200 (ii) ₹250 (iii) ₹350 (iv) ₹370

(e) A person travels 35 km in Mumbai. The amount he must pay is
(a) ₹350 (ii) ₹352 (iii) ₹358 (iv) ₹365

18) Solve for x: $\frac{12x-8}{x-5}=8$

19) The n^{th} term of an AP is $a_n = 7-4n$. The c.d is ___

20) The equation $2x^2+kx+3$ has two equal roots, then the value of k is
(a) $\pm\sqrt{6}$ (b) ± 4 (c) $\pm 3\sqrt{2}$ (d) $\pm 2\sqrt{6}$

21) The sum of roots of the quadratic equation $2x^2-9x-5=0$ is ___

22) $(n-1)^{th}$ term of the sequence $a, a+d, a+2d, \ldots$ is ___

23) Check whether $(y-2)^2+1 = 2y^2-3$ is a quadratic equation. Justify.

24) If a pair of linear equations is not consistent, then the lines will be ___

25) The roots of the equation $3x^2+2x-1=0$ are ___
~~25~~ (a) real and distinct (b) real and equal
(c) not real but equal (d) imaginary.

26) If p, q, r and s are in A.P, then
(a) $r-q=-p$ (b) $q-p=s-r$ (c) $q-p=r-s$ (d) $p-q=r-s$

27) Find the value of y which satisfies the equation
$1+\frac{y^2}{13}=\sqrt{\frac{27}{169}+1}$

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28) Solve for x and y: $43x + 67y = -24$
$67x + 43y = 24$

29) Solve for x: $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$; $x \neq -\frac{3}{2}, 3$

30) A train covered a certain distance at a uniform speed. If the train would have been 10 km/hr faster, it would have taken 2 hours less than the scheduled time. And if the train were slower by 10 km/hr; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

31) A motor boat whose speed is 20 km/hr in still water, takes 1 hour more to go 48 km upstream than to return downstream to the same spot. Find the speed of the stream.

Case-Study

31) (i) There are 50 houses in the first row, 58 houses in the second row, 66 houses in the third row and so on in the same increasing pattern. If the first plot has 25 rows of houses, how many houses are there in the first plot?

(ii) The total houses in the second plot is 1065. The first row has 21 houses and each row has one house more than the row in front of it. How many rows of houses are there in the plot?

(iii) There are 3 houses in the first row, 8 houses in the second row, 13 houses in the third row and (so on) 253 houses in the last row. Find the no. of houses in the 20^{th} row from the last row.

Case-Study

32) Ravi saved his pocket money in a piggy bank. He saves Rs 12 in the first month, Rs 15 in the second month, Rs 18 in the third month and continues to save in this manner.

(i) In which month will Ravi be able to save Rs 72?

(ii) How much amount did he save in the 13^{th} month?

(iii) The ratio of 7^{th} and 3^{rd} terms of an AP is 12 : 5. Find the ratio of its 13^{th} and 4^{th} terms.

33) Places A and B are 80 km apart from each other on a highway. A car starts from A and another from B at the same time. If they move in the same direction,

they meet in 8 hours and if they move in opposite directions, they meet in 1 hour 20 minutes. Find the speed of the two cars.

34) A man sold a chair and a table together for Rs 1520 thereby making a profit of 25% on chair and 10% on table. By selling them together for Rs 1535, he would have made a profit of 10% on the chair and 25% on the table. Find the C.P of each.

35) Solve for x: $\dfrac{x+1}{x-1} + \dfrac{x-2}{x+2} = 4 - \dfrac{2x+3}{x-2}$; $x \neq 1, -2, 2$

36) A train travelling at a uniform speed for 360 km would have taken 48 minutes less to travel the same distance if its speed were 5 km/hr more. Find the original speed of the train.

37) The required solution of $4x^2 - 25x = 0$ are

(a) $x=0, x=\frac{12}{7}$ (b) $x=0, x=\frac{25}{4}$ (c) $x=1, x=\frac{5}{9}$ (d) $x=1, x=\frac{12}{7}$

38) ~~The value~~ In a class, the teacher asked every student to write an example of AP.

Case-Study

Two friends Riya and Joe wrote their progression as $-15, -11, -7, \ldots$ and $150, 154, 158, \ldots$ respectively. Now, the teacher asks various students of class the following questions based on these progressions.

(a) Find 20th term of Riya's progression

(b) Find the product of C.ds of two progressions

(c) Find the sum of first 15 terms of Joe's progressions

(d) Which term of Riya's progression will have the value 33?

39) Seven years ago, Tom's age was five times the square of Ritu's age. Three years hence, Ritu's age will be $\frac{2}{5}$ of Tom's age. Find their present ages.

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40) If the zeroes of quadratic equation $mx^2+nx+p=0$ are equal; $p\neq 0$ then $p=$ ___

41) A natural number when decreased by 1, becomes equal to 20 times its reciprocal. The number is ___
(a) 3 (b) 4 (c) 5 (d) 6

42) If $ux+vy=u^2-v^2$ and $vx+uy=0$, then find the value of $u+v$.

43) Solve for z : $4z^2-4p^2z+(p^4-q^4)=0$

44) If -5 is a root of quadratic equation $2y^2+my-15=0$ and another quadratic equation $m(y^2+y)+n=0$ has equal roots. Find the value of n.

45) If twice the area of a smaller square is subtracted from the area of a larger square, the result is $14\,cm^2$. However, if twice the area of the larger square is added to three times the area of the smaller square, the result is $203\,cm^2$. Find the sides of both the squares.

46) Prove that $3-2\sqrt{7}$ is irrational, given $\sqrt{7}$ is irrational

47) Solve graphically $x-y=1$; $2x+y=8$. Shade the region bounded by these lines and y-axis. Also find its area.

48) Solve using factorisation: $\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$; $a+b\neq 0$

49) If α,β are zeroes of quadratic polynomial $5x^2+5x+1$, find the value of (i) $\alpha^2+\beta^2$

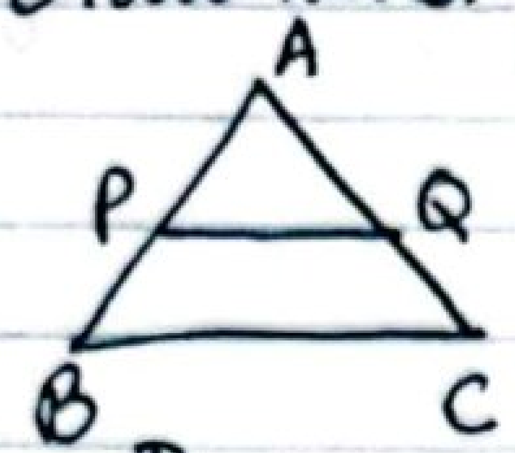
(ii) $\alpha^{-1}+\beta^{-1}$

50) Check whether 6^n can end with the digit 0 for any natural number n.

TRIANGLES

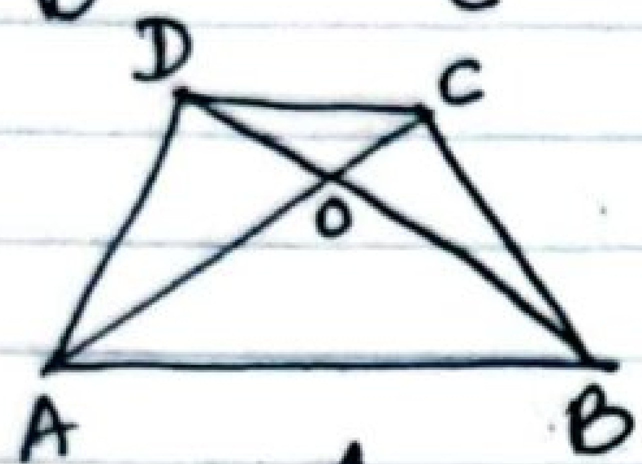
51) State and prove Basic Proportionality theorem.

52)



If $PQ \parallel BC$, then P.T $\frac{AP}{PB} = \frac{AQ}{QC}$

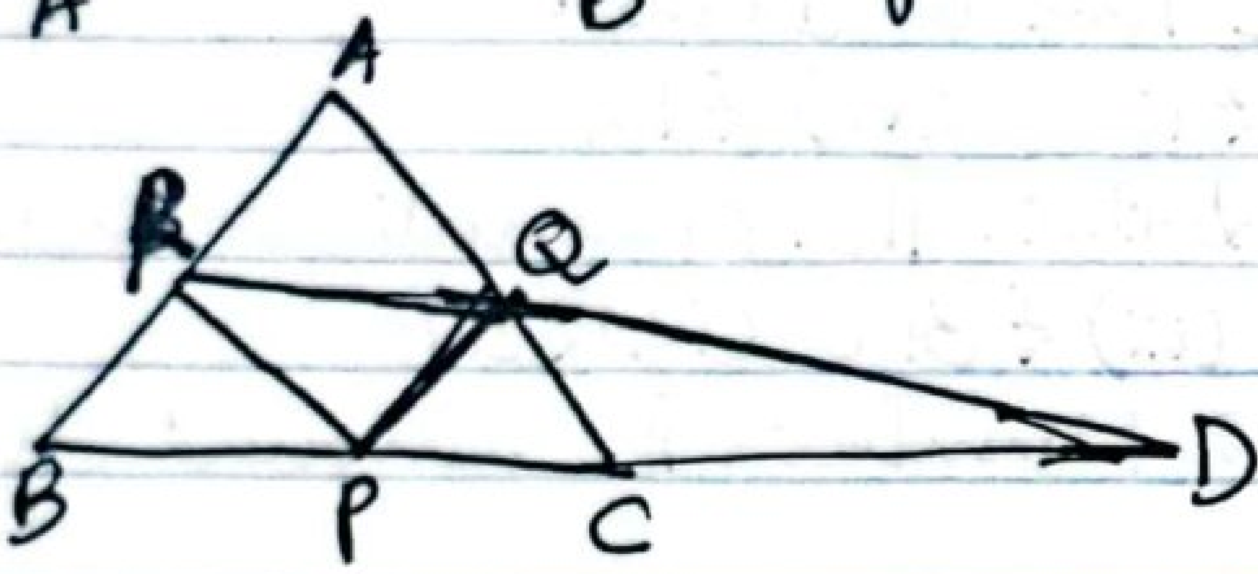
53)



$OA = x+5$; $OB = x-1$; $OC = x+3$; $OD = x-2$

If $AB \parallel DC$, find the value of x

54)



$PQ \parallel AB$; $PR \parallel CA$.

If $PD = 12cm$, find $BD \times CD$

55) ABCD is a trapezium in which $AB \parallel DC$ and P, Q are points on AD and BC respectively, such that $PQ \parallel DC$, if $PD = 18cm$, $BQ = 35cm$ and $QC = 15cm$, find AD

56) ABCD is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$

57) In a ΔPQR, $ST \parallel QR$ and $\frac{PS}{SQ} = \frac{3}{5}$ and $PR = 28cm$, find PT

58) In ΔRST, if $XY \parallel ST$ such that $RX = b$, $XS = b-2$, $RY = b+2$ and $YT = b-1$, then the value of b is __

(a) 3 (b) 4 (c) 5 (d) 3.5

Assertion - Reason

59) A: If a line divides two sides of a Δ in the same ratio then it is parallel to the third side.

R: ABCD is a trapezium in which $AB \parallel DC$ and P, Q are points on AD and BC respectively, such that $PQ \parallel DC$ if $PD = 18cm$, $BQ = 35cm$ and $QC = 15cm$, then $AD = 60cm$

(a) (b) (c) (d)

60) In a ΔPQR, $ST \parallel QR$ such that $PS = \frac{1}{5} SQ$ and $PT = 3.5cm$. find PR.

X Revision (PT-2) (Answers)

1) Let the roots be α and β.

$$\alpha+\beta = -\frac{b}{a}$$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2} = \frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2} = \frac{\left(-\frac{b}{a}\right)^2-2\times\frac{c}{a}}{\left(\frac{c}{a}\right)^2}$$

$$= \frac{\frac{b^2}{a^2}-\frac{2c\times a}{a\times a}}{\frac{c^2}{a^2}} = \frac{b^2-2ac}{\cancel{a^2}_{1}}\times\frac{\cancel{a^2}^{1}}{c^2}$$

$$= \frac{b^2-2ac}{c^2}$$

ATQ, $\alpha+\beta = \frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$$\Rightarrow -\frac{b}{a} = \frac{b^2-2ac}{c^2}$$

$$\Rightarrow -bc^2 = ab^2-2a^2c$$

$$\Rightarrow \underline{\underline{2a^2c = ab^2+bc^2}}$$

2) Let the terms be $a-3d, a-d, a+d, a+3d$.

ATQ, $a-\cancel{3d}+a-\cancel{d}+a+\cancel{d}+a+\cancel{3d} = 32$

$$4a = 32$$

$$\boxed{a = 8}$$

Also, $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

$$\Rightarrow \frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$$

$$\Rightarrow 15a^2-135d^2 = 7a^2-7d^2$$

$$\Rightarrow 8a^2 = 128d^2$$

$$\Rightarrow 8\times 64 = 128d^2$$

$$d^2 = \frac{\cancel{8\times64}^{1}}{\cancel{128}_{2}} = 4$$

$$\boxed{d = \pm 2}$$

When $a=8$ and $d=2$, the numbers are $2, 6, 10, 14$
When $a=8$ and $d=-2$, the numbers are $14, 10, 6, 2$.

3) Let the usual speed of plane be x km/hr.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

ATQ, $\dfrac{1500}{x} - \dfrac{1500}{x+100} = \dfrac{30}{60}$

$$1500\left[\frac{1}{x} - \frac{1}{x+100}\right] = \frac{1}{2}$$

$$1500\left[\frac{\cancel{x}+100-\cancel{x}}{x^2+100x}\right] = \frac{1}{2}$$

$$100 \times 1500 \times 2 = x^2+100x$$

$$x^2+100x-300000=0$$

$$(x-500)(x+600)=0$$

$$x = 500, -600$$

$$\begin{array}{r|l} 5 & 300000 \\ 3 & 60000 \\ 5 & 20000 \\ 2 & 400 \\ 2 & 200 \\ & 100 \end{array}$$

S	P
100	−30000

$-500, 600$

x cannot be −ve, ∴ required value of $x = 500$

Hence, the usual speed of plane is 500 km/hr.

4) Let the given equations be of the form $ax^2+bx+c=0$
where $a = k+4$; $b = k+1$; $c = 1$.

For equal roots, $b^2-4ac=0$

$$(k+1)^2-4(k+4)=0$$

$$k^2+2k+1-4k-16=0$$

$$k^2-2k-15=0$$

$$(k+3)(k-5)=0.$$

S	P	
−2	−15	3, −5

$$\therefore k = -3, 5$$

5) $\left(\dfrac{n+1}{2}\right)^{th}$ term $= 30$; $n = 11$

6^{th} term $= 30$

$$a_6 = a+5d = 30$$

$$S_{11} = \frac{11}{2}[2a+10d]$$

$$= \frac{11}{2} \times 2[a+5d] = 11 \times 30 = \underline{\underline{330}}$$

6) Since difference between two consecutive terms in an AP is a constant, $a_2 - a_1 = a_3 - a_2$

$$\Rightarrow 2k-1-k-9 = \cancel{2k}+7-\cancel{2k}+1$$

$$\Rightarrow k-10 = 8$$

$$k = 18$$

7) $\dfrac{1 \times (x-3)}{(x-2)(x-1)} + \dfrac{1 \times (x-1)}{(x-2)(x-3)} = \dfrac{2}{3}$

$$\frac{x-3+x-1}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$(2x-4)3 = 2(x^2-3x+2)(x-3)$$

$$6x-12 = 2(x^3-3x^2-3x^2+9x+2x-6)$$

$$6x-12 = 2(x^3-6x^2+11x-6)$$

$$6x\cancel{-12} = 2x^3-12x^2+22x\cancel{-12}$$

$$2x^3-12x^2+16x = 0$$

$$2x(x^2-6x+8) = 0$$

$$(x-4)(x-2) = 0$$

$$x = 2, 4$$

S   P -4

-6 8 < -2

x cannot be 2, $\therefore$ required value of $x = 4$

8) $a_6 = 800 \Rightarrow a+5d = 800$

$a_9 = 1130 \Rightarrow a+8d = 1130$

$(-)$, $\quad -3d = -330$

$$d = 110$$

From eq: (1), $a + 550 = 800$

$$a = 250$$

(i) $a_2 = a+d = 250+110 = 360$ (b)

(ii) $a_4 = a+3d = 250+330 = 580$

$\therefore$ increase in production $= 580-250 = 330$ (c)

(iii) $a_{10} = a+9d = 250+990 = 1240$ (d)

(iv) $n = ?$ $a_n = 1460$

$a+(n-1)d = 1460$

$250+(n-1)110 = 1460$

$110(n-1) = 1210$

$n-1 = 11$

$n = 12^{th}$ year (d)

9) $\sqrt{3}x^2 - 2x - 8\sqrt{3} = 0$

S	P
-2	$-24 < \begin{smallmatrix}-6 \\ 4\end{smallmatrix}$

$\sqrt{3}x^2 + 4x - 6x - 8\sqrt{3} = 0$

$x(\sqrt{3}x+4) - 2\sqrt{3}(\sqrt{3}x+4) = 0$

$(x - 2\sqrt{3})(\sqrt{3}x+4) = 0$

$\therefore x = 2\sqrt{3}$ or $x = -\dfrac{4}{\sqrt{3}}$

10) $\cancel{2}^1 \times \dfrac{9}{\cancel{4}_2} - \dfrac{3k}{2} - 6 = 0 \quad \times 2$

$9 - 3k - 12 = 0$

$-3k = 3$

$k = -1$

11) Let the given equation be of the form $Ax^2 + Bx + C = 0$; where $A = 9$, $B = -9(a+b)$, $C = 2a^2 + 5ab + 2b^2$

$B^2 - 4AC = 81(a+b)^2 - 4 \times 9(2a^2 + 5ab + 2b^2)$

$= 81(a^2 + b^2 + 2ab) - 36(2a^2 + 5ab + 2b^2)$

$= 81a^2 + 81b^2 + 162ab - 72a^2 - 180ab - 72b^2$

$= 9a^2 + 9b^2 - 18ab$

$= 9(a^2 + b^2 - 2ab) = 9(a-b)^2$

$\therefore x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \dfrac{9(a+b) \pm 3(a-b)}{18}$

$x = \dfrac{9a + 9b + 3a - 3b}{18} = \dfrac{12a + 6b}{18} = \dfrac{2a+b}{3}$

$x = \dfrac{9a + 9b - 3a + 3b}{18} = \dfrac{6a + 12b}{18} = \dfrac{a+2b}{3}$

12) Let the digit in the tens place be x and that in the ones place be y.

ATQ, $10x + y = 4(x+y) \Rightarrow 10x + y = 4x + 4y$

$\Rightarrow 6x = 3y$

$\Rightarrow 2x = y \rightarrow (1)$

$10x + y = 3xy$

$\Rightarrow 10x + 2x = 3x \times 2x$

$\Rightarrow \cancel{12}^{2} \cancel{x}^{1} = \cancel{6}^{1} x^{\cancel{2}\,1}$

$\boxed{x = 2}$

From eq: (1), $\boxed{y = 4}$

Hence the number is 24

13) Since the difference between two consecutive terms is a constant in an AP,

$$a_2 - a_1 = a_3 - a_2$$
$$m + 20 - 4m = 3m - 2 - m - 20$$
$$-3m + 20 = 2m - 22$$
$$-5m = -42$$
$$m = \frac{42}{5}$$

14) $S_{13} = 182 \Rightarrow \frac{\cancel{13}^{1}}{2}[2a + 12d] = \cancel{182}^{14} \Rightarrow a + 6d = 14 \rightarrow (1)$

$S_{19} = 380 \Rightarrow \frac{\cancel{19}^{1}}{2}[2a + 18d] = \cancel{380}^{20} \Rightarrow a + 9d = 20 \rightarrow (2)$

$(-), \; -3d = -6$

$\boxed{d = 2}$

From eq: (1), $a + 12 = 14$

$\boxed{a = 2}$

$\left[S_n = \frac{n}{2}[2a + (n-1)d] \right]$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[4 + 2(n-1)] = \frac{n}{2}[4 + 2n - 2]$

$= \frac{n}{2}(2 + 2n) = \underline{\underline{n(n+1)}}$

15) $S_6 = 42 \Rightarrow \frac{6}{2}[2a + 5d] = 42$

$\Rightarrow 2a + 5d = 14 \rightarrow (1)$

$\frac{a_{10}}{a_{30}} = \frac{1}{3} \Rightarrow \frac{a + 9d}{a + 29d} = \frac{1}{3}$

$\Rightarrow 3a + 27d = a + 29d$

$\Rightarrow 2a = 2d$

$a = d \rightarrow (2)$

From eq: (1), $2a + 5a = 14$

$7a = 14$

$\boxed{a = 2}$

$\boxed{d = 2}$

$\therefore a_{13} = a + 12d$

$= 2 + 24$

$= \underline{\underline{26}}$

16) (i) $\frac{1-3b}{3} - \frac{1}{3} = \frac{1}{3} - b - \frac{1}{3} = -b$ (b)

(ii) $a = \frac{1}{3}, d = -b$

$a_6 = a+5d = \cancel{\frac{1\times b}{3\times b} - \frac{5\times 3}{b\times 3} = \frac{b-15}{3b}}$

$= \frac{1}{3} - 5b = \frac{1-15b}{3}$ (d)

(iii) $a_n = \frac{1-21b}{3}$

$a+(n-1)d = \frac{1-21b}{3}$

$\cancel{\frac{1}{3}} - b(n-1) = \cancel{\frac{1}{3}} - 7b$

$n-1 = 7$

$n = 8$th (b)

(iv) $a_3 + a_5 = a+2d+a+4d = 2a+6d = \frac{2}{3} - 6b = \frac{2-18b}{3}$ (a)

~~(v) $a_2 - a_6 = a+d-a-5d = -4d = 4b$~~

(v) $a_6 - a_2 = a+5d-a-d = 4d = -4b$ (d)

17) Let the fixed charge be ₹x and charge for distance covered be ₹y.

ATQ, Situation I, $x+10y = 100$
$x+15y = 145$
(−), $-5y = -45$
$y = 9$

$x+90 = 100$
$x = 10$

~~Sit~~ Situation II, $x+8y = 88$
$x+14y = 148$
(−), $-6y = -60$
$y = 10$

$x+80 = 88$
$x = 8$

(a) (ii)

(b) ₹8 (i)

(c) ₹9 (ii)

(d) $10 + 40 \times 9 = 10 + 360 = ₹370$ (iv)

(e) $8 + 35 \times 10 = 350 + 8 = ₹358$ (iii)

18) $12x - 8 = 8x - 40$

$4x = -32$

$x = -8$

19) $a_1 = 7 - 4 = 3$

$a_2 = 7 - 8 = -1$

$\therefore d = a_2 - a_1 = -1 - 3 = -4$

20) $a = 2, b = k, c = 3$

For equal roots, $b^2 - 4ac = 0$

$k^2 - 24 = 0$

$k = \sqrt{24} = \pm 2\sqrt{6}$ (d)

$2 \,|\, 24$, $2 \,|\, 12$, $2 \,|\, 6$, 3

21) $\alpha + \beta = -\frac{b}{a} = \frac{9}{2}$

22) $a_{n-1} = a + (n-1-1)d = a + (n-2)d$

23) $y^2 + 4 - 4y + 1 = 2y^2 - 3$

$-y^2 - 4y + 8 = 0$, is a quadratic equation.

24) parallel

25) $a = 3, b = 2, c = -1$

$b^2 - 4ac = 4 + 12 = 16 > 0$ (a)

26) $\cancel{q - p = r - q}$

$\cancel{2q = r + p}$

$q - p = r - q = s - r$

(b)

27)

$1 + \frac{y^2}{13} = \sqrt{\frac{27 + 169}{169}} = \sqrt{\frac{196}{169}} = \frac{14}{13}$

$\frac{y^2}{13} = \frac{14}{13} - 1 = \frac{1}{13}$

$y^2 = 1$

$y = \pm 1$

28). (1)+(2), $110x+110y=0 \Rightarrow x+y=0$

(1)−(2), $-24x+24y=-48 \Rightarrow x-y=2$

(+), $2x=2$

$\boxed{x=1 \quad y=-1}$

29) $\dfrac{2x}{(x-3)}^{\times 2x+3} + \dfrac{1}{(2x+3)}^{\times(x-3)} + \dfrac{3x+9}{(x-3)(2x+3)} = 0$

$\Rightarrow \dfrac{2x(2x+3)+(x-3)+3x+9}{(x-3)(2x+3)} = 0$

$\Rightarrow \dfrac{4x^2+6x+x-3+3x+9}{2x^2+3x-6x-9} = 0$

$\Rightarrow 4x^2+10x+6=0$ S 5 P 6 < 2, 3

$\Rightarrow 2x^2+5x+3=0$

$\Rightarrow 2x^2+2x+3x+3=0$

$\Rightarrow 2x(x+1)+3(x+1)=0$

$\Rightarrow (2x+3)(x+1)=0$

$\therefore x = -\frac{3}{2}, -1$

x cannot be $-3/2$, $\therefore$ required value of $x=-1$

30) Let the usual speed of the train be x km/hr and the usual time taken be y hours

Distance = speed × time

ATQ, $(x+10)(y-2)=xy$

$\Rightarrow \cancel{xy}-2x+10y-20=\cancel{xy}$

$(\div -2) \Rightarrow x-5y+10=0$

$x-5y=-10 \rightarrow (1)$

Also, $(x-10)(y+3)=xy$

$\Rightarrow \cancel{xy}+3x-10y-30=\cancel{xy}$

$\Rightarrow 3x-10y=30 \rightarrow (2)$

(1)×3, $3x-15y=-30$

(2), $\overset{(-)}{3x}\overset{(+)}{-10y}=\overset{(-)}{30}$

(−), $-5y=-60$

$\boxed{y=12}$

$x=-10+60=50$

$\boxed{x=50}$

Hence, the distance covered by the train $= x\times y = 50\times 12$

$= \underline{600\text{ km}}$

Case - Study:-

31) 50, 58, 66, ... forms an AP with $a = 50$ and $d = 8$

(i) $n = 25$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{25} = \frac{25}{2}[2\times50+24\times8] = \frac{25}{2}[100+192] = \frac{25}{\cancel{2}_1}\times\cancel{292}^{146}$

$= 3650$ houses//

(ii) $S_n = 1065$

$a = 21$

$d = 1$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow 1065 = \frac{n}{2}[2\times21+(n-1)]$

$\Rightarrow 2130 = n[42-1+n]$

$\Rightarrow 2130 = n(41+n)$

$\Rightarrow n^2+41n-2130=0$

$\Rightarrow (n+71)(n-30)=0$

$\therefore n = -71, 30$

5	2130
3	426
2	142
	71

S 41, P -2130 < 71, -30

n cannot be negative, $\therefore$ required value of $n = 30$

Hence, No. of rows $= 30$//

(iii) 3, 8, 13, ... 253 forms an A.P with $a = 3, d = 5$

20^{th} term from the last row $= l-(n-1)d$

$= 253 - 19\times5$

$= 253 - 95$

$= 158$ houses//

31) Let the speed of stream be y km/hr.

ATQ, $\frac{48}{20-y} - \frac{48}{20+y} = 1$

$\Rightarrow 48\left[\frac{\cancel{20}+y-\cancel{20}+y}{400-y^2}\right] = 1$

$\Rightarrow 48\times2y = 400-y^2$

$\Rightarrow y^2+96y-400=0$

S 96, P -400 < 100, -4

$\Rightarrow (y+100)(y-4)=0$

$y = -100, 4$

y cannot be -ve.

$\therefore$ Required value of $y = 4$

Hence, speed of stream $= 4$ km/hr

Case-Study :-

32) 12, 15, 18, ... forms an AP with $a = 12, d = 3$

(i) $a_n = 72$

$\Rightarrow a + (n-1)d = 72$

$\Rightarrow 12 + (n-1)3 = 72$

$n - 1 = 20$

$\boxed{n = 21}$

$\therefore$ In 21^{st} month, he saved Rs.72

(ii) $a_{13} = a + 12d = 12 + 12 \times 3 = 12 + 36 = ₹48$

$\therefore$ He saved ₹48 in 13^{th} month.

(iii) $\frac{a_7}{a_3} = \frac{12}{5}$

$\Rightarrow \frac{a+6d}{a+2d} = \frac{12}{5}$

$\Rightarrow 5a + 30d = 12a + 24d$

$\Rightarrow -7a = -6d$

$\boxed{a = \frac{6}{7}d}$

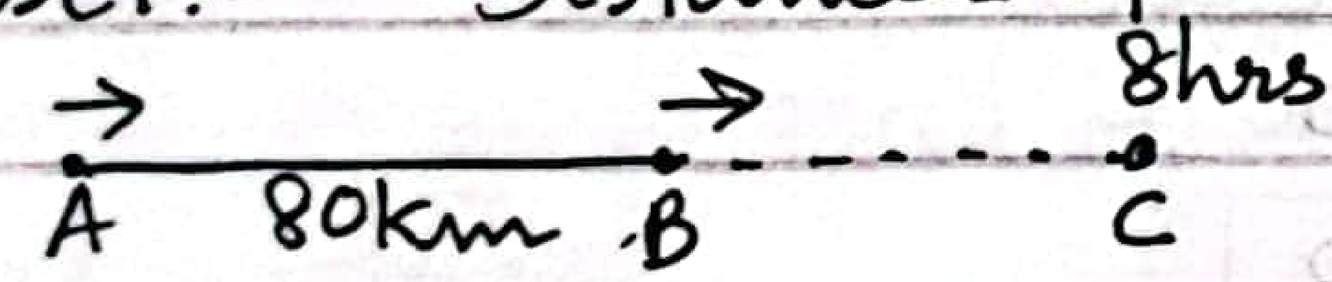
$\frac{a_{13}}{a_4} = \frac{a+12d}{a+3d}$

$= \frac{\frac{6}{7}d + 12d}{\frac{6}{7}d + 3d} = \frac{6d + 84d}{6d + 21d} = \frac{90d}{27d} = \frac{10}{3}$

$\therefore a_{13} : a_4 = 10 : 3$

33) Let the speed of two cars be x km/hr and y km/hr.

Case 1:- Distance = Speed × time

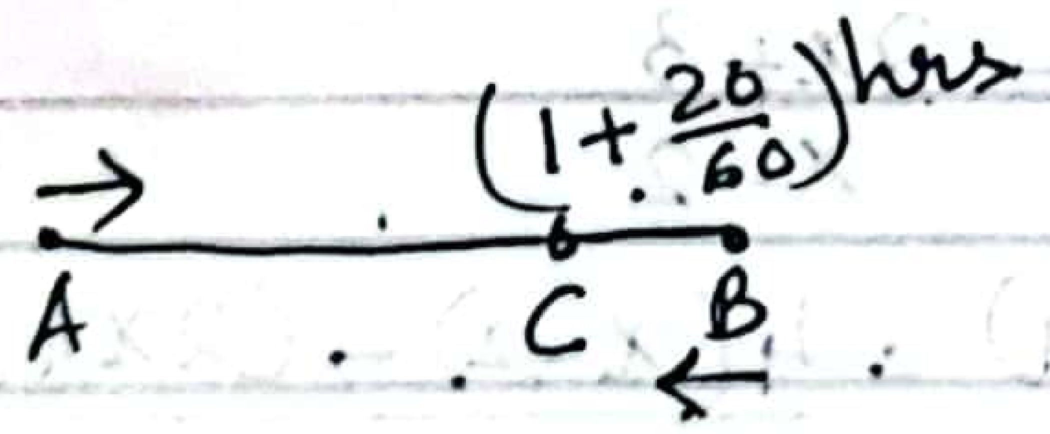


$AB = AC - BC$

$80 = 8x - 8y$

$(\div 8)$, $x - y = 10 \longrightarrow (1)$

Case 2:-



$AB = AC + BC$

$80 = \frac{4}{3}x + \frac{4}{3}y$

$1 + \frac{2}{6} = 1 + \frac{1}{3} = \frac{4}{3}$

$\Rightarrow 240 = 4x + 4y$

$(\div 4)$, $x + y = 60 \longrightarrow (2)$

$(1) + (2)$, $2x = 70$

$\boxed{x = 35}$

$\boxed{y = 25}$

Hence, the speed of two cars are 35 km/hr and 25 km/hr.

34) S.P = C.P + profit % of C.P.

Let the C.P of chair be ₹ x and that of table be ₹ y.

ATQ, $x + 25\%$ of $x + y + 10\%$ of $y = 1520$

$$\Rightarrow x + \frac{25}{100}x + y + \frac{10}{100}y = 1520$$

$$\Rightarrow 125x + 110y = 152000 \longrightarrow (1)$$

Also, $x + 10\%$ of $x + y + 25\%$ of $y = 1535$

$$\Rightarrow x + \frac{10}{100}x + y + \frac{25}{100}y = 1535$$

$$\Rightarrow 110x + 125y = 153500 \longrightarrow (2)$$

(1) + (2), $235x + 235y = 305500$

$(\div 235)$ $\quad x + y = 1300 \longrightarrow (3)$

(1) − (2), $15x - 15y = -1500$

$(\div 15)$ $\quad x - y = -100 \longrightarrow (4)$

(3) + (4), $2x = 1200$

$$\boxed{\begin{aligned} x &= 600 \\ y &= 700 \end{aligned}}$$

Hence, C.P of chair = ₹600

C.P of table = ₹700

35) $$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$$

$$\frac{(x+1)(x+2) + (x-2)(x-1)}{(x-1)(x+2)} = \frac{4(x-2) - (2x+3)}{x-2}$$

$$\frac{x^2 + \cancel{2x} + \cancel{x} + 2 + x^2 - \cancel{x} - \cancel{2x} + 2}{x^2 + 2x - x - 2} = \frac{4x - 8 - 2x - 3}{x-2}$$

$$\frac{2x^2 + 4}{x^2 + x - 2} = \frac{2x - 11}{x - 2}$$

$$(2x^2+4)(x-2) = (2x-11)(x^2+x-2)$$

$$\cancel{2x^3} - 4x^2 + 4x - 8 = \cancel{2x^3} + 2x^2 - 4x - 11x^2 - 11x + 22$$

$$-4x^2 + 4x - 8 = -9x^2 - 15x + 22$$

$$5x^2 + 19x - 30 = 0$$

S	P
19	-150

$25, -6$

$$\begin{array}{r|l} 5 & 150 \\ 5 & 30 \\ & 6 \end{array}$$

$$5x^2 + 25x - 6x - 30 = 0$$

$$5x(x+5) - 6(x+5) = 0$$

$$(5x-6)(x+5) = 0$$

$$x = \frac{6}{5}, -5$$

36) Let the original speed of the train be x km/hr.

$$\frac{360}{x} - \frac{360}{x+5} = \frac{\cancel{48}\,4}{\cancel{60}\,5}$$

$$\Rightarrow 360\left[\frac{\cancel{x}+5-\cancel{x}}{x^2+5x}\right] = \frac{4}{5}$$

$$\Rightarrow \overset{90}{\cancel{360}} \times 25 = \overset{1}{\cancel{4}}(x^2+5x)$$

$$\Rightarrow x^2 + 5x - 2250 = 0$$

$$\Rightarrow (x-45)(x+50) = 0$$

$$\therefore x = 45, -50$$

S	P
5	-2250

$-45, 50$

$$\begin{array}{r|l} 5 & 2250 \\ 5 & 450 \\ 5 & 90 \\ 3 & 18 \\ 3 & 6 \\ & 2 \end{array}$$

x cannot be -ve, $\therefore$ required value of $x = 45$

Hence, original speed of the train = 45 km/hr

37) $4x^2 - 25x = 0$

$x(4x - 25) = 0$

$x = 0, \frac{25}{4}$ (b)

38) Case-Study :-

(a) $a = -15$

$d = -11 + 15 = 4$

$a_{20} = a + 19d = -15 + 19 \times 4 = -15 + 76 = 61$

(b) $d_1 = 4$

$d_2 = 154 - 150 = 4$

$\therefore$ product $= 4 \times 4 = 16$

(c) $a = 150, d = 4$

$$S_{15} = \frac{n}{2}[2a + 14d] = \frac{15}{2}[300 + 14 \times 4] = \frac{15}{2} \times 356 = 15 \times 178 = 2670$$

(d) $a_n = 33$

$a+(n-1)d = 33$

$\Rightarrow -15+4(n-1) = 33$

$n-1 = 12$

$n = 13$

39)

	Tom	Ritu
7 years ago,	$5x^2$	x
present age,	$5x^2+7$	$x+7$
After 3 years,	$5x^2+10$	$x+10$

ATQ, $x+10 = \frac{2}{5}(5x^2+10)$

$5x+50 = 10x^2+20$

$10x^2-5x-30=0$

$2x^2-x-6=0$ S: -1, P: -12 $<$ -4, 3

$2x^2-4x+3x-6=0$

$2x(x-2)+3(x-2)=0$

$(2x+3)(x-2)=0$

$\therefore x = -\frac{3}{2}, 2$

x cannot be -ve $\therefore$ required value of $x=2$

Hence, present age of Tom $= 5\times4+7 = 27$ years

Ritu $= 2+7 = 9$ years

40) $a=m, b=n, c=p$

$b^2-4ac=0$

$n^2-4mp=0$

$p = \frac{n^2}{4m}$

41) Let the number be x

$x-1 = \frac{20}{x}$

$x^2-x-20=0$ S: -1, P: -20 $<$ $+4$, -5

$(x+4)(x-5)=0$

$x = 5, -4$

$\therefore$ the number is 5

(c)

42) $ux + vy = u^2 - v^2 \longrightarrow (1)$

$vx + uy = 0 \longrightarrow (2)$

$(1)-(2),\ (u-v)x + (v-u)y = u^2 - v^2$

$\cancel{(u-v)}x - \cancel{(u-v)}y = (u+v)\cancel{(u-v)}$

$\therefore \underline{\underline{u+v = x-y}}$

43) $4z^2 - 4p^2z + (p^4 - q^4) = 0$

$a = 4,\ b = -4p^2,\ c = p^4 - q^4$

$b^2 - 4ac = 16p^4 - 16(p^4 - q^4)$

$= \cancel{16p^4} - \cancel{16p^4} + 16q^4 = 16q^4$

$\therefore z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{4p^2 \pm 4q^2}{8} = \dfrac{p^2 \pm q^2}{2}$

$\therefore z = \dfrac{p^2+q^2}{2}$ or $\dfrac{p^2-q^2}{2}$ //

44) when $y = -5$, $2 \times 25 - 5m - 15 = 0$

$-5m = -35$

$m = 7$ //

Then, $7(y^2 + y) + n = 0$

$7y^2 + 7y + n = 0$

$a = 7,\ b = 7,\ c = n$

for equal roots, $b^2 - 4ac = 0$

$49 - 4 \times 7 \times n = 0$

$-28n = -49$

$n = \dfrac{7}{4}$ //

45) Let the sides of two squares be x cm and y cm. $(x > y)$

ATQ; $x^2 - 2y^2 = 14 \longrightarrow (1)$

$2x^2 + 3y^2 = 203 \longrightarrow (2)$

$(1)\times 2,\ 2x^2 - 4y^2 = 28$

$(2),\ 2x^2 + 3y^2 = 203$

$(-),\ -7y^2 = -175$

$y^2 = 25$

$y = \pm 5$ cm

y cannot be -ve.

$\therefore \boxed{y = 5}$

$x^2 = 14 + 50 = 64$

$x = \pm 8$

x cannot be -ve.

$\therefore \boxed{x = 8}$

Hence sides are 8cm and 5cm.

46) Let $3-2\sqrt{7}$ is a rational number.

Then, $3-2\sqrt{7} = \frac{p}{q}$; where p and q are co-prime integers and $q \neq 0$

$\Rightarrow 2\sqrt{7} = 3 - \frac{p}{q} = \frac{3q-p}{q}$

$\Rightarrow \sqrt{7} = \frac{3q-p}{2q}$; which is a rational number since p, q are integers.

Then, $\sqrt{7}$ is also a rational number.

But this contradicts the fact that $\sqrt{7}$ is an irrational number. This contradiction arises due to our wrong assumption that $3-2\sqrt{7}$ is rational.

Hence, $3-2\sqrt{7}$ is irrational.

47) $x - y = 1$

$\Rightarrow y = x - 1$

x	0	1	2
y	-1	0	1

$2x + y = 8$

$y = 8 - 2x$

x	0	1	4
y	8	6	0

(graph)

48) $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

$\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$\frac{\cancel{x}-a-b-\cancel{x}}{x(a+b+x)} = \frac{b+a}{ab}$

$\frac{-\cancel{(a+b)}^{-1}}{x(a+b)+x^2} = \frac{\cancel{a+b}^{1}}{ab}$

$\therefore x^2 + x(a+b) + ab = 0$

$(x+a)(x+b) = 0$

$\therefore x = -a, -b$ //

49) Let $p(x) = 5x^2 + 5x + 1$ be of the form ax^2+bx+c where $a = 5, b = 5, c = 1$

$$\alpha + \beta = -\frac{b}{a} = \frac{-5}{5} = -1$$

$$\alpha\beta = \frac{c}{a} = \frac{1}{5}$$

(i) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (-1)^2 - 2 \times \frac{1}{5} = 1 - \frac{2}{5} = \underline{\underline{\frac{3}{5}}}$

(ii) $\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-1}{1} \times 5 = \underline{\underline{-5}}$

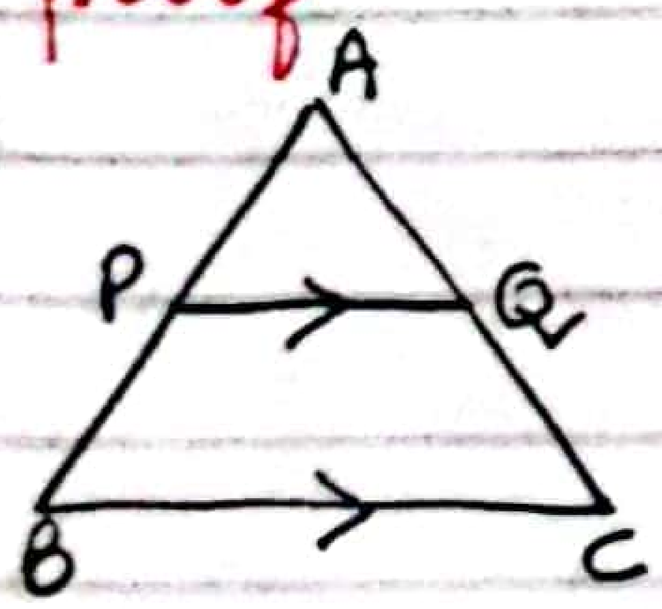
50) $6^n = (2 \times 3)^n = 2^n \times 3^n$

thus the prime factorisation of 6^n does not contain 5 as a prime factor. According to fundamental theorem of arithmetic, the prime factorisation is unique and no other factors occur in the prime factorisation of 6^n. Hence, 6^n cannot end with the digit 0 for any natural number n.

TRIANGLES

51) Proof

52)

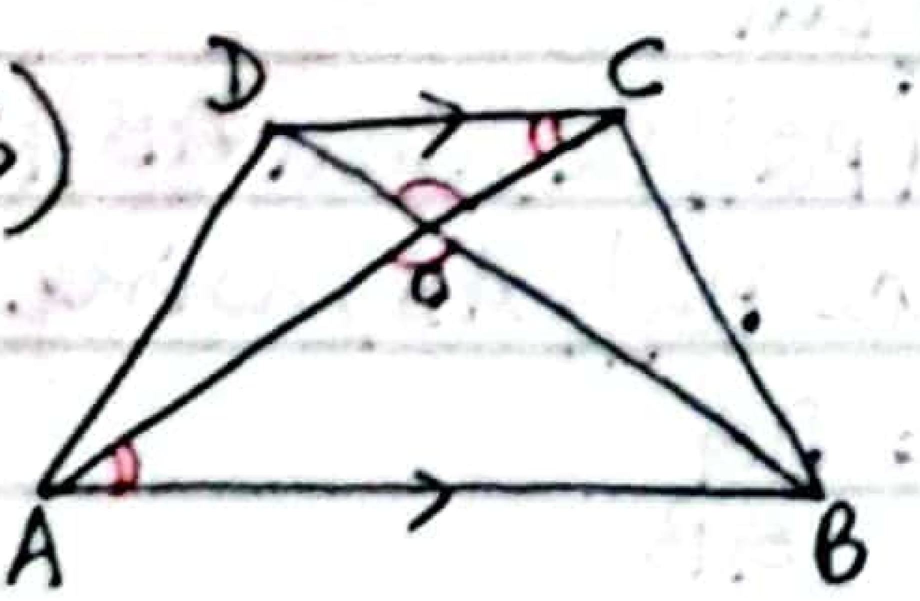


Given: in $\triangle ABC$, $PQ \parallel BC$

To prove: $\frac{AP}{PB} = \frac{AQ}{QC}$

Proof :- in $\triangle ABC$, since $PQ \parallel BC$, using Basic Proportionality theorem, $\frac{AP}{PB} = \frac{AQ}{QC}$. Hence Proved.

53)



In $\triangle OCD$ and $\triangle OAB$, $\angle DOC = \angle BOA$ (VOA)

$\angle OCD = \angle OAB$ (alternate interior $\angle$s)

$\therefore \triangle OCD \sim \triangle OAB$ (AA similarity)

Thus, $\frac{OC}{OA} = \frac{OD}{OB}$ (corresponding sides of similar $\triangle$s are proportional)

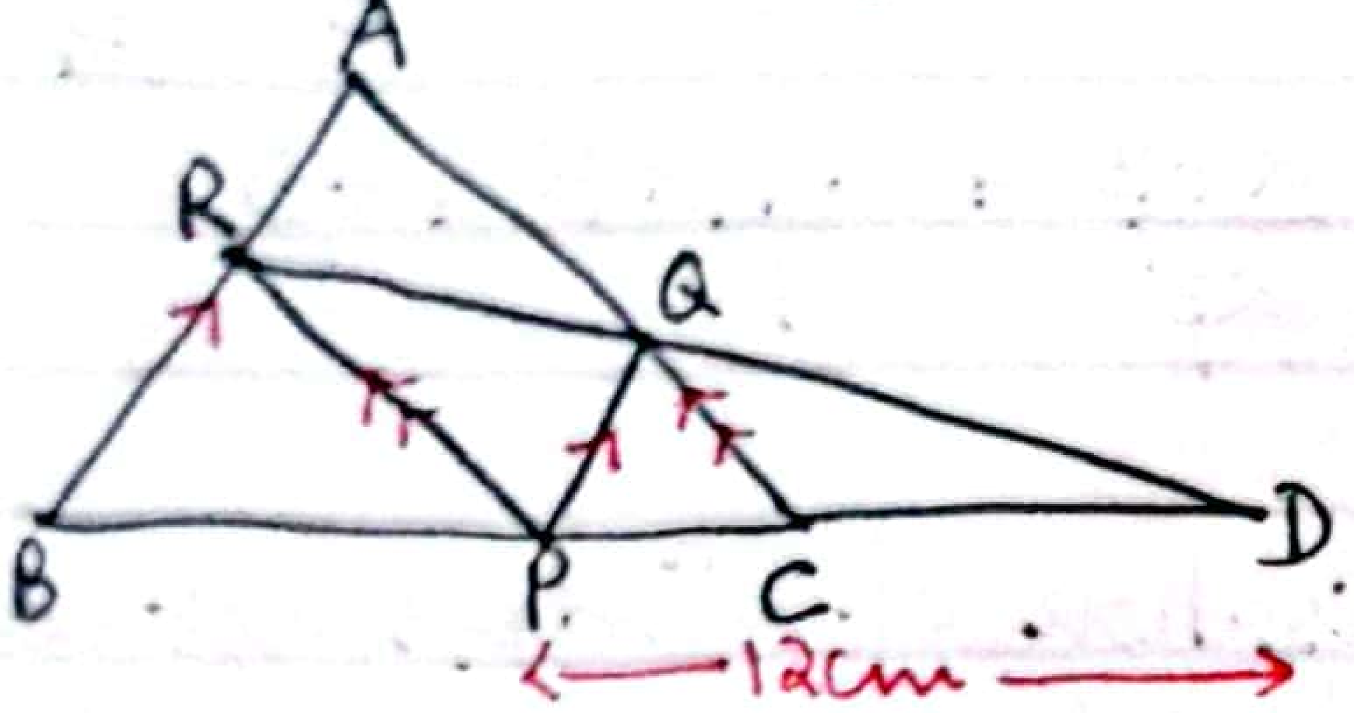
$$\Rightarrow \frac{x+3}{x+5} = \frac{x-2}{x-1}$$

$$\Rightarrow (x+3)(x-1) = (x-2)(x+5)$$

$$\Rightarrow \cancel{x^2} + 2x - 3 = \cancel{x^2} + 3x - 10$$

$$\Rightarrow -x = -7$$

$$\boxed{x = 7}$$

54) 

Given: $PQ \parallel AB$; $PR \parallel CA$.
$PD = 12\text{ cm}$

To find: $BD \times CD$.

In $\triangle BRD$, since $PQ \parallel BR$, using Thales theorem,

$$\frac{PD}{BP} = \frac{QD}{RQ} \Rightarrow \frac{BP}{PD} + 1 = \frac{RQ}{QD} + 1$$

$$\Rightarrow \frac{BD}{PD} = \frac{RD}{QD} \rightarrow (1)$$

In $\triangle RDP$, since $QC \parallel RP$,

$$\frac{CD}{PC} = \frac{QD}{QR}$$

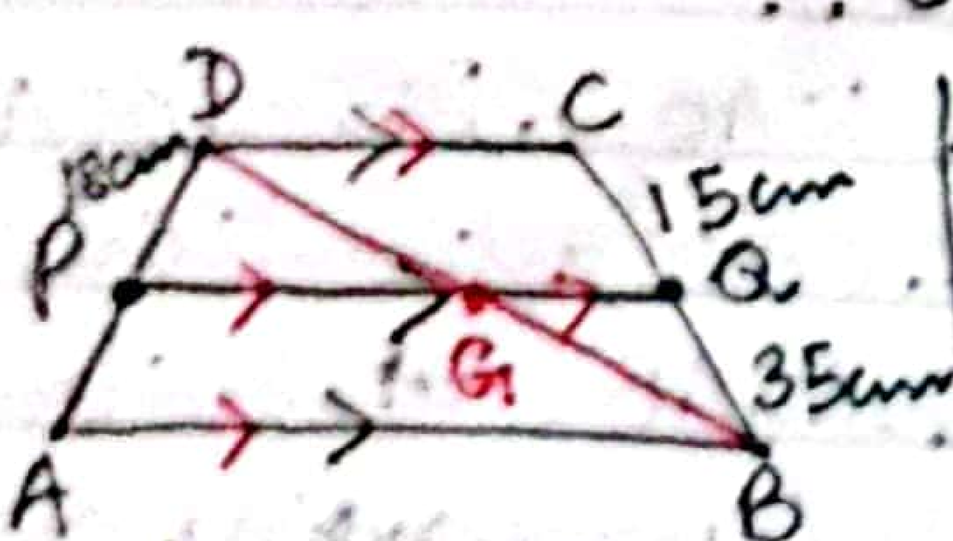
$$\Rightarrow \frac{PC}{CD} + 1 = \frac{QR}{QD} + 1$$

$$\Rightarrow \frac{PD}{CD} = \frac{RD}{QD} \rightarrow (2)$$

From (1) and (2), $\frac{BD}{PD} = \frac{PD}{CD}$

$$\Rightarrow PD^2 = BD \times CD$$

$$\therefore BD \times CD = 12^2 = \underline{144}\text{ cm}^2$$

55) 

Construction:- Join BD to meet PQ at G.

Since $AB \parallel DC$ and $PQ \parallel DC$, then $AB \parallel DC \parallel PQ$

In $\triangle DAB$, since $PG \parallel AB$, using Thales theorem, $\frac{DP}{PA} = \frac{DG}{GB}$

$$\Rightarrow \frac{18}{PA} = \frac{DG}{GB} \rightarrow (1)$$

Similarly, in $\triangle DBC$, since $GQ \parallel DC$, $\frac{DG}{GB} = \frac{CQ}{QB}$

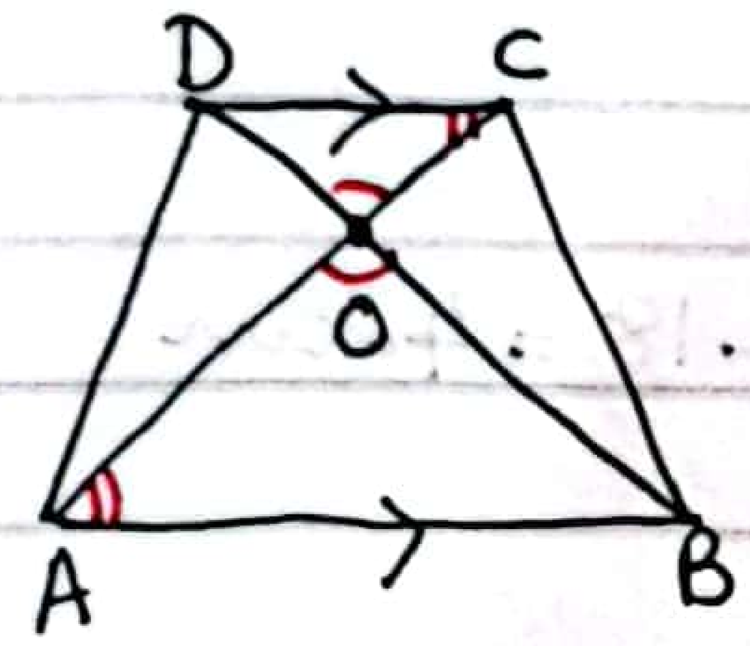
$$\Rightarrow \frac{DG}{GB} = \frac{15}{35} \rightarrow (2)$$

From (1) and (2), $\frac{18}{PA} = \frac{15}{35}$

$$\Rightarrow PA = \frac{18 \times \cancel{35}^{7}}{\cancel{15}_{3}} = 42\,cm$$

$$\therefore AD = 18 + 42 = \underline{60\,cm}$$

56)



Given: in trapezium ABCD, diagonals AC and BD intersect at O.

To prove: $\frac{AO}{BO} = \frac{CO}{DO}$

Proof:- In $\triangle AOB$ and $\triangle COD$, $\angle AOB = \angle COD$ (VOA)

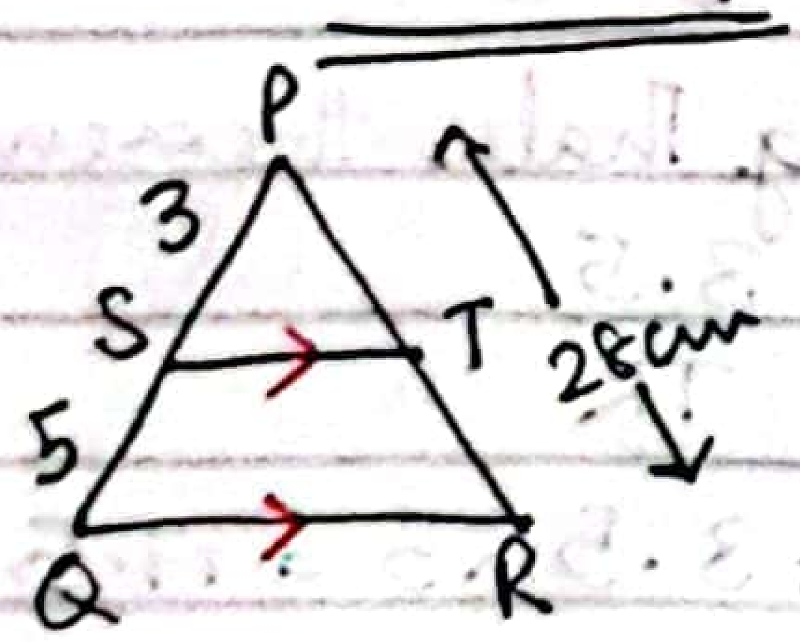
$\angle OAB = \angle OCD$ (alternate interior angles, $AB \parallel DC$)

$\therefore \triangle AOB \sim \triangle COD$ (AA Similarity)

Thus, $\frac{AO}{OC} = \frac{OB}{OD}$ [$\because$ corresponding sides of similar $\triangle$s are in proportion]

$$\Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$$ Hence Proved.

57)



Since $ST \parallel QR$, using Thales theorem,

$$\frac{PS}{SQ} = \frac{PT}{TR}$$

$$\Rightarrow \frac{PS}{SQ} = \frac{PT}{TR}$$

$$\Rightarrow \frac{PS}{SQ} = \frac{PT}{PR - PT}$$

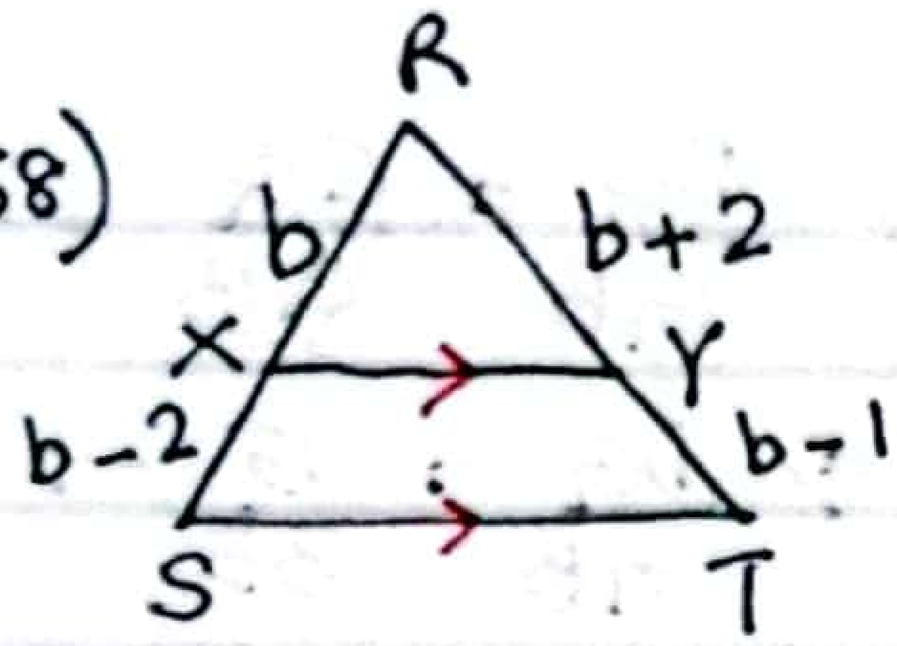
$$\Rightarrow \frac{3}{5} = \frac{PT}{28 - PT}$$

$3(28 - PT) = 5PT$

$84 - 3PT = 5PT$

$8PT = 84$

$PT = 10.5\,cm$

58) 

Since $XY \parallel ST$, using Thales theorem,

$$\frac{RX}{XS} = \frac{RY}{YT}$$

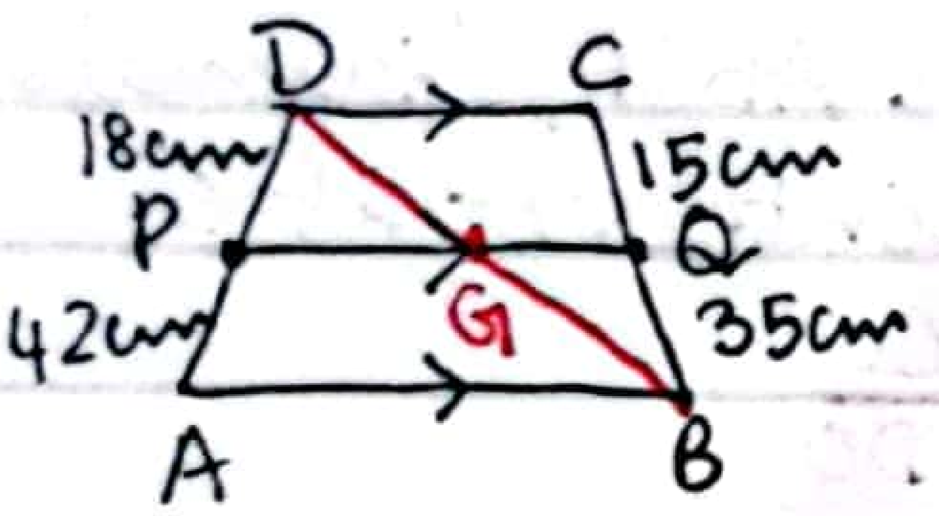
$$\Rightarrow \frac{b}{b-2} = \frac{b+2}{b-1}$$

$$\Rightarrow \cancel{b^2} - b = \cancel{b^2} - 4$$

$$\therefore \boxed{b = 4}$$

59) Assertion: True

Reason:



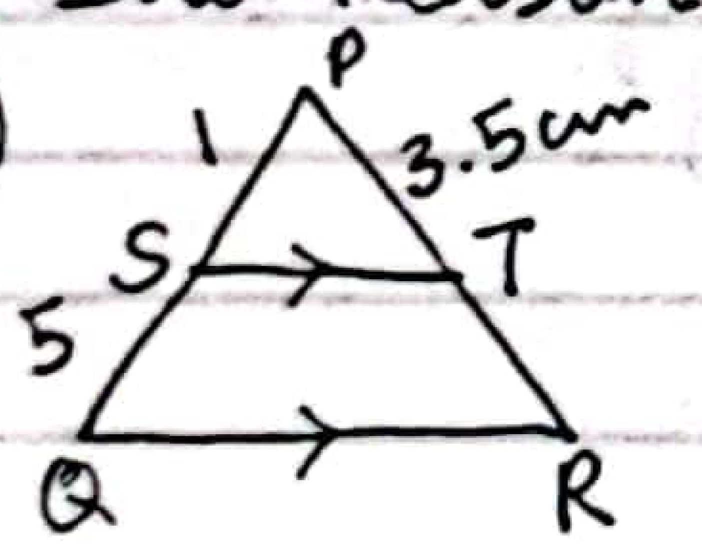
$AP = AD - PD = 60 - 18 = 42cm$

$$\frac{DP}{PA} = \frac{18}{42} = \frac{3}{7}$$

$$\frac{CQ}{QB} = \frac{15}{35} = \frac{3}{7}$$

$\therefore$ using converse of Thales theorem $PQ \parallel DC$ and $PQ \parallel AB$. True

(b) But Reason is not the correct explaination of assertion.

60) 

$$PS = \frac{1}{5} SQ \Rightarrow \frac{PS}{SQ} = \frac{1}{5}$$

Since $ST \parallel QR$, using Thales theorem,

$$\frac{PS}{SQ} = \frac{PT}{TR} \Rightarrow \frac{1}{5} = \frac{3.5}{TR}$$

$$\therefore TR = 3.5 \times 5 = 17.5 \text{ cm}$$

$$\therefore PR = PT + TR = 3.5 + 17.5 = \underline{\underline{21cm}}$$