IX Extra Questions – POLYNOMIALS

1) If $(x+a)$ is a factor of polynomials x^2+px+q and x^2+mx+n, prove that $a=\dfrac{n-q}{m-p}$

2) Factorise (i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

(iii) $25x^2+16y^2+4z^2-40xy+16yz-20xz$

3) Expand : (i) $(2x+1)^3$ (iv) $\left(x-\frac{2}{3}y\right)^3$

(ii) $(2a-3b)^3$ (v) $\left(\frac{1}{x}+\frac{y}{3}\right)^3$

(iii) $(0.1x-0.2y)^3$

4) Evaluate : (i) $(998)^3$ (ii) $(102)^3$ (iii) 101×102

5) Factorise : (i) $8a^3+b^3+12a^2b+6ab^2$

(ii) $2\sqrt{2}a^3+3\sqrt{3}b^3+6\sqrt{3}a^2b+9\sqrt{2}ab^2$

(iii) $8a^3-b^3-12a^2b+6ab^2$

(iv) $27a^3+\frac{1}{64b^3}+\frac{27a^2}{4b}+\frac{9a}{16b^2}$

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

(vi) $64a^3-27b^3-144a^2b+108ab^2$

(vii) $1-64a^3-12a+48a^2$

(viii) $8p^3+\frac{12}{5}p^2+\frac{6}{25}p+\frac{1}{125}$

(ix) $27-125a^3-135a+225a^2$

6) Divide (i) (x^4+1) by $(x-1)$

7) What must be subtracted from $4x^4-2x^3-6x^2+x-5$ so that the result is exactly divisible by $2x^2+x-1$?

8) What must be added to $x^4 + 2x^3 - 2x^2 + x - 1$ so that the result is exactly divisible by $x^2 + 2x - 3$.

9) What must be subtracted from $x^4 + 3x^3 + 4x^2 - 3x - 6$ to get $3x^3 + 4x^2 - x + 3$?

10) What must be added to $2x^2 - 5x + 6$ to get $x^3 - 3x^2 + 3x - 5$?

11) Verify (i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

(iii) $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x+y+z)\left[(x-y)^2 + (y-z)^2 + (z-x)^2\right]$

12) Factorise: (i) $27y^3 + 125z^3$

(ii) $84 - 2r - 2r^2$

(iii) $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$

(iv) $1 + 64x^3$

~~(v) $25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz$~~

(v) $a^{12}y^4 - a^4y^{12}$

(vi) $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$

(vii) $a^7 - ab^6$

(viii) $x(x-y)^3 + 3x^2y(x-y)$

(ix) $a^3 + b^3 + a + b$

(x) $3a^3b - 243ab^3$

(xi) $x^4 - 625$

(x) $27x^3 - (3x-y)^3$

(xi) $16x^2 + 4y^2 + 9z^2 - 16xy - 12yz + 24xz$

(x) $24\sqrt{3}x^3 - 125y^3$

(xi) $x^2 + \frac{1}{x^2} - 2 - 3x + \frac{3}{x}$

(xii) $27x^3 + y^3 + z^3 - 9xyz$

(xiii) $\left(9x - \frac{1}{5}\right)^2 - \left(x + \frac{1}{3}\right)^2$

13) Simplify: $\dfrac{186\times186\times186+14\times14\times14}{186\times186-186\times14+14\times14}$

13) Factorise : $\dfrac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x)^3}$

14) If $p=2-a$, P.T $a^3+6ap+p^3-8=0$

15) $125x^3-27y^3+8+90xy$: Factorise.

16) Factorise : a^6-b^6

17) Without actually calculating cubes: find

(i) $(-12)^3+7^3+5^3$

(ii) $\left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3-\left(\frac{5}{6}\right)^3$

(iii) $(0.2)^3+(0.1)^3-(0.3)^3$

18) If $x+y+z=0$, then find the value of $\frac{x^2}{yz}+\frac{y^2}{zx}+\frac{z^2}{xy}$

19) If $x+\frac{1}{x}=2$, then $x^2+\frac{1}{x^2}=$ ____

20) If $x+\frac{1}{x}=7$, then $x^3+\frac{1}{x^3}=$ ____

21) If $x^2+\frac{1}{x^2}=38$, find $x^3-\frac{1}{x^3}$

22) If $a+b=10$ and $a^2+b^2=58$, find a^3+b^3

23) If $a^2+b^2+c^2=90$ and $a+b+c=20$, find $ab+bc+ca$

24) If $a^{\frac{1}{3}}+b^{\frac{1}{3}}+c^{\frac{1}{3}}=0$, the value of $a+b+c=$ ____

25) Factorise x^8-y^8

IX POLYNOMIALS – extra questions (Answers)

1) Let $p(x) = x^2 + px + q$ and $f(x) = x^2 + mx + n$.
Since $(x+a)$ is a factor of $p(x)$, $p(-a) = 0$
$\Rightarrow a^2 - pa + q = 0 \rightarrow (1)$
Also, since $(x+a)$ is a factor of $f(x)$, $f(-a) = 0$
$\Rightarrow a^2 - ma + n = 0 \rightarrow (2)$
From (1) and (2), $\cancel{a^2} - pa + q = \cancel{a^2} - ma + n$
$ma - pa = n - q$
$a(m-p) = n - q$

$\therefore a = \dfrac{n-q}{m-p}$

2) (i) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2$
$4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$
$= (2x)^2 + (3y)^2 + (-4z)^2 + 2 \times 2x \times 3y + 2 \times 3y \times -4z + 2 \times -4z \times 2x$
$= (2x + 3y - 4z)^2 = (2x + 3y - 4z)(2x + 3y - 4z)$

(ii) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2$
$2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$
$= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2 \times -\sqrt{2}x \times y + 2 \times y \times 2\sqrt{2}z + 2 \times 2\sqrt{2}z \times \sqrt{2}x$
$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2 = (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$

(iii) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2$
$25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz$
$= (-5x)^2 + (4y)^2 + (2z)^2 + 2 \times -5x \times 4y + 2 \times 4y \times 2z + 2 \times 2z \times -5x$
$= (-5x + 4y + 2z)^2 = (-5x + 4y + 2z)(-5x + 4y + 2z)$

3) (i) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(2x+1)^3 = (2x)^3 + 3 \times (2x)^2 \times 1 + 3 \times 2x \times 1^2 \quad (1)^3$
$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$
$(2a - 3b)^3 = (2a)^3 - 3 \times (2a)^2 \times 3b + 3 \times 2a \times (3b)^2 - (3b)^3$
$= 8a^3 - 36a^2b + 54ab^2 - 27b^3$

(iii) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
$(0.1x - 0.2y)^3 = (0.1x)^3 - 3 \times (0.1x)^2 \times 0.2y + 3 \times 0.1x \times (0.2y)^2 - (0.2y)^3$
$= 0.001x^3 - 0.006x^2y + 0.012xy^2 - 0.008y^3$

(iv) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(x-\frac{2}{3}y)^3 = x^3 - \cancel{3}\times x^2 \times \frac{2}{\cancel{3}}y + 3\times x\times\left(\frac{2}{3}y\right)^2 - \left(\frac{2}{3}y\right)^3$

$= x^3 - 2x^2y + \cancel{3}x \times \frac{4}{\cancel{9}\,3}y^2 - \frac{8}{27}y^3$

$= x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3$

(v) $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$\left(\frac{1}{x}+\frac{y}{3}\right)^3 = \left(\frac{1}{x}\right)^3 + \cancel{3}\times\left(\frac{1}{x}\right)^2\times\frac{y}{\cancel{3}} + 3\times\frac{1}{x}\times\left(\frac{y}{3}\right)^2 + \left(\frac{y}{3}\right)^3$

$= \frac{1}{x^3} + \frac{y}{x^2} + \frac{3y^2}{9x} + \frac{y^3}{27}$

$= \frac{1}{x^3}+\frac{y}{x^2}+\frac{y^2}{3x}+\frac{y^3}{27}$

4) (i) $(998)^3 = (1000-2)^3$ $[(a-b)^3 = a^3-3a^2b+3ab^2+b^3]$

$= (1000)^3 - 3\times(1000)^2\times 2 + 3\times1000\times(2)^2 - (2)^3$

$= 1000000000 - 6000000 + 12000 - 8 = 994011992$

(ii) $(102)^3 = (100+2)^3$ $[(a+b)^3 = a^3+3a^2b+3ab^2+b^3]$

$= (100)^3 + 3\times100^2\times2 + 3\times100\times2^2 + 2^3$

$= 1000000+60000+1200+8 = 1061208$

(iii) $101\times102 = (100+1)(100+2)$ $[(x+a)(x+b) = x^2+(a+b)x+ab]$

$=(100)^2+(1+2)\times100+1\times2$

$=10000+200+2 = 10202$

5) (i) $x^3+y^3+3x^2y+3xy^2 = (x+y)^3$

$8a^3+b^3+12a^2b+6ab^2 = (2a)^3+(b)^3+3\times(2a)^2\times b+3\times2a\times b^2$

$=(2a+b)^3 = (2a+b)(2a+b)(2a+b)$

(ii) $x^3+y^3+3x^2y+3xy^2 = (x+y)^3$

$2\sqrt{2}a^3+3\sqrt{3}b^3+6\sqrt{3}a^2b+9\sqrt{2}ab^2$

$=(\sqrt{2}a)^3+(\sqrt{3}b)^3+3\times(\sqrt{2}a)^2\times\sqrt{3}b+3\times\sqrt{2}a\times(\sqrt{3}b)^2$

$=(\sqrt{2}a+\sqrt{3}b)^3 = (\sqrt{2}a+\sqrt{3}b)(\sqrt{2}a+\sqrt{3}b)(\sqrt{2}a+\sqrt{3}b)$

(iii) $x^3-y^3-3x^2y+3xy^2$

$8a^3-b^3-12a^2b+6ab^2$

$=(2a)^3-(b)^3-3\times(2a)^2\times b+3\times2a\times b^2$

$=(2a-b)^3=(2a-b)(2a-b)(2a-b)$

(iv) $x^3+y^3+3x^2y+3xy^2=(x+y)^3$

$27a^3+\frac{1}{64b^3}+\frac{27a^2}{4b}+\frac{9a}{16b^2}$

$=(3a)^3+\left(\frac{1}{4b}\right)^3+3\times(3a)^2\times\frac{1}{4b}+3\times3a\times\left(\frac{1}{4b}\right)^2$

$=\left(3a+\frac{1}{4b}\right)^3=\left(3a+\frac{1}{4b}\right)\left(3a+\frac{1}{4b}\right)\left(3a+\frac{1}{4b}\right)$ //

(v) $a^3-b^3-3a^2b+3ab^2=(a-b)^3$

$27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

$=(3p)^3-\left(\frac{1}{6}\right)^3-3\times(3p)^2\times\frac{1}{6}+3\times3p\times\left(\frac{1}{6}\right)^2$

$=\left(3p-\frac{1}{6}\right)^3=\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)$ //

(vi) $x^3-y^3-3x^2y+3xy^2=(x-y)^3$

$64a^3-27b^3-144a^2b+108ab^2$

$=(4a)^3-(3b)^3-3\times(4a)^2\times3b+3\times4a\times(3b)^2$

$=(4a-3b)^3=(4a-3b)(4a-3b)(4a-3b)$ //

(vii) $a^3-b^3-3a^2b+3ab^2=(a-b)^3$

$1-64a^3-12a+48a^2$

$=(1)^3-(4a)^3-3\times(1)^2\times4a+3\times1\times(4a)^2$

$=(1-4a)^3=(1-4a)(1-4a)(1-4a)$ //

(viii) $a^3+3a^2b+3ab^2+b^3=(a+b)^3$

$8p^3+\frac{12}{5}p^2+\frac{6}{25}p+\frac{1}{125}$

$=(2p)^3+3\times(2p)^2\times\frac{1}{5}+3\times2p\times\left(\frac{1}{5}\right)^2+\left(\frac{1}{5}\right)^3$

$=\left(2p+\frac{1}{5}\right)^3=\left(2p+\frac{1}{5}\right)\left(2p+\frac{1}{5}\right)\left(2p+\frac{1}{5}\right)$ //

(ix) $x^3-y^3-3x^2y+3xy^2=(x+y)^3$

$27-125a^3-135a+225a^2$

$=(3)^3-(5a)^3-3\times(3)^2\times5a+3\times3\times(5a)^2$

$=(3-5a)^3$

$=(3-5a)(3-5a)(3-5a)$ //

6) On dividing $(x^4+0x^3+0x^2+0x+1)$ by $(x-1)$,

$$
\begin{array}{r}
x^3+x^2+x+1 \\
x-1 \,\big)\, x^4+0x^3+0x^2+0x+1 \\
\overset{(-)}{}x^4 \overset{(+)}{-} x^3 \\
\hline
x^3+0x^2+0x+1 \\
\overset{(-)}{}x^3 \overset{(+)}{-} x^2 \\
\hline
x^2+0x+1 \\
\overset{(-)}{}x^2 \overset{(+)}{-} x \\
\hline
x+1 \\
\overset{(-)}{}x \overset{(+)}{-} 1 \\
\hline
\underline{\underline{2}}
\end{array}
$$

quotient, $q(x) = x^3+x^2+x+1$

remainder, $r(x) = 2$

7) Let $p(x) = 4x^4-2x^3-6x^2+x-5$

On dividing $p(x)$ by $2x^2+x-1$,

$$
\begin{array}{r}
2x^2-2x-1 \\
2x^2+x-1 \,\big)\, 4x^4-2x^3-6x^2+x-5 \\
\overset{(-)}{}4x^4 \overset{(-)}{+} 2x^3 \overset{(+)}{-} 2x^2 \\
\hline
-4x^3-4x^2+x-5 \\
\overset{(+)}{-}4x^3 \overset{(+)}{-} 2x^2 \overset{(-)}{+} 2x \\
\hline
-2x^2-x-5 \\
\overset{(+)}{-}2x^2 \overset{(+)}{-} x \overset{(-)}{+} 1 \\
\hline
-\underline{\underline{6}}
\end{array}
$$

$\therefore$ The required polynomial to be subtracted $= -6$//

8) Let $p(x) = x^4+2x^3-2x^2+x-1$

On dividing $p(x)$ by x^2+2x-3,

$$
\begin{array}{r}
x^2+1 \\
x^2+2x-3 \,\big)\, x^4+2x^3-2x^2+x-1 \\
\overset{(-)}{}x^4 \overset{(-)}{+} 2x^3 \overset{(+)}{-} 3x^2 \\
\hline
x^2+x-1 \\
\overset{(-)}{}x^2 \overset{(-)}{+} 2x \overset{(+)}{-} 3 \\
\hline
-x+2
\end{array}
$$

$\therefore$ The required polynomial to be added $= x-\underline{\underline{2}}$

9) Let the polynomial to be subtracted be X.

Then, $x^4+3x^3+4x^2-3x-6-X = 3x^3+4x^2-x+3$

$\therefore X = x^4+\cancel{3x^3}+\cancel{4x^2}-3x-6-\cancel{3x^3}-\cancel{4x^2}+x-3$

$= x^4-2x-9$

10) Let the polynomial to be added be X

Then, $2x^2-5x+6+X = x^3-3x^2+3x-5$

$\therefore X = x^3-3x^2+3x-5-2x^2+5x-6$

$= x^3-4x^2+8x-11$

11) (i) RHS, $(x+y)(x^2-xy+y^2)$

$= x^3-\cancel{x^2y}+\cancel{xy^2}+\cancel{x^2y}-\cancel{xy^2}+y^3$

$= x^3+y^3$, LHS

$\therefore$ LHS = RHS. Hence verified

(ii) RHS, $(x-y)(x^2+xy+y^2)$

$= x^3+\cancel{x^2y}+\cancel{xy^2}-\cancel{x^2y}-\cancel{xy^2}-y^3$

$= x^3-y^3$, LHS

$\therefore$ LHS = RHS. Hence verified

(iii) $x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$= \frac{1}{2}(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2zx)$

$= \frac{1}{2}(x+y+z)((x^2+y^2-2xy)+(y^2+z^2-2yz)+(z^2+x^2-2zx))$

$= \frac{1}{2}(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$

$=$ RHS

$\therefore$ LHS = RHS. Hence verified

12) (i) $27y^3+125z^3 = (3y)^3+(5z)^3$ $[a^3+b^3 = (a+b)(a^2-ab+b^2)]$

$= (3y+5z)(9y^2+25z^2-15yz)$

(ii) $84 - 2r - 2r^2$

$= 2(42 - r - r^2)$

$= -2(r^2 + r - 42)$

$= -2(r-6)(r+7)$

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$1 \quad -42 < {-6 \atop 7}$

(iii) $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$ $\qquad a^2 - b^2 = (a+b)(a-b)$

$= \left(2x + \frac{1}{3}^{\times 2} + x - \frac{1}{2}^{\times 3}\right)\left(2x + \frac{1}{3}^{\times 2} - x + \frac{1}{2}^{\times 3}\right)$

$= \left(3x - \frac{1}{6}\right)\left(x + \frac{5}{6}\right)$

(iv) $1^3 + 64x^3 = (1)^3 + (4x)^3$ $\qquad a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$= (1+4x)(1+16x^2-4x)$

(v) $a^{12}y^4 - a^4y^{12} = a^4y^4(a^8 - y^8)$ $\qquad a^2 - b^2 = (a+b)(a-b)$

$= a^4y^4(a^4+y^4)(a^4-y^4)$

$= a^4y^4(a^4+y^4)(a^2+y^2)(a^2-y^2)$

$= a^4y^4(a^4+y^4)(a^2+y^2)(a+y)(a-y)$

(vi) $x^2 + \frac{1}{x^2} + 2 - 2x - \frac{2}{x}$ $= \left(x^2 + \frac{1}{x^2} + 2\right) - 2\left(x + \frac{1}{x}\right)$

$= \left(x + \frac{1}{x}\right)^2 - 2\left(x + \frac{1}{x}\right)$

$= \left(x + \frac{1}{x}\right)\left[x + \frac{1}{x} - 2\right]$

$= \left(x + \frac{1}{x}\right)\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$

$= \left(x + \frac{1}{x}\right)\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$

(vii) $a^7 - ab^6 = a(a^6 - b^6)$

$= a((a^3)^2 - (b^3)^2)$ $a^2 - b^2 = (a+b)(a-b)$

$= a[a^3 + b^3][a^3 - b^3]$

$= a(a+b)(a^2 - ab + b^2)(a-b)(a^2 + ab + b^2)$

$= a(a+b)(a-b)(a^2 - ab + b^2)(a^2 + ab + b^2)$

(viii) $x(x-y)^3 + 3x^2y(x-y)$

$x[(x-y)^3 + 3xy(x-y)]$

$x(x^3 - y^3)$ $[a^3 - b^3 = (a-b)^3 + 3ab(a-b)]$

$= x(x-y)(x^2 - xy + y^2)$

(ix) $a^3 + b^3 + a + b$

$= (a+b)(a^2 - ab + b^2) + (a+b)$

$= (a+b)(a^2 - ab + b^2 + 1)$

(x) $3a^3b - 243ab^3 = 3ab(a^2 - 81b^2)$ $[a^2 - b^2 = (a+b)(a-b)]$

$= 3ab(a+9b)(a-9b)$

(xi) $x^4 - 625 = (x^2)^2 - (25)^2$ $a^2 - b^2 = (a+b)(a-b)$

$= (x^2 + 25)(x^2 - 25)$

$= (x^2 + 25)(x+5)(x-5)$

(x) $24\sqrt{3}x^3 - 125y^3$

$(2\sqrt{3}x)^3 - (5y)^3$ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$= (2\sqrt{3} - 5y)(12 + 10\sqrt{3}xy + 25y^2)$

(xi) $x^2 + \frac{1}{x^2} - 2 - 3x + \frac{3}{x}$

$= \left(x^2 + \frac{1}{x^2} - 2\right) - 3\left(x - \frac{1}{x}\right)$

$= \left(x - \frac{1}{x}\right)^2 - 3\left(x - \frac{1}{x}\right)$

$= \left(x - \frac{1}{x}\right)\left(x - \frac{1}{x} - 3\right)$

(xii) $27x^3+y^3+z^3-9xyz = (3x)^3+(y)^3+(z)^3-3\times 3x\times y\times z$

$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= (3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz)$

(xiii) $\left(9x-\frac{1}{5}\right)^2-\left(x+\frac{1}{3}\right)^2$ $\quad a^2-b^2=(a+b)(a-b)$

$=\left(9x-\frac{1}{5}^{\times 3}+x+\frac{1}{3}^{\times 5}\right)\left(9x-\frac{1}{5}^{\times 3}-x-\frac{1}{3}^{\times 5}\right)$

$=\left(10x+\frac{2}{15}\right)\left(8x-\frac{8}{15}\right)$

13) $\frac{186\times186\times186+14\times14\times14}{186\times186-186\times14+14\times14} = \frac{186^3+14^3}{186^2-186\times14+14^2}$ $\quad a^3+b^3=(a+b)(a^2-ab+b^2)$

$=\frac{(186+14)\cancel{(186^2-186\times14+14^2)}}{\cancel{186^2-186\times14+14^2}}$

$=186+14=\underline{\underline{200}}$

13) If $a+b+c=0$, then $a^3+b^3+c^3=3abc$

checking: $\cancel{x^2}-\cancel{y^2}+\cancel{y^2}-\cancel{z^2}+\cancel{z^2}-\cancel{x^2}=0$

$\cancel{x}-\cancel{y}+\cancel{y}-\cancel{z}+\cancel{z}-\cancel{x}=0$

$\therefore \frac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x)^3} = \frac{3(x^2-y^2)\times(y^2-z^2)\times(z^2-x^2)}{3(x-y)(y-z)(z-x)}$

$=\frac{\cancel{3}(x+y)\cancel{(x-y)}\times(y+z)\cancel{(y-z)}\times(z+x)\cancel{(z-x)}}{\cancel{3}\cancel{(x-y)}\cancel{(y-z)}\cancel{(z-x)}}$

$=(x+y)(y+z)(z+x)$

14) $x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$$a^3+6ap+p^3-8 = (a)^3+(p)^3+(-2)^3-3\times a\times p\times(-2)$$
$$=(a+p-2)(a^2+p^2+4-ap+2p+2a)$$
$$=(\cancel{a}-\cancel{2}+\cancel{2}-\cancel{a})(a^2+p^2+4-ap+2p+2a)$$
$$=0\times(a^2+p^2+4-ap+2p+2a)$$
$$=\underline{\underline{0}}$$

15) $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$$125x^3-27y^3+8+90xy = (5x)^3+(-3y)^3+(2)^3-3\times5x\times(-3y)\times2$$
$$=(5x-3y+2)(25x^2+9y^2+4+15xy+6y-10x)$$

16) $$a^6-b^6 = (a^3)^2-(b^3)^2$$
$$=(a^3+b^3)(a^3-b^3)$$
$$=\underline{\underline{(a+b)(a^2-ab+b^2)(a-b)(a^2+ab+b^2)}}$$

17) (i) If $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Checking:- $-12+7+5=-12+12=0$

$\therefore (-12)^3+(7)^3+(5)^3 = 3\times-12\times7\times5 = \underline{\underline{-1260}}$

(ii) If $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Checking:- $\frac{1^{\times3}}{2_{\times3}}+\frac{1^{\times2}}{3_{\times2}}-\frac{5}{6} = \frac{3+2-5}{6} = \frac{0}{6} = 0$

$\therefore \left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3+\left(\frac{-5}{6}\right)^3 = 3\times\frac{1}{2}\times\frac{1}{3}\times\frac{-5}{6} = \underline{\underline{-\frac{5}{12}}}$

(iii) If $a+b+c=0$, then $a^3+b^3+c^3=3abc$

~~Checking:- $(0.2)^3+(0.1)^3$~~

Checking:- $0.2+0.1-0.3=0.3-0.3=0$

$\therefore (0.2)^3+(0.1)^3+(-0.3)^3 = 3\times0.2\times0.1\times-0.3$
$$=\underline{\underline{-0.018}}$$

18) If $x+y+z=0$, then $x^3+y^3+z^3=3xyz$

$$\frac{x^2\times x}{yz\times x}+\frac{y^2\times y}{zx\times y}+\frac{z^2\times z}{xy\times z}=\frac{x^3+y^3+z^3}{xyz}=\frac{3xyz}{xyz}=\underline{\underline{3}}$$

19) $a^2+b^2=(a+b)^2-2ab$

$$x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2$$

$$=(2)^2-2=4-2=\underline{\underline{2}}$$

20) $a^3+b^3=(a+b)^3-3ab(a+b)$

$$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)$$

$$=(7)^3-3\times 7$$

$$=343-21=\underline{\underline{322}}$$

21) $(a-b)^2=a^2+b^2-2ab$

$$\left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2$$

$$=38-2=36$$

$$\therefore x-\frac{1}{x}=\sqrt{36}=6$$

Now, $x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)^3+3\left(x-\frac{1}{x}\right)$

$$=6^3+3\times 6$$

$$=216+18$$

$$=\underline{\underline{234}}$$

22) $(a+b)^2=a^2+b^2+2ab$

$$(10)^2=58+2ab$$

$$2ab=100-58=42$$

$$ab=21$$

$a^3+b^3=(a+b)^3-3ab(a+b)$

$$=(10)^3-3\times 21\times 10$$

$$=1000-630$$

$$=\underline{\underline{370}}$$

23) $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

$$(20)^2=90+2(ab+bc+ca)$$

$$400-90=2(ab+bc+ca)$$

$$\therefore ab+bc+ca=\frac{310}{2}=\underline{\underline{155}}$$

24) If $a+b+c=0$, then $a^3+b^3+c^3=3abc$

If $a^{\frac{1}{3}}+b^{\frac{1}{3}}+c^{\frac{1}{3}}=0$, then $\left(a^{\frac{1}{3}}\right)^3+\left(b^{\frac{1}{3}}\right)^3+\left(c^{\frac{1}{3}}\right)^3=3\times a^{\frac{1}{3}}\times b^{\frac{1}{3}}\times c^{\frac{1}{3}}$

$$\Rightarrow a+b+c=\underline{\underline{3(abc)^{1/3}}}$$

25) $x^8 - y^8 = (x^4)^2 - (y^4)^2 = (x^4+y^4)(x^4-y^4)$

$= (x^4+y^4)(x^2+y^2)(x^2-y^2)$

$= \underline{(x^4+y^4)(x^2+y^2)}(x+y)(x-y)$