IX Homework-6

1) Find the value of k so that the polynomial x^3+3x^2-kx-3 has one factor as $x+3$.
2) Find the value of k if $x-2$ is a factor of $f(x) = x^2+kx+2k$.
3) Find the value of a for which $(x-1)$ is a factor of the polynomial $a^2x^3-4ax+4a-1$
4) For what value of k, is the polynomial $p(x) = 2x^3-kx^2+3x+10$ exactly divisible by $x+2$?
5) Find the value of k if $(x-1)$ is a factor of $p(x) = 2x^2+kx+\sqrt{2}$
6) Find the value of a if $2x^4-ax^3+4x^2-x+2$ is divisible by $2x+1$.
7) If $f(x)$ be a polynomial such that $f\left(-\frac{1}{3}\right)=0$, then one of the factors of $f(x)$ is ___
8) If $f(x) = x^2-5x+7$, evaluate $f(2) - f(-1) + f\left(\frac{1}{3}\right)$
9) If $f(x) = 5x^2-4x+5$, find $f(1) + f(-1) + f(0)$

10) If $x = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $y = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, find the value of x^2+y^2
11) If $x = 4-\sqrt{15}$, then find the value of $\left(x+\frac{1}{x}\right)^2$.
12) Rationalise the denominator of $\frac{1}{(\sqrt{2}+\sqrt{3})-\sqrt{4}}$
13) If $x = 3-2\sqrt{2}$, find the value of $\sqrt{x}+\frac{1}{\sqrt{x}}$
14) Simplify: $\frac{4}{(2187)^{-3/7}} - \frac{5}{(256)^{-1/4}} + \frac{2}{(1331^2)^{-1/3}}$
15) Express $0.00323232\ldots$ in the form $\frac{p}{q}$

IX Homework - 6 (Answers)

1) Let $p(x) = x^3 + 3x^2 - kx - 3$.
Since $x+3$ is a factor of $p(x)$, $p(-3) = 0$

$\Rightarrow (-3)^3 + 3(-3)^2 - k(-3) - 3 = 0$

$\Rightarrow -\cancel{27} + \cancel{27} + 3k - 3 = 0$

$3k = 3$

$\boxed{k = 1}$

2) Since $x-2$ is a factor of $f(x)$, then $f(2) = 0$

$\Rightarrow (2)^2 + k \times 2 + 2k = 0$

$\Rightarrow 4 + 2k + 2k = 0$

$4k = -4$

$\boxed{k = -1}$

3) Let $p(x) = a^2x^3 - 4ax + 4a - 1$.
Since $(x-1)$ is a factor of $p(x)$, then $p(1) = 0$

$\Rightarrow a^2(1)^3 - 4a \times 1 + 4a - 1 = 0$

$\Rightarrow a^2 - \cancel{4a} + \cancel{4a} - 1 = 0$

$a^2 = 1$

$\boxed{a = \pm 1}$

4) Since $p(x)$ is exactly divisible by $x+2$, $p(-2) = 0$

$\Rightarrow 2(-2)^3 - k(-2)^2 + 3(-2) + 10 = 0$

$\Rightarrow -16 - 4k - 6 + 10 = 0$

$\Rightarrow -4k = 12$

$\boxed{k = -3}$

5) Since $(x-1)$ is a factor of $p(x)$, then $p(1) = 0$

$\Rightarrow 2 + k + \sqrt{2} = 0$

$\boxed{k = -2 - \sqrt{2}}$

6) Let $p(x) = 2x^4 - ax^3 + 4x^2 - x + 2$.
Since $p(x)$ is divisible by $2x+1$, $p\left(-\frac{1}{2}\right) = 0$

$\Rightarrow 2\left(\frac{-1}{2}\right)^4 - a\left(\frac{-1}{2}\right)^3 + 4\left(\frac{-1}{2}\right)^2 - \left(\frac{-1}{2}\right) + 2 = 0$

$\Rightarrow \overset{1}{\cancel{2}} \times \frac{1}{\underset{8}{\cancel{16}}} + \frac{a}{8} + \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{4}}} + \frac{1}{2} + 2 = 0$

$$\Rightarrow \frac{1+a+8+4+16}{8}=0$$

$$\Rightarrow a+29=0$$

$$\boxed{a=-29}$$

7) $x+\frac{1}{3}$ or $3x+1$

8) $f(2)=(2)^2-5\times2+7=4-10+7=11-10=1$

$f(-1)=(-1)^2-5\times(-1)+7=1+5+7=13$

$f(\frac{1}{3})=(\frac{1}{3})^2-5\times\frac{1}{3}+7=\frac{1}{9}-\frac{5^{\times3}}{3_{\times3}}+7^{\times9}=\frac{1-15+63}{9}=\frac{49}{9}$

$\therefore f(2)-f(-1)+f(\frac{1}{3})=1-13+\frac{49}{9}=-12+\frac{49}{9}=\frac{-108+49}{9}=\underline{\underline{\frac{-59}{9}}}$

9) $f(1)=5-4+5=6$

$f(-1)=5+4+5=14$

$f(0)=5$

$\therefore f(1)+f(-1)+f(0)=6+14+5=\underline{\underline{25}}$

10) $x=\frac{\sqrt5+1}{\sqrt5-1}=\frac{(\sqrt5+1)^2}{(\sqrt5)^2-1^2}=\frac{5+1+2\sqrt5}{5-1}=\frac{6+2\sqrt5}{4}=\frac{2(3+\sqrt5)}{4}$

$=\frac{3+\sqrt5}{2}$

$y=\frac{\sqrt5-1}{\sqrt5+1}=\frac{(\sqrt5-1)^2}{(\sqrt5)^2-1^2}=\frac{5+1-2\sqrt5}{5-1}=\frac{6-2\sqrt5}{4}=\frac{3-\sqrt5}{2}$

$\therefore x^2+y^2=(x+y)^2-2xy$

$=\left(\frac{3+\sqrt5}{2}+\frac{3-\sqrt5}{2}\right)^2-2\left(\frac{3+\sqrt5}{2}\right)\times\left(\frac{3-\sqrt5}{2}\right)$

$=\left(\frac{\cancel{6}^{3}}{\cancel{2}_{1}}\right)^2-2\left(\frac{9-5}{4}\right)=9-2=\underline{\underline{7}}$

11) $x=4-\sqrt{15}$

$\frac{1}{x}=\frac{1}{4-\sqrt{15}}=\frac{4+\sqrt{15}}{(4)^2-(\sqrt{15})^2}=\frac{4+\sqrt{15}}{16-15}=4+\sqrt{15}$

$\therefore\left(x+\frac{1}{x}\right)^2=(4-\cancel{\sqrt{15}}+4+\cancel{\sqrt{15}})^2=8^2=\underline{\underline{64}}$

12) $$\frac{1}{\left[(\sqrt{2}+\sqrt{3})-\sqrt{4}\right]\times\left[(\sqrt{2}+\sqrt{3})+\sqrt{4}\right]}\times\left[(\sqrt{2}+\sqrt{3})+\sqrt{4}\right] = \frac{\sqrt{2}+\sqrt{3}+\sqrt{4}}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{4})^2}$$

$$= \frac{\sqrt{2}+\sqrt{3}+\sqrt{4}}{2+3+2\sqrt{6}-4} = \frac{\sqrt{2}+\sqrt{3}+2}{1+2\sqrt{6}} = \frac{(\sqrt{2}+\sqrt{3}+2)(1-2\sqrt{6})}{1^2-(2\sqrt{6})^2}$$

$$= \frac{\sqrt{2}-2\sqrt{12}+\sqrt{3}-2\sqrt{18}+2-4\sqrt{6}}{1-24}$$

$$\begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array} \quad \begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$$= \frac{\sqrt{2}-4\sqrt{3}+\sqrt{3}-6\sqrt{2}+2-4\sqrt{6}}{-23}$$

$$= \frac{-5\sqrt{2}-3\sqrt{3}-4\sqrt{6}+2}{-23} = \frac{5\sqrt{2}+3\sqrt{3}+4\sqrt{6}-2}{23}$$

13) $$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = x+\frac{1}{x}+2 \longrightarrow (1)$$

$$x = 3-2\sqrt{2}$$

$$\frac{1}{x} = \frac{1}{3-2\sqrt{2}} = \frac{3+2\sqrt{2}}{3^2-(2\sqrt{2})^2} = \frac{3+2\sqrt{2}}{9-8} = 3+2\sqrt{2}$$

$$\therefore \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = 3-2\sqrt{2}+3+2\sqrt{2}+2 = 8$$

$$\therefore \sqrt{x}+\frac{1}{\sqrt{x}} = \sqrt{8} = 2\sqrt{2}$$

14) $$\frac{4}{3^{\cancel{7}^1\times-\frac{3}{\cancel{7}_1}}} - \frac{5}{4^{\cancel{4}^1\times-\frac{1}{\cancel{4}_1}}} + \frac{2}{11^{\cancel{3}^1\times 2\times-\frac{1}{\cancel{3}_1}}} = \frac{4}{3^{-3}} - \frac{5}{4^{-1}} + \frac{2}{11^{-2}}$$

$$= 4\times 3^3 - 5\times 4 + 2\times 11^2$$

$$= 108-20+242 = 330$$

15) Let $x = 0.00323232\ldots$

$$100x = 0.32323232\ldots$$

$$10000x = 32.323232\ldots$$

$$9900x = 32$$

$$x = \frac{\cancel{32}\,8}{\cancel{9900}\,2475} = \frac{8}{2475}$$

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