X Holiday Homework (Answers) POLYNOMIALS

1) If α and β are the zeroes of $f(x) = x^2 - 5x + 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$.

Soln:- Let the given polynomial $f(x) = x^2 - 5x + 4$ be of the form $ax^2 + bx + c$; where $a = 1, b = -5, c = 4$.

Then, $\alpha + \beta = -\frac{b}{a} = 5$

$$\alpha\beta = \frac{c}{a} = 4$$

$$\therefore \frac{1 \times \beta}{\alpha \times \beta} + \frac{1 \times \alpha}{\beta \times \alpha} - 2\alpha\beta = \frac{\beta + \alpha}{\alpha\beta} - 2\alpha\beta = \frac{5}{4} - 2 \times 4$$

$$= \frac{5}{4} - 8 = \frac{5 - 32}{4} = \frac{-27}{4}$$

2) If α and β are the zeroes of $p(y) = 5y^2 - 7y + 1$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

Soln:- Let the given polynomial $p(y) = 5y^2 - 7y + 1$ be of the form $ay^2 + by + c$; where $a = 5, b = -7, c = 1$.

Then, $\alpha + \beta = \frac{-b}{a} = \frac{7}{5}$

$$\alpha\beta = \frac{c}{a} = \frac{1}{5}$$

$$\therefore \frac{1 \times \beta}{\alpha \times \beta} + \frac{1 \times \alpha}{\beta \times \alpha} = \frac{\beta + \alpha}{\alpha\beta} = \frac{7}{5} \times \frac{5}{1} = 7$$

3) If α and β are the zeroes of $f(x) = x^2 - x - 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$.

Soln:- Let the given polynomial $f(x) = x^2 - x - 4$ be of the form $ax^2 + bx + c$; where $a = 1, b = -1, c = -4$.

Then $\alpha + \beta = -\frac{b}{a} = 1$

$$\alpha\beta = \frac{c}{a} = -4$$

$$\therefore \frac{1 \times \beta}{\alpha \times \beta} + \frac{1 \times \alpha}{\beta \times \alpha} - \alpha\beta = \frac{\beta + \alpha}{\alpha\beta} - \alpha\beta$$

$$= \frac{1}{-4} + 4 = -\frac{1}{4} + 4 = \frac{15}{4}$$

4) If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 + x - 2$, find the value of $\frac{1}{\alpha} - \frac{1}{\beta}$.

Soln:- Let the given polynomial $f(x) = x^2 + x - 2$ be of the form $ax^2 + bx + c$; where $a = 1$, $b = 1$ and $c = -2$.

$$\alpha + \beta = -\frac{b}{a} = -1$$

$$\alpha\beta = \frac{c}{a} = -2$$

$$\frac{1\times\beta}{\alpha\times\beta} - \frac{1\times\alpha}{\beta\times\alpha} = \frac{\beta - \alpha}{\alpha\beta} = \sqrt{\frac{(\alpha+\beta)^2 - 4\alpha\beta}{\alpha\beta}} = \sqrt{\frac{1+8}{-2}} = \frac{\sqrt{9}}{-2} = \underline{\underline{-\frac{3}{2}}}$$

5) If one zero of the quadratic polynomial $f(x) = 4x^2 - 8kx - 9$ is negative of the other, find the value of k.

Soln:- Let the given polynomial $f(x) = 4x^2 - 8kx - 9$ be of the form $ax^2 + bx + c$; where $a = 4$, $b = -8k$, $c = -9$.

$$\alpha + (-\alpha) = -\frac{b}{a} = \frac{8k}{4} = 2k$$

$$\Rightarrow 0 = 2k$$

$\therefore \boxed{k = 0}$

6) If the sum of the zeroes of the quadratic polynomial $f(t) = kt^2 + 2t + 3k$ is equal to their product, find the value of k.

Soln:- Let $f(t) = kt^2 + 2t + 3k$ be of the form $ax^2 + bx + c$; where $a = k$, $b = 2$, $c = 3k$

$$\alpha + \beta = -\frac{b}{a} = \frac{-2}{k}$$

$$\alpha\beta = \frac{c}{a} = \frac{3\cancel{k}}{\cancel{k}} = 3$$

ATQ, $\alpha + \beta = \alpha\beta$

$$\Rightarrow \frac{-2}{k} = 3$$

$\therefore \boxed{k = \frac{-2}{3}}$

7) If α and β are the zeroes of the quadratic polynomial $p(x) = 4x^2 - 5x - 1$, find the value of $\alpha^2\beta + \alpha\beta^2$.

Soln:- Let $p(x) = 4x^2 - 5x - 1$ be of the form $ax^2 + bx + c$; where $a = 4$, $b = -5$, $c = -1$.

$$\alpha + \beta = -\frac{b}{a} = \frac{5}{4}$$

$$\alpha\beta = \frac{c}{a} = \frac{-1}{4}$$

$$\therefore \alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha+\beta) = \frac{-1}{4} \times \frac{5}{4} = \underline{\underline{\frac{-5}{16}}}$$

8) If α and β are the zeroes of the quadratic polynomial $f(t) = t^2 - 4t + 3$, find the value of $\alpha^4\beta^3 + \alpha^3\beta^4$.

Soln:- Let $f(t) = t^2 - 4t + 3$ be of the form $at^2 + bt + c$; where $a = 1$, $b = -4$, $c = 3$.

$$\alpha + \beta = -\frac{b}{a} = 4$$

$$\alpha\beta = \frac{c}{a} = 3$$

$$\therefore \alpha^4\beta^3 + \alpha^3\beta^4 = \alpha^3\beta^3(\alpha+\beta)$$
$$= (\alpha\beta)^3(\alpha+\beta)$$
$$= 27 \times 4$$
$$= \underline{\underline{108}}$$

9) If α and β are the zeroes of the quadratic polynomial $f(x) = 6x^2 + x - 2$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.

Soln:- Let $f(x) = 6x^2 + x - 2$ be of the form $ax^2 + bx + c$; where $a = 6$, $b = 1$, $c = -2$.

$$\alpha + \beta = -\frac{b}{a} = -\frac{1}{6}$$

$$\alpha\beta = \frac{c}{a} = \frac{-\cancel{2}^{1}}{\cancel{6}_{3}} = \frac{-1}{3}$$

$$\therefore \frac{\alpha^{\times\alpha}}{\beta_{\times\alpha}} + \frac{\beta^{\times\beta}}{\alpha_{\times\beta}} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$
$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$
$$= \frac{\frac{1}{36} + \frac{2\times 12}{3\times 12}}{-\frac{1}{3}}$$
$$= \frac{25}{\cancel{36}_{12}} \times -\frac{\cancel{3}^{1}}{1}$$
$$= -\underline{\underline{\frac{25}{12}}}$$

10) If α and β are the zeroes of $p(s) = 3s^2 - 6s + 4$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$.

Soln:- Let $p(s) = 3s^2 - 6s + 4$ be of the form $as^2 + bs + c$; where $a = 3$, $b = -6$, $c = 4$.

Then $\alpha + \beta = \frac{-b}{a} = \frac{6}{3} = 2$

$$\alpha\beta = \frac{c}{a} = \frac{4}{3}$$

$$\therefore \frac{\alpha^{\times\alpha}}{\beta_{\times\alpha}} + \frac{\beta^{\times\beta}}{\alpha_{\times\beta}} + 2\left(\frac{1^{\times\beta}}{\alpha_{\times\beta}} + \frac{1^{\times\alpha}}{\beta_{\times\alpha}}\right) + 3\alpha\beta = \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{\alpha+\beta}{\alpha\beta}\right) + 3\alpha\beta$$
$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\alpha+\beta}{\alpha\beta}\right) + 3\alpha\beta$$

$$= \frac{4 - \frac{8}{3}}{\frac{4}{3}} + 2\left(\frac{2}{4/3}\right) + \cancel{3} \times \frac{4}{\cancel{3}}$$

$$= \frac{\cancel{4}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} + \cancel{2} \times \cancel{2} \times \frac{3}{\cancel{4}} + 4 = 1 + 3 + 4 = \underline{\underline{8}}$$

11) If the squared difference of the zeroes of the quadratic polynomial $f(x) = x^2 + px + 45$ is equal to 144, find the value of p.

Soln:- Let $f(x) = x^2 + px + 45$ be of the form $ax^2 + bx + c$; where $a = 1$, $b = p$, $c = 45$ and α, β be the zeroes.

Then, $\alpha + \beta = -\frac{b}{a} = -p$

$\alpha\beta = \frac{c}{a} = 45$

Given, $(\alpha - \beta)^2 = 144$

$\Rightarrow (\alpha+\beta)^2 - 4\alpha\beta = 144$

$\Rightarrow p^2 - 4 \times 45 = 144$

$\Rightarrow p^2 - 180 = 144$

$p^2 = 324$

$\boxed{p = \pm 18}$

12) If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - px + q$, prove that $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{p^4}{q^2} - \frac{4p^2}{q} + 2$

Soln:- Let $f(x) = x^2 - px + q$ be of the form $ax^2 + bx + c$; where $a = 1$, $b = -p$, $c = q$.

Then, $\alpha + \beta = -\frac{b}{a} = p$

$\alpha\beta = \frac{c}{a} = q$

$$\therefore \frac{\alpha^{2 \times \alpha^2}}{\beta^2 \times \alpha^2} + \frac{\beta^{2 \times \beta^2}}{\alpha^2 \times \beta^2} = \frac{\alpha^4 + \beta^4}{\alpha^2\beta^2} = \frac{(\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2}{(\alpha\beta)^2} = \frac{[(\alpha+\beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2}{(\alpha\beta)^2}$$

$$= \frac{(p^2 - 2q)^2 - 2q^2}{q^2} = \frac{p^4 + 4q^2 - 4p^2q - 2q^2}{q^2}$$

$$= \frac{p^4 + 2q^2 - 4p^2q}{q^2} = \frac{p^4}{q^2} + 2 - \frac{4p^2}{q}$$

$$= \frac{p^2}{q^2} - \frac{4p^2}{q} + 2$$

13) If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - p(x+1) - c$, Show that $(\alpha+1)(\beta+1) = 1-c$.

Soln:- Let $f(x) = x^2 - px - p - c$ be of the form ax^2+bx+c; where $a = 1$, $b = -p$, $c = -p-c$.

$$\alpha+\beta = -\frac{b}{a} = p$$

$$\alpha\beta = \frac{c}{a} = -p-c$$

$$\therefore (\alpha+1)(\beta+1) = \alpha\beta + (\alpha+\beta)+1$$
$$= -\cancel{p}-c+\cancel{p}+1 = \underline{\underline{1-c}}$$

14) If α and β are the zeroes of a quadratic polynomial such that $\alpha+\beta = 24$ and $\alpha-\beta = 8$, find a quadratic polynomial having α and β as its zeroes.

Soln:-

$$\alpha+\beta = 24 \rightarrow (1)$$
$$\alpha-\beta = 8 \rightarrow (2)$$

$(1)+(2)$, $2\alpha = 32$

$$\alpha = 16$$
$$\beta = 8$$
$$\therefore \alpha\beta = 16 \times 8 = 128$$

$\therefore$ The required polynomial is $k[x^2 - (\alpha+\beta)x + \alpha\beta]$; where k is any non-zero real number

$$= k[x^2 - 24x + 128]$$
$$= \underline{\underline{x^2 - 24x + 128}}\text{; where } k = 1$$

15) If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 1$, find a quadratic polynomial whose zeroes are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$.

Soln:- Let $f(x) = x^2 + 0x - 1$ be form the form ax^2+bx+c; where $a=1$, $b=0$, $c=-1$.

$$\alpha+\beta = -\frac{b}{a} = 0$$

$$\alpha\beta = \frac{c}{a} = -1$$

For the required polynomial,

sum of zeroes $= \frac{2\alpha \times \alpha}{\beta \times \alpha} + \frac{2\beta \times \beta}{\alpha \times \beta} = \frac{2(\alpha^2+\beta^2)}{\alpha\beta} = \frac{2[(\alpha+\beta)^2 - 2\alpha\beta]}{\alpha\beta}$

$$= \frac{2[0+2]}{-1} = \underline{\underline{-4}}$$

Product of zeroes $= \frac{2\alpha}{\beta} \times \frac{2\beta}{\alpha} = 4$

$\therefore$ The required polynomial is

$k[x^2 - (\text{sum of zeroes})x + \text{product of zeroes}]$; where k is any non-zero real number.

$= k[x^2 + 4x + 4]$

$= x^2 + 4x + 4$; $k = 1$

16) If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - 3x - 2$, find a quadratic polynomial whose zeroes are $\frac{1}{2\alpha+\beta}$ and $\frac{1}{2\beta+\alpha}$.

Soln:- Let α and β be the zeroes of $f(x) = x^2 - 3x - 2$ which is of the form $ax^2 + bx + c$; where $a = 1, b = -3, c = -2$.

$\alpha + \beta = \frac{-b}{a} = 3$

$\alpha\beta = \frac{c}{a} = -2$

For the new polynomial :-

Sum of zeroes $= \frac{1}{2\alpha+\beta} + \frac{1}{2\beta+\alpha} = \frac{2\beta+\alpha+2\alpha+\beta}{(2\alpha+\beta)(2\beta+\alpha)}$

$= \frac{3\beta + 3\alpha}{4\alpha\beta + 2\alpha^2 + 2\beta^2 + \alpha\beta}$

$= \frac{3(\alpha+\beta)}{5\alpha\beta + 2[(\alpha+\beta)^2 - 2\alpha\beta]} = \frac{9}{-10 + 2[9+4]}$

$= \frac{9}{-10+26} = \frac{9}{16}$

Product of zeroes $= \frac{1}{2\alpha+\beta} \times \frac{1}{2\beta+\alpha} = \frac{1}{4\alpha\beta + 2(\alpha^2+\beta^2) + \alpha\beta} = \frac{1}{16}$

$\therefore$ The required polynomial is $k[x^2 - (\text{sum of zeroes})x + \text{product of zeroes}]$; where k is any non-zero real number.

$= k[x^2 - \frac{9}{16}x + \frac{1}{16}] = \frac{k}{16}[16x^2 - 9x + 1] = 16x^2 - 9x + 1$; where $k = 16$

17) If α and β are the zeroes of the polynomial $f(x) = x^2+px+q$, form a polynomial whose zeroes are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$.

Soln:- $\alpha+\beta = -p$

$\alpha\beta = q$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$
$= p^2 - 4q$

For the new polynomial:-

Sum of zeroes $= (\alpha+\beta)^2 + (\alpha-\beta)^2 = \alpha^2+\beta^2+\cancel{2\alpha\beta}+\alpha^2+\beta^2-\cancel{2\alpha\beta}$
$= 2(\alpha^2+\beta^2)$
$= 2[(\alpha+\beta)^2 - 2\alpha\beta]$
$= 2[p^2-2q]$

Product of zeroes $= (\alpha+\beta)^2 \times (\alpha-\beta)^2 = p^2(p^2-4q)$

$\therefore$ The required polynomial is
k {x^2 − (sum of zeroes)x + product of zeroes}; where k is any non-zero real number

$= k\{x^2 - 2(p^2-2q)x + p^2(p^2-4q)\}$

$= x^2 - 2(p^2-2q)x + p^2(p^2-4q)$; where $k=1$

18) If α and β are the zeroes of the quadratic polynomial $f(x) = x^2-2x+3$, find a polynomial whose roots are
(i) $\alpha+2, \beta+2$ (ii) $\dfrac{\alpha-1}{\alpha+1}, \dfrac{\beta-1}{\beta+1}$

Soln:- (i) Let $f(x) = x^2-2x+3$ be of the form ax^2+bx+c; where $a=1, b=-2, c=3$.

$\alpha+\beta = -\dfrac{b}{a} = 2$

$\alpha\beta = \dfrac{c}{a} = 3$

For new polynomial:-

Sum of zeroes $= \alpha+2+\beta+2 = (\alpha+\beta)+4 = 2+4 = 6$

product of zeroes $= (\alpha+2)(\beta+2) = \alpha\beta + 2\alpha + 2\beta + 4$
$= \alpha\beta + 2(\alpha+\beta) + 4$
$= 3+4+4 = 11$

$\therefore$ The required polynomial is k[x^2 − (sum of zeroes)x + product of zeroes]; where k is any non-zero real number.

$$= k[x^2 - 6x + 11] = x^2 - 6x + 11 \text{; where } k = 1$$

(ii) For the new polynomial:-

Sum of zeroes $= \dfrac{\alpha - 1}{\alpha + 1} + \dfrac{\beta - 1}{\beta + 1} = \dfrac{(\alpha-1)(\beta+1) + (\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)}$

$$= \frac{\alpha\beta + \cancel{\alpha} - \cancel{\beta} - 1 + \alpha\beta + \cancel{\beta} - \cancel{\alpha} - 1}{\alpha\beta + (\alpha + \beta) + 1}$$

$$= \frac{2\alpha\beta - 2}{\alpha\beta + (\alpha+\beta) + 1} = \frac{6 - 2}{3 + 2 + 1} = \frac{4}{6} = \frac{2}{3}$$

product of zeroes $= \left(\dfrac{\alpha - 1}{\alpha + 1}\right)\left(\dfrac{\beta - 1}{\beta + 1}\right) = \dfrac{\alpha\beta - \alpha - \beta + 1}{\alpha\beta + \alpha + \beta + 1}$

$$= \frac{\alpha\beta - (\alpha+\beta) + 1}{\alpha\beta + (\alpha+\beta) + 1} = \frac{3 - 2 + 1}{3 + 2 + 1} = \frac{2}{6} = \frac{1}{3}$$

$\therefore$ The required polynomial is $k[x^2 - (\text{sum of zeroes})x + \text{product of zeroes}]$; where k is any non-zero real number.

$$= k\left[x^2 - \frac{2}{3}x + \frac{1}{3}\right] = x^2 - \frac{2}{3}x + \frac{1}{3} \text{; } k = 1$$

19) If α and β are the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$, then evaluate

(i) $\alpha - \beta$

(ii) $\dfrac{1}{\alpha} - \dfrac{1}{\beta}$

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} - 2\alpha\beta$

(iv) $\alpha^2\beta + \alpha\beta^2$

(v) $\alpha^4 + \beta^4$

(vi) $\dfrac{1}{a\alpha + b} + \dfrac{1}{a\beta + b}$

(vii) $\dfrac{\beta}{a\alpha + b} + \dfrac{\alpha}{a\beta + b}$

(viii) $a\left(\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha}\right) + b\left(\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}\right)$

Soln:- $\alpha + \beta = -\dfrac{b}{a}$

$\alpha\beta = \dfrac{c}{a}$

(i) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = \dfrac{b^2}{a^2} - \dfrac{4c^{\times a}}{a_{\times a}} = \dfrac{b^2 - 4ac}{a^2}$

$\therefore (\alpha - \beta) = \dfrac{\sqrt{b^2 - 4ac}}{a}$

(ii) $\dfrac{1^{\times\beta}}{\alpha\times\beta}-\dfrac{1^{\times\alpha}}{\beta\times\alpha} = \dfrac{\beta-\alpha}{\alpha\beta} = \dfrac{\sqrt{b^2-4ac}}{\cancel{a}_1}\times\dfrac{\cancel{a}^1}{c}$

$= \dfrac{\sqrt{b^2-4ac}}{c}$

$(\beta-\alpha)^2 = (\beta+\alpha)^2 - 4\alpha\beta$

$= \dfrac{b^2}{a^2} - \dfrac{4c}{a}$

$= \dfrac{b^2-4ac}{a^2}$

$\therefore \beta-\alpha = \dfrac{\sqrt{b^2-4ac}}{a}$

(iii) $\dfrac{1}{\alpha}+\dfrac{1}{\beta}-2\alpha\beta = \dfrac{\beta+\alpha}{\alpha\beta} - 2\alpha\beta = \dfrac{\frac{-b}{\cancel{a}1}}{\frac{c}{\cancel{a}1}} - 2\dfrac{c}{a}$

$= -\dfrac{b}{c} - \dfrac{2c}{a} = -\left(\dfrac{b}{c}+\dfrac{2c}{a}\right)$

(iv) $\alpha^2\beta+\alpha\beta^2 = \alpha\beta(\alpha+\beta) = \dfrac{c}{a}\times -\dfrac{b}{a} = \dfrac{-bc}{a^2}$

(v) $\alpha^4+\beta^4 = (\alpha^2)^2+(\beta^2)^2 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$

$= \left[(\alpha+\beta)^2 - 2\alpha\beta\right]^2 - 2(\alpha\beta)^2$

$= \left(\dfrac{b^2}{a^2} - \dfrac{2c\times a}{a\times a}\right)^2 - \dfrac{2c^2}{a^2}$

$= \dfrac{(b^2-2ac)^2}{a^4} - \dfrac{2c^2\times a^2}{a^2\times a^2}$

$= \dfrac{(b^2-2ac)^2 - 2a^2c^2}{a^4}$

(vi) $\dfrac{1}{a\alpha+b}+\dfrac{1}{a\beta+b} = \dfrac{a\beta+b+a\alpha+b}{(a\alpha+b)(a\beta+b)} = \dfrac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}$

$= \dfrac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab(\alpha+\beta)+b^2} = \dfrac{\cancel{a}^1\times\frac{-b}{\cancel{a}_1}+2b}{a^{\cancel{2}1}\times\frac{c}{\cancel{a}_1}+\cancel{a}^1b\times\frac{-b}{\cancel{a}_1}+b^2}$

$= \dfrac{-b+2b}{ac-\cancel{b^2}+\cancel{b^2}} = \dfrac{b}{ac}$

(vii) $\dfrac{\beta}{a\alpha+b}+\dfrac{\alpha}{a\beta+b} = \dfrac{\beta(a\beta+b)+\alpha(a\alpha+b)}{(a\alpha+b)(a\beta+b)} = \dfrac{a\beta^2+b\beta+a\alpha^2+b\alpha}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}$

$= \dfrac{a(\alpha^2+\beta^2)+b(\alpha+\beta)}{a^2\alpha\beta+ab(\alpha+\beta)+b^2} = \dfrac{a\left[(\alpha+\beta)^2-2\alpha\beta\right]+b(\alpha+\beta)}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$

$$= \frac{a\left[\frac{b^2}{a^2} - \frac{2c}{a}\right] + b \times -\frac{b}{a}}{a^2 \times \frac{c}{a} + ab \times -\frac{b}{a} + b^2} = \frac{\cancel{a}\left[\frac{b^2-2ac}{a^{\cancel{2}}}\right] - \frac{b^2}{a}}{ac - \cancel{b^2} + \cancel{b^2}}$$

$$= \frac{\cancel{b^2} - 2ac - \cancel{b^2}}{a^2c} = \frac{-2\cancel{a}\cancel{c}}{a^{2}\cancel{c}} = \underline{\underline{-\frac{2}{a}}}$$

(viii) $a\left[\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right] + b\left[\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right] = a\left[\frac{\alpha^3+\beta^3}{\alpha\beta}\right] + b\left[\frac{\alpha^2+\beta^2}{\alpha\beta}\right]$

$$= a\left[\frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}\right] + b\left[\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}\right]$$

$$= a\left[\frac{\frac{-b^3}{a^3} - 3 \times \frac{c}{a} \times -\frac{b}{a}}{\frac{c}{a}}\right] + b\left[\frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c}{a}}\right]$$

$$= \cancel{a}\left[\frac{-b^3 + 3abc}{a^{\cancel{3}}}\right] \times \frac{\cancel{a}}{c} + b\left[\frac{b^2 - 2ac}{a^{\cancel{2}}}\right] \times \frac{\cancel{a}}{c}$$

$$= \frac{-b^3 + 3abc}{ac} + \frac{b^3 - 2abc}{ac} = \frac{-\cancel{b^3} + 3abc + \cancel{b^3} - 2abc}{ac}$$

$$= \frac{abc}{a\cancel{c}} = \underline{\underline{b}}$$