X Test-5 (Real Numbers and Polynomials)

Section-A (1 mark each)

1) If α and β are the zeroes of a polynomial $f(y) = py^2 - 2y + 3p$ and $\alpha + \beta = \alpha\beta$, then p is __
(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $-\frac{1}{3}$ (d) $\frac{1}{3}$

2) The number of polynomials having zeroes as -2 and 5 is
(a) 1 (b) 3 (c) more than 3 (d) 2

3) The greatest number which when divides 1251, 15628 and 9377 leaves remainder 1, 3 and 2 respectively is
(a) 575 (b) 450 (c) 750 (d) 625

4) The LCM of two numbers is 2400. Which of the following cannot be their HCF?
(a) 300 (b) 400 (c) 500 (d) 600

5) If the zeroes of quadratic polynomial are 1, 1 then the polynomial can be (a) x^2+x+1 (b) x^2-2x+1 (c) x^2+3x+2 (d) x^2+2x+2

6) Assertion: if product of two numbers is 2890 and their HCF is 17, then their LCM is 450.
Reason: LCM is always greater than HCF.

Section-B (2 marks)

7) If one of the zeroes of polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

Section-C (3 marks)

8) Find the zeroes of the quadratic polynomial $7x^2 - \frac{11}{3}x - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

Section-D (5 marks)

9) If α and β are the zeroes of $p(x) = 3x^2 + 2x + 1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$

Section-E (4 marks)

10) A seminar is being conducted by an Educational Organisation where the participants will be educators of different subjects. The number of participants in Hindi, English and Maths are 60, 84 and 108 respectively.

(i) In each room the same number of participants are to be seated and all of them being in the same Subject, hence maximum number of participants that can accommodated in each room are

(a) 14 (b) 12 (c) 16 (d) 18

(ii) The LCM of 60, 84 and 108 is

(a) 3780 (b) 4780 (c) 4680 (b) 3680

(iii) What is the minimum number of rooms required during the event?

(a) 11 (b) 31 (c) 41 (d) 21

(iv) 108 can be prime factorised as

(a) $2^3 \times 3^2$ (b) $2^3 \times 3^3$ (c) $2^2 \times 3^2$ (d) $2^2 \times 3^3$

CamScanner

X Test - 5 (Answers)

1) $f(y) = py^2 - 2y + 3p$; $a = p$, $b = -2$, $c = 3p$

$\alpha + \beta = \alpha\beta$

$\Rightarrow -\frac{b}{a} = \frac{c}{a}$

$\Rightarrow \frac{2}{\cancel{p}_1} = \frac{3p}{\cancel{p}_1}$

$\therefore p = \frac{2}{3}$ (a)

2) more than 3 (c)

3) $1251 - 1 = 1250 = 5^4 \times 2$

$15628 - 3 = 15625 = 5^6$

$9377 - 2 = 9375 = 5^5 \times 3$

HCF $= 5^4 = 625$ (d)

$$\begin{array}{r|l} 5 & 1250 \\ 5 & 250 \\ 5 & 50 \\ 5 & 10 \\ & 2 \end{array} \qquad \begin{array}{r|l} 5 & 15625 \\ 5 & 3125 \\ 5 & 625 \\ 5 & 125 \\ 5 & 25 \\ & 5 \end{array} \qquad \begin{array}{r|l} 5 & 9375 \\ 5 & 1875 \\ 5 & 375 \\ 5 & 75 \\ 5 & 15 \\ & 3 \end{array}$$

4) 500 (c)

5) Sum of zeroes $= 1 + 1 = 2$

product of zeroes $= 1 \times 1 = 1$

$\therefore$ The required polynomial is $x^2 - (\text{Sum of zeroes})x + \text{product of zeroes}$

$= x^2 - 2x + 1$ (b)

6) HCF $\times$ LCM = product of numbers

$450 \times 17 = 7650 \neq 2890$

$$\begin{array}{r} 45 \\ \times\ 17 \\ \hline 315 \\ 45\ \ \\ \hline 7650 \end{array}$$

(d) Assertion is wrong and reason is correct.

7) Let the zeroes be α and 7α and $p(x) = 3x^2 - 8x + 2k + 1$ be of the form $ax^2 + bx + c$; $a = 3$, $b = -8$, $c = 2k + 1$

Sum of zeroes $= 7\alpha + \alpha = -\frac{b}{a}$

$\Rightarrow 8\alpha = \frac{8}{3}$

$\alpha = 1/3$

Product of zeroes

$= 7\alpha \times \alpha = \frac{c}{a}$

$\Rightarrow 7 \times \frac{1}{3} \times \frac{1}{\cancel{3}_1} = \frac{2k+1}{\cancel{3}_1}$

$\Rightarrow 2k = \frac{7}{3} - 1 = \frac{4}{3}$

$k = \frac{2}{3}$

8) Let $p(x) = 7x^2 - \frac{11}{3}x - \frac{2}{3} = 21x^2 - 11x - 2$ be of the form $ax^2 + bx + c$; where $a = 21$, $b = -11$, $c = -2$ and α, β be the zeroes

$7x^2 - \frac{11}{3}x - \frac{2}{3} = 0$

$\Rightarrow 21x^2 - 11x - 2 = 0$

$\Rightarrow 21x^2 - 14x + 3x - 2 = 0$

$\Rightarrow 7x(3x - 2) + 1(3x - 2) = 0$

$\Rightarrow (7x + 1)(3x - 2) = 0$

S: -11 P: -42 ⟨ -14, 3

$\therefore x = -\frac{1}{7}, \frac{2}{3}$ are the zeroes of $p(x)$.

Verification :- let $\alpha = -\frac{1}{7}$, $\beta = \frac{2}{3}$

$\alpha + \beta = -\frac{1}{7} + \frac{2}{3} = \frac{-3+14}{21} = \frac{11}{21} = -\left(\frac{-11}{21}\right) = -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$\alpha\beta = -\frac{1}{7} \times \frac{2}{3} = \frac{-2}{21} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence verified.

9) Let $p(x) = 3x^2 + 2x + 1$ be of the form $ax^2 + bx + c$; where $a = 3$, $b = 2$, $c = 1$.

$\alpha + \beta = -\frac{b}{a} = \frac{-2}{3}$

$\alpha\beta = \frac{c}{a} = \frac{1}{3}$

For the new polynomial: zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$

CS CamScanner

Sum of zeroes $= \dfrac{1-\alpha}{1+\alpha} + \dfrac{1-\beta}{1+\beta}$

$$= \frac{(1-\alpha)(1+\beta) + (1-\beta)(1+\alpha)}{(1+\alpha)(1+\beta)}$$

$$= \frac{1+\cancel{\beta}-\cancel{\alpha}-\alpha\beta+1+\cancel{\alpha}-\cancel{\beta}-\alpha\beta}{(1+\alpha)(1+\beta)}$$

$$= \frac{2-2\alpha\beta}{1+(\alpha+\beta)+\alpha\beta} = \frac{2-2\times\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}}$$

$$= \frac{\frac{6-2}{\cancel{3}\,1}}{\frac{3-2+1}{\cancel{3}\,1}} = \frac{4}{2} = \underline{\underline{2}}$$

Product of zeroes $= \left(\dfrac{1-\alpha}{1+\alpha}\right)\left(\dfrac{1-\beta}{1+\beta}\right)$

$$= \frac{1-\beta-\alpha+\alpha\beta}{1+\alpha+\beta+\alpha\beta}$$

$$= \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta} = \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}}$$

$$= \frac{\frac{3+2+1}{\cancel{3}\,1}}{\frac{3-2+1}{\cancel{3}\,1}} = \frac{6}{2} = \underline{\underline{3}}$$

$\therefore$ The required polynomial is $k[x^2 - (\text{Sum of zeroes})x + \text{product of zeroes}]$;

Where k is any non-zero real number.

$= k[x^2 - 2x + 3]$

$= \underline{x^2 - 2x + 3}$; where $k = 1$

10) $60 = 5 \times 3 \times 2^2$

$84 = 7 \times 3 \times 2^2$

$108 = 2^2 \times 3^3$

$$\begin{array}{r|l} 5 & 60 \\ \hline 3 & 12 \\ \hline 2 & 4 \\ \hline & 2 \end{array} \qquad \begin{array}{r|l} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

(i) Maximum number of participants in each room
$= HCF(60, 84, 108)$
$= 2^2 \times 3 = 12$ participants in each room (b)

(ii) $LCM(60, 84, 108) = 5 \times 7 \times 2^2 \times 3^3 = 3780$ (a)

(iii) Minimum number of rooms required $= \frac{60}{12} + \frac{84}{12} + \frac{108}{12}$
$= 5 + 7 + 9$
$= 21$ rooms (d)

(iv) $108 = 2^2 \times 3^3$ (d)