Test-3

1) If two positive integers p and q can be expressed as

① $p = a^4 b^3$ and $q = a^2 b^4$; a, b being prime numbers,

then LCM (p, q) is ——

2) The product of LCM and HCF of two natural numbers
② is 24. The difference of two numbers is 2. Find the numbers.

3) Two bells ring at intervals of 72 seconds and 50 seconds respectively. If they first ~~beep~~ bring together at 12 ~~noon~~ mid-night, at what time will they ring again for the second time?
③

④ 4) Prove that $2-5\sqrt{3}$ is irrational; given that $\sqrt{3}$ is irrational.

Answers

1) $a^4 b^4$

2) Let the numbers be x and y.
We know that product of LCM and HCF is equal to product of numbers, then LCM × HCF = 24

$$\Rightarrow xy = 24 \rightarrow (1)$$

Also, $x - y = 2$

$$\Rightarrow x = 2 + y \rightarrow (2)$$

On substituting eq: (2) in eq: (1), $(2+y)y = 24$

$$\Rightarrow y^2 + 2y - 24 = 0$$

$$\Rightarrow (y+6)(y-4) = 0$$

$$\therefore y = -6, 4$$

S P
2 −24
6, −4

Ignoring negative value of y, required value of y = 4
Hence, the numbers are 4 and 6 (2+y)

3) $72 = 2^3 \times 3^2$
$50 = 5^2 \times 2$
$LCM(72, 50) = 2^3 \times 3^2 \times 5^2 = 72 \times 25 = 1800$ seconds = 30 minutes
Hence, the bells ring again at 12:30 am

4) Let us assume $2 - 5\sqrt{3}$ is rational.
Then $2 - 5\sqrt{3} = \frac{a}{b}$; where a and b are co-prime integers and $b \neq 0$

$$-5\sqrt{3} = \frac{a}{b} - 2$$

$$5\sqrt{3} = 2 - \frac{a}{b} = \frac{2b-a}{b}$$

$$\sqrt{3} = \frac{2b-a}{5b}$$

Since a and b are integers, $\frac{2b-a}{5b}$ is a rational no. Then, $\sqrt{3}$ is also a rational no. But this contradicts the fact that $\sqrt{3}$ is irrational. This contradiction arises due to our wrong assumption that $2-5\sqrt{3}$ is rational. Hence $2-5\sqrt{3}$ is an irrational number.