X Arithmetic Progression (Worksheet - 1)

1) How many three-digit numbers are there between 97 to 201, Which are divisible by 3? 33

2) Nidhi saves ₹ 2 on first day of the month, ₹4 on second day, ₹6 on third day and so on. What will be her savings in the month of February 2012? ₹870

3) Ram asks the labour to dig a well upto a depth of 10 m. Labour charges ₹150 for first metre and ₹50 for each subsequent metres. Labour claims ₹550 for the whole work. What should be the actual amount to be paid to the labour? ₹600

4) The ratio of the sums of first m and n terms of an AP is $m^2 : n^2$. Show that the ratio of the mth and nth terms is $(2m-1) : (2n-1)$

5) The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last terms to the product of the two middle terms is 7:15. Find the numbers. 2, 6, 10, 14 / 14, 10, 6, 2

6) The sum of the first p, q, r terms of an A.P are a, b and c respectively, Show that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

7) If the m^{th} term of an AP is $\frac{1}{n}$ and the n^{th} term is $\frac{1}{m}$, then Show that the Sum of mn terms is $\frac{1}{2}(mn+1)$.

8) Solve the equation : $(-4)+(-1)+2+\cdots+x=437$ 50

9) Jaspal Singh repays his total loan of ₹ 118000 by paying every month starting with the first instalment of ₹1000. If he increases the instalment by ₹100 every month, then what amount will be paid by him in the 30th instalment? What amount of loan does he still have to pay after 30th instalment? ₹3900, ₹44500

10) Each year, a tree grows 5cm less than it did the preceding year. If it grew by 1m in the first year, then in how many years will it have ceased growing? 21 years

11) The sum of first n terms of three APs are S_1, S_2 and S_3. The first term of each AP is unity and their common differences are 1, 2 and 3 respectively.
P.T $S_1+S_3=2S_2$

12) The sum of first n, $2n$ and $3n$ terms of an AP are S_1, S_2 and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$

13) find the common difference of an AP whose first term is 5 and the sum of its first 4 terms is half the sum of the next 4 terms. 2

14) If the nth terms of the two A.Ps 9, 7, 5, ... and 24, 21, 18, ... are the same, then find the value of n. Also find that term. 16, −21

15) Find the no. of terms in the sequence $20, 19\frac{1}{3}, 18\frac{2}{3}, \ldots$ of which the sum is 300. Explain the double answer. 25 or 36

16) If four numbers are in A.P such that their sum is 50 and the greatest number is 4 times the least, then find the numbers. 5, 10, 15, 20

17) The sum of 5th and 9th terms of an AP is 72 and sum of 7th and 12th terms is 97. Find the A.P 6, 11, 16, 21, ...

18) Sum of the first n terms of an A.P is $5n^2 - 3n$. Find the AP and also find its 16th term. 2, 12, 22, ... 152

19) If the 12th term of an AP is 213 and the sum of its four terms is 24, then what is the sum of its first 10 terms? $\frac{13560}{9}$

20) The sum of the first three terms of an AP is 33. If the product of the first and the third term exceeds the second term by 29, then find the A.P 2, 11, 20, ... 20, 11, 2, ...

21) Split 207 into three parts such that these are in A.P and the product of the two smaller parts is 4623 67, 69, 71

22) Show that the sum of an AP whose first term is a, second term is b and the last term is c is equal to

$$\frac{(b+c-2a)(a+c)}{2(b-a)}$$

23) An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. find the AP 3, 7, 11, 15, ...

24) Find the sum of all three-digit numbers which leaves remainder 2, when divided by 3. 164850

25) Find the sum of the integers between 100 and 200, that are not divisible by 9. 13167

26) If the sum of first p terms of an AP is same as the sum of its first q terms, then S.T the sum of its first $(p+q)$ terms is 0.

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27) The ratio of the 11th term to the 18th term of an AP is 2 : 3. Find the ratio of 5th term to 21st term and also the ratio of the sum of the first 5 terms to sum of the first 21 terms. 1 : 3 ; 5 : 49

28) If pth, qth and rth terms of an AP are a, b and c respt. then show that $(a-b)r+(b-c)p+(c-a)q=0$

29) If the mth term of an A.P is $\frac{1}{n}$ and nth term is $\frac{1}{m}$, then S.T its mn th term is 1

30) The sum of first n, 2n and 3n terms of an A.P are S_1, S_2 and S_3 respt. P.T $S_3 = 3(S_2 - S_1)$

31) If S_n denotes the sum of first n terms of an AP, then S.T the common difference, d is given by $d = S_n - 2S_{n-1} + S_{n-2}$

32) Find the sum of those integers between 1 and 500, which are multiples of 2 as well as of 5. 49, 12250

34) The angles of a Δ are in A.P, the greatest angle is twice the least. Find all the angles of the Δ 40°, 60°, 80°

35) If $\frac{1+3+5+\cdots \text{ upto } n \text{ terms}}{2+5+8+\cdots \text{ upto } 8 \text{ terms}} = 9$, find the value of n. 30

36) The eighth term of an AP is half its second term and the eleventh term exceeds one-third of its fourth term by 1. Find the 15th term. 3

37) The 16th term of an AP is 1 more than twice its 8th term. If the 12th term is 47, find its nth term. 4n−1

38) Find k, so that k^2+4k+8, $2k^2+3k+6$ and $3k^2+4k+4$ are three consecutive terms of an AP 0

39) Find the 4th term from the end of AP: −11, −8, −5, ... 49 40

40) If k−1, k+3 and 3k−1 are in AP, then find the value of k 4

41) Find the C.d of AP: $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$ −1

42) If the ratio of the sum of first n terms of the two A.Ps is (7n+1) : (4n+27), then find the ratio of their 9th terms 24 : 19

43) In a garden bed, there are 23 rose plants in the first row, 21 are in the 2nd, 19 in 3rd row and so on. There are 5 plants in the last row. How many rows are there of rose plants? Also find the total no. of rose plants in the garden 10, 140

44) A thief runs with a uniform speed of 100m/min. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100m/min in the first min and increases his speed by 10m/min every succeeding minute. After how many minutes, the policeman will catch the thief. 6 min

X AP (worksheet-1 Answers)

1) 102, 105, ... 198 are the required numbers in AP where $a = 102$, $d = 105 - 102 = 3$, $a_n = 198$

$a_n = a + (n-1)d$

$\Rightarrow 198 = 102 + 3(n-1)$

$\Rightarrow 96 = 3(n-1)$

$\therefore n - 1 = 32$

$n = 33$

$\therefore$ There are 33 required no.s divisible by 3.

2) ~~Let~~ The savings Rs 2, Rs 4, Rs 6, ... form an A.P with $a = 2$, $d = 4 - 2 = 2$

$n = 29$ (2012 is a leap year)

$\therefore$ Her total savings on the month of february 2012,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{28} = \frac{29}{2}[2 \times 2 + 28 \times 2] = \frac{29}{2}[4 + 56]$

$= \frac{29}{\cancel{2}} \times \cancel{60} = 29 \times 30$

$= \underline{\underline{\text{Rs } 870}}$

3) The labour charges ₹150, ₹150+50, ₹150+2×50, ...

$\Rightarrow$ ₹150, ₹200, ₹250, ... forms an A.P with $a = 150$, $d = 50$

$n = 10$

$a_n = a + (n-1)d$

$\Rightarrow a_{10} = a + 9d = 150 + 9 \times 50$

$= 150 + 450 = \text{Rs } 600.$

Hence, ₹600 should be paid to the labour.

4) $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\frac{S_m}{S_n} = \frac{m^2}{n^2} \Rightarrow \frac{\frac{\cancel{m}}{\cancel{2}}[2a + (m-1)d]}{\frac{\cancel{n}}{\cancel{2}}[2a + (n-1)d]} = \frac{m^{\cancel{2}}}{n^{\cancel{2}}}$$

$\Rightarrow n[2a+(m-1)d] = m[2a+(n-1)d]$
$\Rightarrow 2an + n(m-1)d = 2am + m(n-1)d$
$\Rightarrow 2an - 2am + [n(m-1) - m(n-1)]d = 0$
$\Rightarrow 2a(n-m) + (\cancel{nm} - n - \cancel{mn} + m)d = 0$
$\Rightarrow 2a(n-m) + (m-n)d = 0$
$\Rightarrow (m-n)d = -2a(n-m)$
$\Rightarrow (m-n)d = 2a(m-n)$
$\therefore \underline{d = 2a}$

Thus, $\dfrac{a_m}{a_n} = \dfrac{a+(m-1)d}{a+(n-1)d} = \dfrac{a+2a(m-1)}{a+2a(n-1)}$

$= \dfrac{a+2am-2a}{a+2an-2a} = \dfrac{2am-a}{2an-a}$

$= \dfrac{\cancel{a}(2m-1)}{\cancel{a}(2n-1)} = \dfrac{2m-1}{2n-1}$ //

$\therefore a_m : a_n = (2m-1) : (2n-1)$

5) Let the four consecutive numbers in AP be $a-3d$, $a-d$, $a+d$ and $a+3d$.

ATQ, $a\cancel{-3d} + a\cancel{-d} + a\cancel{+d} + a\cancel{+3d} = 32$
$4a = 32$
$a = 8$ //

Also, $\dfrac{(a-3d)(a+3d)}{(a-d)(a+d)} = \dfrac{7}{15}$

$\Rightarrow \dfrac{a^2-9d^2}{a^2-d^2} = \dfrac{7}{15}$

$\Rightarrow (a^2-9d^2)15 = 7(a^2-d^2)$
$\Rightarrow 15a^2 - 135d^2 = 7a^2 - 7d^2$
$\Rightarrow 8a^2 - 128d^2 = 0$
$\div 8 \Rightarrow a^2 - 16d^2 = 0$
$\Rightarrow 64 - 16d^2 = 0$
$-16d^2 = -64$
$d^2 = 4$
$d = \pm 2$

When $a=8, d=2$, the numbers are 2, 6, 10, 14

When $a=8, d=-2$ the numbers are 14, 10, 6, 2

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6) $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_p = \frac{p}{2}[2a+(p-1)d] = a$

$\Rightarrow \frac{1}{2}[2a+(p-1)d] = \frac{a}{p}$

$\therefore \frac{a}{p}(q-r) = \frac{q-r}{2}[2a+(p-1)d] \rightarrow (1)$

$S_q = \frac{q}{2}[2a+(q-1)d] = b$

$\Rightarrow \frac{1}{2}[2a+(q-1)d] = \frac{b}{q}$

$\therefore \frac{b}{q}(r-p) = \frac{r-p}{2}[2a+(q-1)d] \longrightarrow (2)$

$S_r = \frac{r}{2}[2a+(r-1)d] = c$

$\Rightarrow \frac{1}{2}[2a+(r-1)d] = \frac{c}{r}$

$\therefore \frac{c}{r}(p-q) = \frac{p-q}{2}[2a+(r-1)d] \rightarrow (3)$

$(1)+(2)+(3), \frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)$

$= \frac{1}{2}[(q-r)(2a+(p-1)d)+(r-p)(2a+(q-1)d)+(p-q)(2a+(r-1)d)$

$= \frac{1}{2}[\cancel{2aq}+q(p-1)d-\cancel{2ar}-r(p-1)d+\cancel{2ar}+r(q-1)d-\cancel{2ap}-p(q-1)d$
$+\cancel{2ap}+p(r-1)d-\cancel{2aq}-q(r-1)d]$

$= \frac{1}{2}[d(\cancel{pq}-\cancel{q}-\cancel{rp}+\cancel{r}+\cancel{rq}-\cancel{r}-\cancel{pq}+\cancel{p}+\cancel{pr}-\cancel{p}-\cancel{qr}+\cancel{q})]$

$= \frac{1}{2}[d\times 0] = \frac{1}{2}\times 0 = \underline{\underline{0}}$

7) $a_m = \frac{1}{n} \Rightarrow a+(m-1)d = \frac{1}{n} \rightarrow (1)$

$a_n = \frac{1}{m} \Rightarrow a+(n-1)d = \frac{1}{m} \rightarrow (2)$

$(1)-(2), \quad d(m\cancel{-1}-n\cancel{+1}) = \frac{1}{n}-\frac{1}{m}$

$\Rightarrow d\,(\cancel{m-n}) = \frac{\cancel{m-n}}{mn} \Rightarrow d = \underline{\underline{\frac{1}{mn}}}$

From eq:(1), $a+(m-1)\times\frac{1}{mn}=\frac{1}{n}$

$$\Rightarrow a+\cancel{\frac{1}{n}}-\frac{1}{mn}=\cancel{\frac{1}{n}}$$

$$\therefore a=\frac{1}{mn}$$

Thus, $S_{mn}=\frac{mn}{2}[2a+(mn-1)d]$

$$=\frac{mn}{2}\left[\frac{2}{mn}+(mn-1)\frac{1}{mn}\right]$$

$$=\frac{mn}{2}\left[\frac{2}{mn}+1-\frac{1}{mn}\right]=\frac{mn}{2}\left[\frac{1}{mn}+1\right]$$

$$=\frac{\cancel{mn}}{2}\left(\frac{1+mn}{\cancel{mn}}\right)=\frac{1}{2}(mn+1)$$

8) Let a and d be the first term and the common difference.
$a=-4$, $d=a_2-a_1=-1+4=3$, $a_n=x$, $S_n=437$.

$a_n=a+(n-1)d$
$\Rightarrow x=-4+3(n-1)$
$\Rightarrow \frac{x+4}{3}=n-1$

$\therefore n=\frac{x+4}{3}+1=\frac{x+4+3}{3}=\frac{x+7}{3}$ //

$S_n=\frac{n}{2}[a+a_n]$

$\Rightarrow 437=\left(\frac{x+7}{6}\right)(-4+x)=\frac{(x+7)(x-4)}{6}=\frac{x^2+3x-28}{6}$

$\Rightarrow 2622=x^2+3x-28$
$\Rightarrow x^2+3x-2650=0$
$\Rightarrow (x-50)(x+53)=0$
$\therefore x=50,-53$

S	P	
3	−2650	−50, 53

x cannot be −ve, $\therefore$ required value of $x=\underline{50}$

9) $S_n=$ Rs 118000
Rs 1000, Rs 1000+100, Rs 1000+2×100, ... 30 instalments forms an A.P with $a=1000$, $d=100$, $n=30$
Amount of loan paid in 30^{th} instalment, a_{30}
$=a+29d=1000+29\times100=1000+2900=$ Rs 3900 //
Total amount paid in 30 instalments, $S_{30}=\frac{n}{2}[a+a_{30}]$
$=\frac{30}{2}[1000+3900]$
$=15\times4900=$ Rs 73,500

$\therefore$ Amount still need to pay $=118000-73500=$ Rs 44,500

10) $d = -5$

$a = 1m = 100cm$

$a_n = 0$

$\Rightarrow a + (n-1)d = 0$

$\Rightarrow 100 - 5(n-1) = 0$

$\Rightarrow 100 - 5n + 5 = 0$

$-5n = -105$

$n = 21$

Hence, the tree will stop its growth after 21 years.

11) $a = 1$

$d_1 = 1, d_2 = 2, d_3 = 3$

$S_1 = \frac{n}{2}[2a + (n-1)d_1] = \frac{n}{2}[2 + n - 1] = \frac{n}{2}(1+n)$

$S_2 = \frac{n}{2}[2a + (n-1)d_2] = \frac{n}{2}[2 + 2n - 2] = n^2$

$S_3 = \frac{n}{2}[2a + (n-1)d_3] = \frac{n}{2}[2 + 3n - 3] = \frac{n}{2}[3n-1]$

$\therefore S_1 + S_3 = \frac{n}{2}(1+n) + \frac{n}{2}(3n-1) = \frac{n}{2}[\cancel{1} + n + 3n - \cancel{1}]$

$= \frac{n}{2} \times 4n = 2n^2 = \underline{\underline{2S_2}}$

12) Let a and d be the first term and the common difference.

$S_1 = \frac{n}{2}[2a + (n-1)d]$

$S_2 = \frac{2n}{2}[2a + (2n-1)d]$

$S_3 = \frac{3n}{2}[2a + (3n-1)d]$

Then, $S_2 - S_1 = \frac{n}{2}[4a + 2d(2n-1) - 2a - d(n-1)]$

$= \frac{n}{2}[\ a + d(4n - 2 - n + 1)]$

$= \frac{n}{2}[2a + d(3n-1)]$

$\therefore 3(S_2 - S_1) = \frac{3n}{2}[2a + (3n-1)d] = S_3$

13) $a = 5$

$S_4 = \frac{1}{2}(S_8 - S_4)$

$S_4 = \frac{S_8}{2} - \frac{S_4}{2} \Rightarrow S_4 + \frac{S_4}{2} = \frac{S_8}{2}$

$\Rightarrow \frac{3S_4}{\cancel{2}_1} = \frac{S_8}{\cancel{2}_1}$

$\Rightarrow 3 \times \frac{4}{2}[2a + (n-1)d] = \frac{8}{2}[2a + (n-1)d]$

$\Rightarrow 6[10 + 3d] = 4[10 + 7d]$

$\Rightarrow 60 + 18d = 40 + 28d$

$\Rightarrow -10d = -20$

$\underline{\underline{d = 2}}$

14) $a = 9$ | $A = 24$

$d = 7 - 9 = -2$ | $D = 21 - 24 = -3$

ATQ, $a_n = A_n$

$\Rightarrow a + (n-1)d = A + (n-1)D$

$\Rightarrow 9 - 2(n-1) = 24 - 3(n-1)$

$\Rightarrow 9 - 2n + 2 = 24 - 3n + 3$

$\Rightarrow 11 - 2n = 27 - 3n$

$\underline{\underline{n = 16}}$

$\therefore a_{16} = a + 15d = 9 + 15 \times -2 = 9 - 30 = \underline{\underline{-21}}$

15) $a = 20$

$d = \frac{58}{3} - 20 = \frac{58 - 60}{3} = -\frac{2}{3}$

$S_n = 300$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 300$

$\Rightarrow \frac{n}{2}\left[40 + (n-1) \times -\frac{2}{3}\right) = 300$

$\Rightarrow \frac{n}{2}\left[\frac{120 - 2n + 2}{3}\right] = 300$

$\Rightarrow n(122 - 2n) = 1800$

$\Rightarrow -2n^2 + 122n - 1800 = 0$

$\Rightarrow n^2 - 61n + 900 = 0$

$\Rightarrow (n - 36)(n - 25) = 0$

$\therefore n = 25, 36$ //

S P
-61 900 < -25, -36

Since a is +ve and common difference is −ve,

$S_{36} = S_{25} = 300.$

16) Let the four numbers be $a-3d, a-d, a+d$ and $a+3d$.

ATQ, $a-3d+a-d+a+d+a+3d = 50$

$$4a = 50$$

$$a = \frac{50}{4} = \frac{25}{2}$$

Also, $a+3d = 4(a-3d)$

$\Rightarrow a+3d = 4a-12d$

$\Rightarrow -3a = -15d$

$\Rightarrow \cancel{-}\cancel{3}^{1}\times\frac{25}{2} = \cancel{-}\cancel{15}^{5}d$

$\therefore d = \frac{5}{2}$

$\therefore$ The required numbers are $a-3d = \frac{25}{2}-\frac{15}{2} = \frac{10}{2} = 5$

$a-d = \frac{25}{2}-\frac{5}{2} = \frac{20}{2} = 10$

$a+d = \frac{25}{2}+\frac{5}{2} = \frac{0}{2} = 15$

$a+3d = \frac{25}{2}+\frac{15}{2} = \frac{40}{2} = 20$

17) $a_5 + a_9 = 72 \Rightarrow a+4d+a+8d = 72$

$\Rightarrow 2a+12d = 72 \longrightarrow (1)$

$a_7 + a_{12} = 97 \Rightarrow a+6d+a+11d = 97$

$\Rightarrow 2a+17d = 97 \longrightarrow (2)$

$(1)-(2),\ -5d = -25$

$\underline{\underline{d = 5}}$

From eq: (1), $2a+60 = 72$

$2a = 12$

$\underline{\underline{a = 6}}$

$\therefore$ The required A.P is $6, 11, 16, 21, \ldots$

18) $S_n = 5n^2 - 3n$

$S_1 = a_1 = 5-3 = 2$

$S_2 = a_1 + a_2 = 5\times4 - 6 = 20-6 = 14$

$\therefore a_2 = S_2 - S_1 = 14-2 = 12$

$d = a_2 - a_1 = 12-2 = 10$

Hence, the required AP is 2, 12, 22, 32, ...

$a_{16} = a+15d = 2+15\times10 = 2+150 = \underline{\underline{152}}$

19) $a_{12} = 213 \Rightarrow a+11d = 213 \rightarrow (1)$

$S_4 = \frac{4}{2}[2a+3d] = 24$

$\Rightarrow 2a+3d = 12 \longrightarrow (2)$

$(1)\times2,\ 2a+22d = 426$

$(2),\ 2a+3d = 12$

$(-),\ 19d = 414$

$\therefore d = \underline{\underline{\frac{414}{19}}}$

From eq: (1), $a + 11\times\frac{414}{19} = 213$

$a = 213 - \frac{4554}{19} = \frac{4047-4554}{19}$

$a = -\frac{507}{19}$

$\therefore S_{10} = \frac{n}{2}[2a+9d] = \frac{10}{2}\left[2\times-\frac{507}{19} + \frac{9\times414}{19}\right]$

$= 5\left[\frac{-1014+3726}{19}\right] = \frac{5\times2712}{19} = \underline{\underline{\frac{13560}{19}}}$

20) $a_1+a_2+a_3 = 33$

$\Rightarrow a+a+d+a+2d = 33$

$\Rightarrow 3a+3d = 33 \Rightarrow a+d = 11 \rightarrow (1)$

$a+d = 11 \Rightarrow a = 11-d$

Also, $a_1\times a_3 - a_2 = 29$

$\Rightarrow a\times(a+2d) - (a+d) = 29$

$\Rightarrow a^2 + 2ad - (a+d) = 29$

$\Rightarrow a^2 + 2ad - 11 = 29$ [from eq: (1)]

$\Rightarrow a(a+2d) = 40$

$(11-d)(11-d+2d) = 40$

$(11-d)(11+d) = 40$

$121 - d^2 = 40$

$d^2 = 81$

$d = \pm 9$

When $d = 9$, $a = 11 - 9 = 2$
Then, the required AP is $2, 11, 20, 29, \dots$
When $d = -9$, $a = 11 + 9 = 20$
Then, the required AP is $20, 11, 2, -7, \dots$

21) Let the three parts of 207 be $a-d$, a and $a+d$.
Then, $a + a \cancel{-d} + a \cancel{+d} = 207$
$3a = 207$
$a = 69$
Also, $a(a-d) = 4623$
$\Rightarrow 69(69-d) = 4623$
$69 - d = 67$
$d = 69 - 67 = 2$
Hence, the three parts of 207 are $a-d = 69-2 = 67$
$a = 69$
$a+d = 69+2 = 71$

22) $d = b - a$
$a_n = c \Rightarrow a + (n-1)d = c$
$\Rightarrow a + (n-1)(b-a) = c$
$\Rightarrow (n-1) = \dfrac{c-a}{b-a}$
$n = \dfrac{c-a}{b-a} + 1 = \dfrac{c-a+b-a}{b-a}$
$= \dfrac{c+b-2a}{b-a}$
$\therefore S_n = \dfrac{n}{2}[a + a_n] = \dfrac{(c+b-2a)(a+c)}{2(b-a)}$

23) $n = 37$
middle term is $\left(\dfrac{n+1}{2}\right)^{th}$ term $= \left(\dfrac{37+1}{2}\right)^{th} = 19^{th}$ term $= a_{19}$

ATQ, $a_{18} + a_{19} + a_{20} = 225$
$\Rightarrow a + 17d + a + 18d + a + 19d = 225$
$\Rightarrow 3a + 54d = 225$
$a + 18d = 75 \rightarrow (1)$
Also, $a_{35} + a_{36} + a_{37} = 429$
$\Rightarrow a + 34d + a + 35d + a + 36d = 429$
$\Rightarrow 3a + 105d = 429$
$\Rightarrow a + 35d = 143 \rightarrow (2)$

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(1) − (2), $-17d = -68$

$\underline{\underline{d = 4}}$

From eq:(1), $a + 18\times4 = 75 \Rightarrow a = 75 - 72 = 3$//

Hence, the A.P is 3, 7, 11, 15, ...

24) 101, 104, 107, ... 998 forms an A.P with $a = 101$ and $d = 3$.

$a_n = 998 \Rightarrow a + (n-1)d = 998$

$\Rightarrow 101 + (n-1)3 = 998$

$n - 1 = \frac{897}{3} = 299$

$\therefore n = 300$//

$S_{300} = \frac{n}{2}[a + a_n] = \frac{300}{2}[101 + 998] = 150 \times 1099 = \underline{\underline{164850}}$

25) Sum of integers not divisible by 9 = Sum of all integers between 100 and 200 − Sum of integers divisible by 9.

$S_n = \frac{n}{2}[a + a_n]$

The integers between 100 and 200 that are divisible by 9 are

108, 117, 126, ..., 198

$a = 108,\ d = 9,\ a_n = 198$

$a_n = a + (n-1)d = 198$

$\Rightarrow 108 + (n-1)9 = 198$

$n - 1 = \frac{90}{9} = 10$

$\therefore n = 11$//

$\therefore S_{11} = \frac{11}{2}[108 + 198]$

$= \frac{11}{2} \times 306$

$= 11 \times 153 = \underline{\underline{1683}}$

All integers between 100 and 200 are

101, 102, 103, ... 199

$a = 101,\ d = 1,\ a_n = 199$

$n = 199 - 101 + 1 = 199 - 100 = 99$//

$S_{99} = \frac{99}{2}[101 + 199] = \frac{99}{2} \times 300 = 99 \times 150 = \underline{\underline{14850}}$

$\therefore$ The required sum $= 14850 - 1683 = \underline{\underline{13167}}$

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26) To prove: $S_{p+q} = \frac{p+q}{2}[2a+(p+q-1)d] = 0$

$S_p = S_q$

$\Rightarrow \frac{p}{2}[2a+(p-1)d] = \frac{q}{2}[2a+(q-1)d]$

$\Rightarrow 2ap + p(p-1)d - 2aq - q(q-1)d = 0$

$\Rightarrow 2ap - 2aq + (p^2 - p - q^2 + q)d = 0$

$\Rightarrow 2a(p-q) + ((p^2-q^2) - (p-q))d = 0$

$\Rightarrow 2a(p-q) + [(p+q)(p-q) - (p-q)]d = 0$

$\Rightarrow (p-q)[2a + (p+q-1)d] = 0$

$\therefore 2a + (p+q-1)d = 0 \longrightarrow (1)$

Thus, $S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d] = \frac{p+q}{2} \times 0 = \underline{\underline{0}}$

27) $\frac{a_{11}}{a_{18}} = \frac{2}{3} \Rightarrow (a+10d)3 = 2(a+17d)$

$\Rightarrow 3a + 30d = 2a + 34d$

$a = 4d \longrightarrow (1)$

$\therefore$ The required ratio, $\frac{a_5}{a_{21}} = \frac{a+4d}{a+20d} = \frac{4d+4d}{4d+20d} = \frac{8d}{24d} = \frac{1}{3}$

$\therefore \underline{\underline{a_5 : a_{21} = 1:3}}$

Also, $\frac{S_5}{S_{21}} = \frac{\frac{5}{2}[2a+4d]}{\frac{21}{2}[2a+20d]} = \frac{5[2a+a]}{21[2a+5a]} = \frac{5 \times 3a}{21 \times 7a}$

$= \frac{5}{49}$

$\therefore \underline{\underline{S_5 : S_{21} = 5:49}}$

28) $a_p = A + (p-1)d = a \longrightarrow (1)$

$a_q = A + (q-1)d = b \longrightarrow (2)$

(1)−(2), $a - b = d(p-1-q+1) = d(p-q)$

$a_r = A + (r-1)d = c \longrightarrow (3)$

(2)−(3), $b - c = d(q-1-r+1) = d(q-r)$

(3)−(1), $c - a = d(r-1-p+1) = d(r-p)$

$\therefore (a-b)r + (b-c)p + (c-a)q$
$= d(p-q)r + d(q-r)p + d(r-p)q$
$= d(\cancel{pr} - \cancel{qr} + \cancel{qp} - \cancel{rp} + \cancel{rq} - \cancel{pq})$
$= d \times 0 = \underline{0}$

29) To prove: $a_{mn} = a + (mn-1)d = 1$

$a_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n} \rightarrow (1)$

$a_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m} \rightarrow (2)$

(1) − (2), $d(m - \cancel{1} - n + \cancel{1}) = \frac{1}{n} - \frac{1}{m}$

$d(\cancel{m-n}) = \frac{\cancel{m-n}}{mn}$

$\therefore d = \frac{1}{mn}$

From eq: (1), $a + (m-1) \times \frac{1}{mn} = \frac{1}{n}$

$\Rightarrow a + \frac{\cancel{m}}{\cancel{m}n} - \frac{1}{mn} = \frac{1}{n}$

$\Rightarrow a - \frac{1}{mn} = \cancel{\frac{1}{n}} - \cancel{\frac{1}{n}}$

$\therefore a = \frac{1}{mn}$

$\therefore a_{mn} = a + (mn-1)d$
$= \frac{1}{mn} + (mn-1)\frac{1}{mn} = \frac{1}{mn} + 1 - \frac{1}{mn} = \underline{\underline{1}}$

30) $S_n = \frac{n}{2}[2a + (n-1)d] = S_1$

$S_{2n} = \frac{2n}{2}[2a + (2n-1)d] = S_2$

$S_{3n} = \frac{3n}{2}[2a + (3n-1)d] = S_3$

$S_2 - S_1 = \frac{n}{2}[4a + 2(2n-1)d - 2a - (n-1)d]$
$= \frac{n}{2}[2a + [4n - 2 - n + 1]d]$
$= \frac{n}{2}[2a + (3n-1)d]$

$\therefore 3(S_2 - S_1) = \frac{3n}{2}[2a + (3n-1)d] = S_3$

31) $a_n = S_n - S_{n-1} \longrightarrow (1)$

$a_{n-1} = S_{n-1} - S_{n-2} \longrightarrow (2)$

$a_n = S_n - S_{n-1}$
$d = a_n - a_{n-1}$

(1) − (2), $a_n - a_{n-1} = S_n - 2S_{n-1} + S_{n-2}$

$\Rightarrow \quad d = S_n - 2S_{n-1} + S_{n-2}$ //

32) 10, 20, 30, … 490 are the required numbers in A.P with $a = 10, d = 10, a_n = 490$.

$a_n = a + (n-1)d = 490$

$\Rightarrow 10 + (n-1)10 = 490$

$n - 1 = 48$

$n = 49$ //

$\therefore S_{49} = \frac{49}{2}[a + a_n] = \frac{49}{2}[10 + 490] = \frac{49}{2} \times 500 = \underline{\underline{12250}}$

34) Let the angles be $a-d$, a and $a+d$

Then, $a - d + a + a + d = 180°$ (angle sum property of a △)

$3a = 180°$

$a = 60°$

Also, $a + d = 2(a - d)$

$a + d = 2a - 2d$

$-a = -3d$

$a = 3d$

$d = \frac{a}{3} = 20°$

5 | 3600
5 | 720
2 | 144
2 | 72
3 | 36
12

$\therefore$ The angles of the △ are $a - d = 60° - 20° = 40°$ //

$a = 60°$ //

$a + d = 60° + 20° = 80°$ //

35) $S_n = \frac{n}{2}[2a+(n-1)d]$

$$\frac{\frac{n}{\cancel{2}_1}[2\times1+(n-1)2]}{\frac{8}{\cancel{2}_1}[2\times2+(8-1)3]} = 9$$

$$\Rightarrow \frac{n(2+2n-2)}{8(4+3\times7)} = 9$$

$$\Rightarrow \frac{n(2n)}{8(25)} = 9$$

$2n^2 = 9\times8\times25$

$n^2 = 9\times4\times25$

$n = 3\times2\times5$

$= 30$

36) $a_n = a+(n-1)d$

ATQ, $a_8 = \frac{1}{2}\times a_2 \Rightarrow a+7d = \frac{1}{2}(a+d)$

$\Rightarrow 2a+14d = a+d$

$\Rightarrow a+13d = 0 \rightarrow (1)$

Also, $a_{11} - \frac{1}{3}a_4 = 1$

$\Rightarrow a+10d - \frac{a+3d}{3} = 1$

$\Rightarrow 3a+30d-a-3d = 3$

$\Rightarrow 2a+27d = 3 \rightarrow (2)$

$(1)\times2,\ 2a+26d = 0$

$(2),\ 2a+27d = 3$

$(-),\ -d = -3$

$d = 3$

From eq:(1), $a+39 = 0$

$a = -39$

$\therefore a_{15} = a+14d = -39+14\times3 = -39+42 = \underline{\underline{3}}$

37) $a_n = a+(n-1)d$

ATQ, $a_{16} = 2a_8+1 \Rightarrow a+15d = 2(a+7d)+1$

$\Rightarrow a+15d = 2a+14d+1$

$\Rightarrow -a+d = 1 \rightarrow (1)$

Also, $a_{12} = 47 \Rightarrow a+11d = 47 \rightarrow (2)$

$(1)+(2),\ 12d = 48 \Rightarrow d = 4$

From eq:(1), $-a+4=1 \Rightarrow a=3$
Hence, $a_n = a+(n-1) = 3+4(n-1) = 3+4n-4$
$= \underline{\underline{4n-1}}$

38) Since the given terms are in A.P,

$$2k^2+3k+6-k^2-4k-8 = 3k^2+4k+4-2k^2-3k-6$$
$$\cancel{k^2}-k\cancel{-2} = \cancel{k^2}+k\cancel{-2}$$
$$\underline{\underline{k=0}}$$

39) n^{th} term from the end of A P $= l-(n-1)d$
$= 49-(4-1)3$
$= 49-9$
$= \underline{\underline{40}}$

40) Since the given terms are in A.P,
$$\cancel{k}+3\cancel{-k}+1 = 3k-1-k-3$$
$$4 = 2k-4$$
$$2k = 8$$
$$k = 4$$

~~41) Since the given terms are in A.P,~~
$$\cancel{\frac{1-p}{p} - \frac{1}{p} = 2-2p}$$

41) Common difference $= \frac{1-p}{p} - \frac{1}{p} = \frac{1}{p} - 1 - \frac{1}{p} = \underline{\underline{-1}}$

42) $S_n = \frac{n}{2}[2a+(n-1)d]$

$$\frac{\cancel{\frac{n}{2}}[2a+(n-1)d]}{\cancel{\frac{n}{2}}[2A+(n-1)D]} = \frac{7n+1}{4n+27}$$

$$\frac{a+\left(\frac{n-1}{2}\right)d}{A+\left(\frac{n-1}{2}\right)D} = \frac{7n+1}{4n+27} \longrightarrow (1)$$

Now, put $\frac{n-1}{2} = m-1$
$\Rightarrow n-1 = 2m-2$
$\Rightarrow n = 2m-1$

$\frac{a_m}{A_m} = \frac{a+(m-1)d}{A+(m-1)D}$
$= \frac{7(2m-1)+1}{4(2m-1)+27}$ [from eq: (1)]

$$= \frac{14m-7+1}{8m-4+27} = \frac{14m-6}{8m+23}$$

$\therefore$ ratio of 9^{th} terms, $\frac{a_9}{A_9} = \frac{14\times9-6}{8\times9+23} = \frac{126-6}{72+23} = \frac{120}{95}$

$$= \frac{24}{19}.$$

Hence, $a_9 : A_9 = 24:19$

43) The no. of rose plants in 1^{st}, 2^{nd}, 3^{rd}, ... rows are 23, 21, 19, ... 5

$a_n = a+(n-1)d$

$5 = 23-2(n-1)$

$-18 = -2(n-1)$

$n-1 = 9$

$n = 10$

$S_{10} = \frac{n}{2}[a+a_n] = \frac{10}{2}[23+5] = 5\times28 = $ 140 rose plants

44) Let total time to catch the thief be n minutes

Total distance covered by thief $= \text{Speed} \times \text{time}$

$= 100 \times n$

$= 100n$ metres

Total distance covered by policeman

$= 100 + (100+10) + (100 + 2\times10) + \cdots + (n-1)$ terms

$= 100 + 110 + 120 + \cdots\ n-1$ terms

$= \frac{n-1}{2}[200 + (n-2)10]$ $\quad [\because S_n = \frac{n}{2}[2a + (n-1)d]]$

$= (\frac{n-1}{2})(200 + 10n - 20)$

$= \frac{n-1}{2}(180 + 10n)$

$= (n-1)(90 + 5n)$

Since the total ~~time~~ distance covered by both are same,

$100n = (n-1)(90+5n)$

$100n = 90n + 5n^2 - 90 - 5n$

$5n^2 - 15n - 90 = 0$

$n^2 - 3n - 18 = 0$

$(n-6)(n+3) = 0$

$n = 6, -3$

n cannot be -ve, $\therefore$ the required value of $n = 6$

Hence, the time taken to catch the thief = 6 min