IX EW-12

1) What are rational numbers? Give examples.
2) What are irrational numbers? Give examples.
3) All rational numbers and all irrational numbers together make the collection of ____________.
4) Decimal expansion of a rational number is either ____________ or ____________
5) Decimal expansion of an irrational number is ____________
6) If r is a rational number and s is an irrational number, then $r+s$ and $r-s$ are ____________
7) If r is a non-zero rational, then $r \times s$ and $\frac{r}{s}$ are ____________
8) $\sqrt{ab} =$ ___ $\times$ ___

$\sqrt{\frac{a}{b}} =$ ___

$(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) =$ ______

$(a+\sqrt{\ })(a-\sqrt{b}) =$ ______

$(\sqrt{a}+\sqrt{b})^2 =$ ______

9) $a^p \cdot a^q =$ ______

$(a^p)^q =$ ______

$\frac{a^p}{a^q} =$ ______

$a^p b^p =$ ______

10) Which of the following is not equal to $\left[\left(\frac{5}{6}\right)^{\frac{1}{5}}\right]^{-\frac{1}{6}}$?

(a) $\left(\frac{5}{6}\right)^{\frac{1}{5}-\frac{1}{6}}$ (b) $\left(\left(\frac{5}{6}\right)^{-\frac{1}{5}}\right)^{\frac{1}{6}}$ (c) $\left(\frac{6}{5}\right)^{\frac{1}{30}}$ (d) $\left(\frac{5}{6}\right)^{-\frac{1}{30}}$

11) Every rational number is
(a) a natural number
(b) an integer
(c) a real number
(d) a whole number

12) Between two rational numbers
(a) there is no rational number
(b) there is exactly one rational number
(c) there are infinitely many rational numbers
(d) there are only rational numbers and no irrational numbers.

IX EW-12 (Answers)

1) A number is called a rational number, if it can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

eg:- $-\frac{2}{7}, 0$

2) A number which cannot be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called an irrational number.

eg:- $\pi, \sqrt{2}, \sqrt[3]{5}$

3) real numbers.

4) terminating, non-terminating repeating

5) non-terminating non-repeating

6) irrational.

7) irrational

8)

$\sqrt{a} \times \sqrt{b}$	$a^2 - b$
$\frac{\sqrt{a}}{\sqrt{b}}$	$a + b + 2\sqrt{ab}$
$a - b$	

9)

a^{p+q}	a^{p-q}
a^{pq}	$(ab)^p$

10) $\left(\frac{5}{6}\right)^{\frac{1}{5} - \frac{1}{6}}$ (a)

11) a real number (c)

12) there are infinitely many rational numbers (c).

13) Decimal representation of a rational number cannot be
(a) terminating (c) non-terminating repeating
(b) non-terminating (d) non-terminating non-repeating

14) The product of any two irrational numbers is
(a) always an irrational number (c) always an integer
(b) always a rational number (d) sometimes rational, sometimes irrational.

15) The decimal expansion of the number $\sqrt{2}$ is
(a) a finite decimal (c) non-terminating recurring
(b) 1.41421 (d) non-terminating non-recurring

16) which of the following is irrational?
(a) $\sqrt{\frac{4}{9}}$ (b) $\frac{\sqrt{12}}{\sqrt{3}}$ (c) $\sqrt{7}$ (d) $\sqrt{81}$

17) Which of the following is irrational?
(a) 0.14 (b) $0.14\overline{16}$ (c) $0.\overline{1416}$ (d) $0.4014001400014\ldots$

18) A rational number between $\sqrt{2}$ and $\sqrt{3}$ is
(a) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (b) $\frac{\sqrt{2}.\sqrt{3}}{2}$ (c) 1.5 (d) 1.8

19) The value of 1.999... in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is
(a) $\frac{19}{10}$ (b) $\frac{1999}{1000}$ (c) 2 (d) $\frac{1}{9}$

20) $2\sqrt{3}+\sqrt{3}$ is equal to (a) $2\sqrt{6}$ (b) 6 (c) $3\sqrt{3}$ (d) $4\sqrt{6}$

21) $\sqrt{10} \times \sqrt{15}$ is equal to (a) $6\sqrt{5}$ (b) $5\sqrt{6}$ (c) $\sqrt{25}$ (d) $10\sqrt{5}$

22) The number obtained on rationalising the denominator of $\frac{1}{\sqrt{7}-2}$ is (a) $\frac{\sqrt{7}+2}{3}$ (b) $\frac{\sqrt{7}-2}{3}$ (c) $\frac{\sqrt{7}+2}{3}$ (d) $\frac{\sqrt{7}+2}{45}$

23) $\frac{1}{\sqrt{9}-\sqrt{8}}$ is equal to (a) $\frac{1}{2}(3-2\sqrt{2})$ (c) $3-2\sqrt{2}$
(b) $\frac{1}{3+2\sqrt{2}}$ (d) $3+2\sqrt{2}$

24) After rationalising the denominator of $\frac{7}{3\sqrt{3}-2\sqrt{2}}$, we get the denominator as
(a) 13 (b) 19 (c) 5 (d) 35

25) The value of $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$ is equal to (a) $\sqrt{2}$ (b) 2
(c) 4 (d) 8

26) If $\sqrt{2} = 1.4142$, then $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to (a) 2.4142
(b) 5.8282
(c) 0.4142 (d) 0.1718

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13) non-terminating non-repeating (d)
14) sometimes rational, sometimes irrational (d)
15) non-terminating non-recurring (d)
16) $\sqrt{7}$ (c)
17) 0.40140014000014... (d)
18) 1.5 (c)
19) 2 (c)
20) $3\sqrt{3}$ (c)
21) $\sqrt{10} \times \sqrt{15} = \sqrt{2} \times \sqrt{5} \times \sqrt{5} \times \sqrt{3} = 5\sqrt{6}$ (b)
22) $\frac{1}{(\sqrt{7}-2)} \frac{(\sqrt{7}+2)}{(\sqrt{7}+2)} = \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$ (a)

23) $\dfrac{1(\sqrt{9}+\sqrt{8})}{(\sqrt{9}-\sqrt{8})(\sqrt{9}+\sqrt{8})} = \dfrac{3+\sqrt{8}}{9-8} = 3+\sqrt{8} = 3+2\sqrt{2}$ (d)

24) $(3\sqrt{3}-2\sqrt{2})(3\sqrt{3}+2\sqrt{2}) = (3\sqrt{3})^2-(2\sqrt{2})^2 = 27-8 = 19$ (b)

25) $\dfrac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}} = \dfrac{4\sqrt{2}+4\sqrt{3}}{2\sqrt{2}+2\sqrt{3}}$

$$\begin{array}{r|l} 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array} \qquad \begin{array}{r|l} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

$= \dfrac{4(\sqrt{2}+\sqrt{3})}{2(\sqrt{2}+\sqrt{3})} = \dfrac{4}{2} = 2$ (b)

26) $\sqrt{2} = 1.4142$

$\sqrt{\dfrac{(\sqrt{2}-1)(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)}} = \sqrt{\dfrac{(\sqrt{2}-1)^2}{2-1}} = \sqrt{(\sqrt{2}-1)^2} = \sqrt{2}-1 = 1.4142-1$

$= 0.4142$ (c)

27) $\sqrt[4]{\sqrt[3]{2^2}}$ equals to (a) $2^{-\frac{1}{6}}$ (b) 2^{-6} (c) $2^{\frac{1}{6}}$ (d) 2^{6}

28) The product $\sqrt[3]{2} \cdot \sqrt[4]{2} \cdot \sqrt[12]{32}$ equals to

(a) $\sqrt{2}$ (b) 2 (c) $\sqrt[12]{2}$ (d) $\sqrt[12]{32}$

29) The value of $\sqrt[4]{(81)^{-2}}$ is (a) $\frac{1}{9}$ (b) $\frac{1}{3}$ (c) 9 (d) $\frac{1}{81}$

30) The value of $(256)^{0.16} \times (256)^{0.09}$ is

(a) 4 (b) 16 (c) 64 (d) 256.25

31) Which of the following is equal to x?

(a) $x^{\frac{12}{7}} - x^{\frac{5}{7}}$ (b) $\sqrt[12]{(x^4)^{1/3}}$ (c) $\left(\sqrt{x^3}\right)^{\frac{2}{3}}$ (d) $x^{\frac{12}{7}} \times x^{\frac{7}{12}}$

32) Are there two irrational numbers whose sum and product both are rationals? Justify.

33) State whether the following statement is true:
There is a number x such that x^2 is irrational but x^4 is rational. Justify your answer by an example.

34) Let x and y be rational and irrational numbers, respectively. Is $x+y$ necessarily an irrational number? Give an example in support of your answer.

35) Let x be rational and y be irrational. Is xy necessarily irrational? Justify your answer.

36) State whether the following statements are true or false? Justify your answer.

(i) $\frac{\sqrt{2}}{3}$ is a rational number

(ii) There are infinitely many integers between any two integers

(iii) Number of rational numbers between 15 and 18 is finite.

(iv) There are numbers which cannot be written in the form $\frac{p}{q}$, $q \neq 0$, p, q both are integers.

(v) The square of an irrational number is always rational

(vi) $\frac{\sqrt{12}}{\sqrt{3}}$ is not a rational number as $\sqrt{12}$ and $\sqrt{3}$ are not integers.

(vii) $\frac{\sqrt{15}}{\sqrt{3}}$ is written in the form $\frac{p}{q}$, $q \neq 0$ and so it is a rational number.

27) $\sqrt[4]{\sqrt[3]{2^2}} = 2^{2\times\frac{1}{3}\times\frac{1}{4}} = 2^{\frac{2}{12}} = 2^{\frac{1}{6}}$ (c)

28) $2^{\frac{1}{3}} \cdot 2^{\frac{1}{4}} \cdot (2^5)^{\frac{1}{12}} = 2^{\frac{1\times4}{3\times4}+\frac{1\times3}{4\times3}+\frac{5}{12}} = 2^{\frac{4+3+5}{12}} = 2^{\frac{12}{12}} = 2$ (b)

29) $\sqrt[4]{(81)^{-2}} = 9^{2\times-2\times\frac{1}{4}} = 9^{\frac{-4}{4}} = 9^{-1} = \frac{1}{9}$ (a)

30) $(256)^{0.16} \times (256)^{0.09} = (256)^{0.16+0.09} = (256)^{0.25} = (256)^{\frac{1}{4}}$

$= 4^{4\times\frac{1}{4}} = 4$ (a)

31) $\left(\sqrt{x^3}\right)^{\frac{2}{3}}$

$= x^{3\times\frac{1}{2}\times\frac{2}{3}} = x^{\frac{3}{2}\times\frac{2}{3}} = x$ (c)

32) Let the two irrational numbers be $(3+\sqrt{2})$ and $(3-\sqrt{2})$

Sum $= 3+\sqrt{2}+3-\sqrt{2} = 3+3 = 6$, a rational number.

product $= (3+\sqrt{2})(3-\sqrt{2}) = (3)^2 - (\sqrt{2})^2 = 9-2 = 7$, a rational number.

33) Let $x = 5^{\frac{1}{4}}$

$x^2 = \left(5^{\frac{1}{4}}\right)^2 = 5^{\frac{2}{4}} = 5^{\frac{1}{2}} = \sqrt{5}$, an irrational number

$x^4 = \left(5^{\frac{1}{4}}\right)^4 = 5^{\frac{4}{4}} = 5$, a rational number.

34) Yes, $x+y$ will be an irrational number

eg:- Let $x = 5$ and $y = \sqrt{3}$

$x+y = 5+\sqrt{3}$, is an irrational number.

35) No, xy can be rational also.

Let $x = 0$ and $y = \sqrt{5}$, then $xy = 0\times\sqrt{5} = 0$, a rational number.

36) (i) False, division of an irrational number by a non-zero rational number is always an irrational number.

(ii) False, between two consecutive integers, there does not exist any other integer.

(iii) False, between any two rational numbers there exist infinitely many rational numbers.

(iv) True, they are called irrational numbers.

(v) False, it can be sometimes rational, sometimes irrational

eg:- $(\sqrt{2})^2 = 2$, a rational number

$(\sqrt[4]{3})^2 = 3^{\frac{2}{4}} = 3^{\frac{1}{2}} = \sqrt{3}$, an irrational number

(vi) False

$\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2$, a rational number.

(vii) False

$\frac{\sqrt{15}}{\sqrt{3}} = \sqrt{\frac{15}{3}} = \sqrt{5}$, an irrational number which cannot be written in the form $\frac{p}{q}$, $q \neq 0$

37) Classify the following numbers as rational or irrational with justification:

(i) $\sqrt{196}$ (ii) $3\sqrt{18}$ (iii) $\sqrt{\frac{9}{27}}$ (iv) $\frac{\sqrt{28}}{\sqrt{343}}$

(v) $-\sqrt{0.4}$ (vi) $\frac{\sqrt{12}}{\sqrt{75}}$ (vii) 0.5918

(viii) $(1+\sqrt{5})-(4+\sqrt{5})$ (ix) $10.124124\cdots$ (x) $1.010010001\ldots$

38) Locate $\sqrt{13}$ on the number line.

39) Express $0.12\overline{3}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$

40) Simplify: $(3\sqrt{5}-5\sqrt{2})(4\sqrt{5}+3\sqrt{2})$

41) find the value of a in the following :-

$$\frac{6}{3\sqrt{2}-2\sqrt{3}} = 3\sqrt{2} - a\sqrt{3}$$

42) Simplify: $\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$

43) Find which of the variables x, y, z and u represent rational numbers and which irrational numbers.

(i) $x^2 = 5$ (ii) $y^2 = 9$ (iii) $z^2 = 0.04$ (iv) $u^2 = \frac{17}{4}$

44) find three rational numbers between

(i) -1 and -2 (ii) 0.1 and 0.11

(iii) $\frac{5}{7}$ and $\frac{6}{7}$ (iv) $\frac{1}{4}$ and $\frac{1}{5}$

45) Insert a rational number and an irrational number between the following:-

(i) 2 and 3 (ii) 0 and 0.1 (iii) $\frac{1}{3}$ and $\frac{1}{2}$

(iv) $-\frac{2}{5}$ and $\frac{1}{2}$ (v) 0.15 and 0.16 (vi) $\sqrt{2}$ and $\sqrt{3}$

(vii) 2.357 and 3.121 (viii) 0.001 and 0.0001

(ix) 3.623623 and 0.484848 (x) 6.375289 and 6.375738

37) (i) $\sqrt{196} = 14$, a rational number.

(ii) $3\sqrt{18} = 3 \times 3\sqrt{2} = 9\sqrt{2}$, an irrational number.

(iii) $\sqrt{\frac{9}{27}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$, an irrational number

(iv) $\frac{\sqrt{28}}{\sqrt{343}} = \sqrt{\frac{\cancel{28}\,4}{\cancel{343}\,49}} = \frac{2}{7}$, a rational number.

(v) $-\sqrt{0.4} = -\sqrt{\frac{4}{10}} = -\frac{2}{\sqrt{10}} = -\frac{2\sqrt{10}}{10} = -\frac{\sqrt{10}}{5}$, an irrational number.

(vi) $\frac{\sqrt{12}}{\sqrt{75}} = \sqrt{\frac{\cancel{12}\,4}{\cancel{75}\,25}} = \frac{2}{5}$, a rational number.

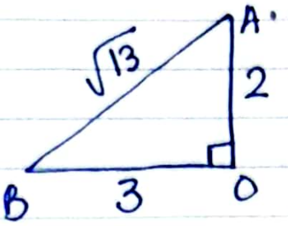
(vii) 0.5918, a terminating decimal expansion. Hence, rational.

(viii) $(1+\sqrt{5}) - (4+\sqrt{5}) = 1 + \cancel{\sqrt{5}} - 4 - \cancel{\sqrt{5}} = -3$, a rational number

(ix) $10.124124\ldots$, a non-terminating repeating decimal expansion. Hence, rational
$= 10.\overline{124}$

(x) $1.010010001\ldots$, a non-terminating non-repeating decimal expansion. Hence, irrational

38)



Using Pythagoras Theorem,

$AB^2 = OA^2 + OB^2$

$= 3^2 + 2^2 = 9 + 4 = 13$

$AB = \sqrt{13}$

39) Let $x = 0.12333\ldots$

$100x = 12.3333\ldots \longrightarrow (1)$

$1000x = 123.3333\ldots \longrightarrow (2)$

(2)-(1), $900x = 111$

$x = \dfrac{\cancel{111}\,37}{\cancel{900}\,300}$

$\therefore\ 0.12\overline{3} = \dfrac{37}{300}$

40) $(3\sqrt{5} - 5\sqrt{2})(4\sqrt{5} + 3\sqrt{2})$

$= 12 \times 5 + 9 \times \sqrt{10} - 20 \times \sqrt{10} - 15 \times 2$

$= 60 + 9\sqrt{10} - 20\sqrt{10} - 30$

$= \underline{\underline{30 - 11\sqrt{10}}}$

41) $\dfrac{6}{3\sqrt{2} - 2\sqrt{3}} = \dfrac{6(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})} = \dfrac{6(3\sqrt{2}+2\sqrt{3})}{18 - 12}$

$= \dfrac{\cancel{6}\,(3\sqrt{2}+2\sqrt{3})}{\cancel{6}} = 3\sqrt{2} + 2\sqrt{3}$

$\therefore\ 3\sqrt{2} + 2\sqrt{3} = 3\sqrt{2} - a\sqrt{3}$

Thus, $\underline{\underline{a = -2}}$

42) $\left[5\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}} = \left[5\left(2^{3 \times \frac{1}{3}} + 3^{3 \times \frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$

$= \left[5(2+3)^3\right]^{\frac{1}{4}} = \left(5 \times 5^3\right)^{\frac{1}{4}} = 5^{4 \times \frac{1}{4}} = \underline{\underline{5}}$

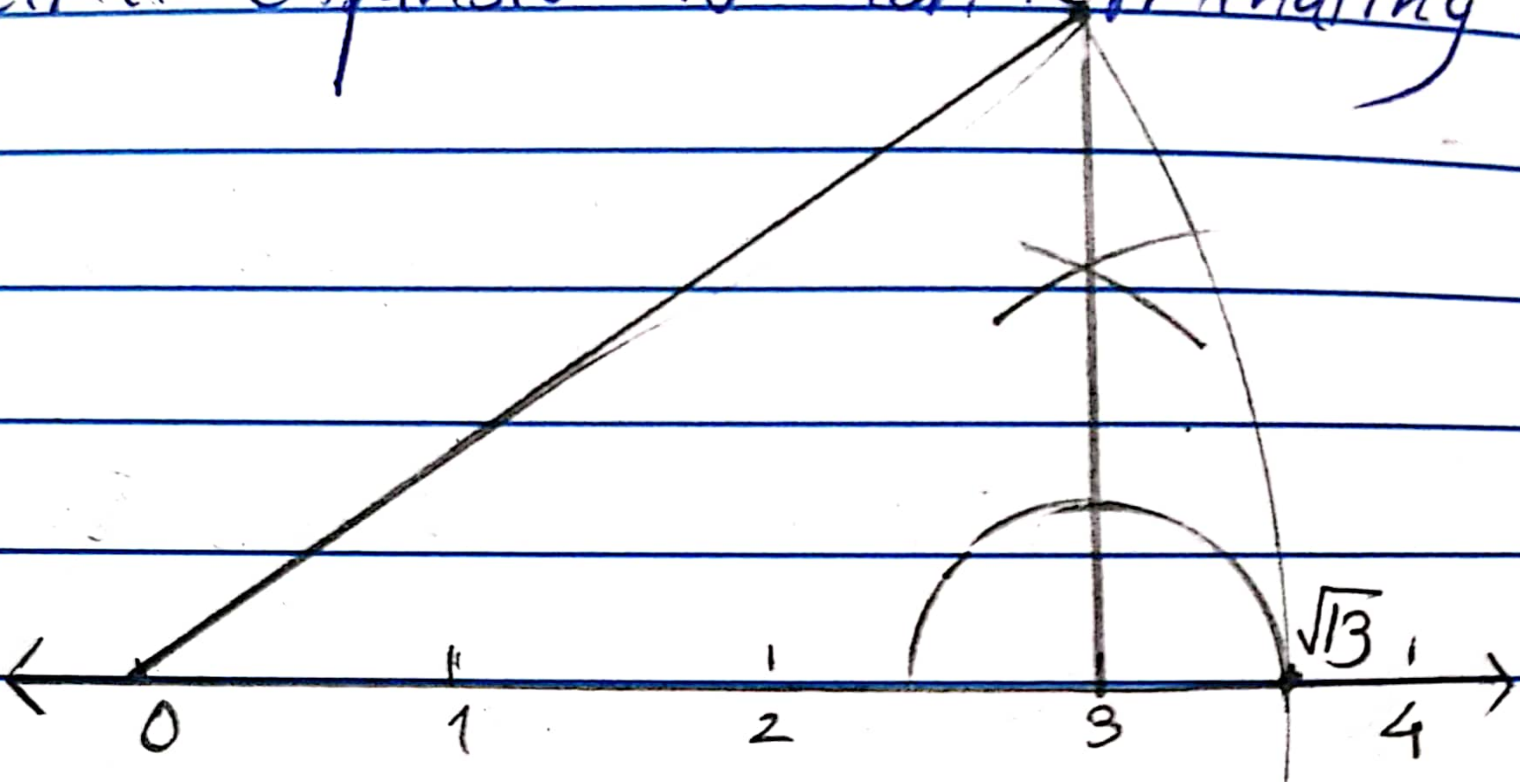
43) (i) $x^2 = 5 \Rightarrow x = \sqrt{5}$, an irrational number.

(ii) $y^2 = 9 \Rightarrow y = \sqrt{9} = 3$, a rational number

(iii) $z^2 = 0.04 \Rightarrow z = \sqrt{0.04} = 0.2$, a rational number

(iv) $u^2 = \dfrac{17}{4} \Rightarrow u = \dfrac{\sqrt{17}}{2}$, an irrational number.

s decimal expansion is non terminating n



44) (i) Three rational numbers between -1 and -2 are $-1.1, -1.2$ and -1.3

(ii) Three rational numbers between 0.1 and 0.11 are 0.101, 0.102 and 0.103

(iii) $\frac{5\times4}{7\times4}$ $\quad$ $\frac{6\times4}{7\times4}$

$\frac{20}{28}$ $\quad$ $\frac{24}{28}$

$\therefore$ Three rational numbers between $\frac{5}{7}$ and $\frac{6}{7}$ are $\cancel{\frac{21}{28}}^{3}_{4}, \cancel{\frac{22}{28}}^{11}_{14}, \frac{23}{28}$

$= \frac{3}{4}, \frac{11}{14}, \frac{23}{28}$

(iv) $\frac{1\times5}{4\times5}$ $\quad$ $\frac{1\times4}{5\times4}$

$\frac{5\times10}{20\times10}$ $\quad$ $\frac{4\times10}{20\times10}$

$\frac{50}{200}$ $\quad$ $\frac{40}{200}$

$\therefore$ Three rational numbers between $\frac{1}{4}$ and $\frac{1}{5}$ are $\frac{41}{200}, \frac{43}{200}, \frac{47}{200}$

45) (i) rational number $\rightarrow$ 2.5
irrational number $\rightarrow$ 2.515115111 5...

(ii) rational number $\rightarrow$ 0.01
irrational number $\rightarrow$ 0.010110111...

(iii) $\frac{1\times2}{3\times2}$ $\quad$ $\frac{1\times3}{2\times3}$

$\frac{2\times10}{6\times10}$ $\quad$ $\frac{3\times10}{6\times10}$

$\frac{20}{60}$ $\quad$ $\frac{30}{60}$

rational number $\rightarrow \cancel{\frac{24}{60}}\frac{4}{10} = 0.4$

irrational number $\rightarrow$ 0.404004000...

(iv) $\frac{-2}{5} = -0.4$

$\frac{1}{2} = 0.5$

rational number $\rightarrow$ 0.2
irrational number $\rightarrow$ 0.202002000...

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(v) rational → 0.151
irrational → 0.1515515551...
(vi) $\sqrt{2} = 1.414\cdots$
$\sqrt{3} = 1.732\cdots$
rational → 1.5
irrational → 1.51511511 1...
(vii) rational → 3
irrational → 3.010010001...
(viii) 0.001 0.0001

$\frac{1 \times 10}{1000 \times 10}$ $\frac{1}{10000}$

$\frac{10}{10000}$ $\frac{1}{10000}$

rational number → $\frac{9}{10000} = 0.0009$

irrational number → 0.00091911911 1...
(ix) rational number → 2
irrational number → 2.101001000...
(x) rational number → 6.375400
irrational number → 6.37540540054000...

46) Represent the following numbers on the number line :
$7, 7.2, -\frac{3}{2}, -\frac{12}{5}$

47) Locate (i) $\sqrt{5}$ (ii) $\sqrt{10}$ (iii) $\sqrt{17}$ on the number line.

48) Represent geometrically the following numbers on the number line.
(i) $\sqrt{4.5}$ (ii) $\sqrt{5.6}$ (iii) $\sqrt{8.1}$ (iv) $\sqrt{2.3}$

49) Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
(i) 0.2 (ii) 0.888... (iii) $5.\overline{2}$ (iv) $0.\overline{001}$ (v) 0.2555...
(vi) $0.1\overline{34}$ (vii) 0.00323232... (viii) 0.404040...

50) Show that $0.142857142857... = \frac{1}{7}$

51) Simplify the following:-

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

(ii) $\frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$

(iii) $\sqrt[4]{12} \times \sqrt[7]{6}$

(iv) $4\sqrt{28} \div 3\sqrt{7} \div \sqrt[3]{7}$

(v) $3\sqrt{3} + 2\sqrt{27} + \frac{7}{\sqrt{3}}$

(vi) $(\sqrt{3} - \sqrt{2})^2$

(vii) $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

(viii) $\frac{3}{\sqrt{8}} + \frac{1}{\sqrt{2}}$

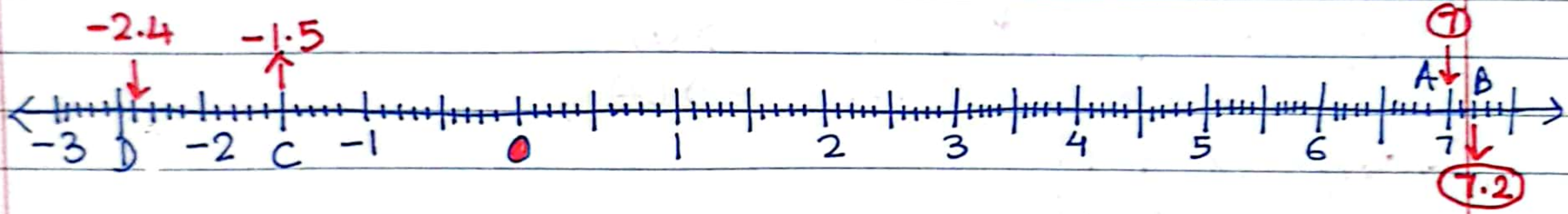
(ix) $\frac{2\sqrt{3}}{3} - \frac{\sqrt{3}}{6}$

52) Rationalise the denominator :
(i) $\frac{2}{3\sqrt{3}}$ (ii) $\frac{\sqrt{40}}{\sqrt{3}}$ (iii) $\frac{3+\sqrt{2}}{4\sqrt{2}}$ (iv) $\frac{16}{\sqrt{41}-5}$ (v) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$
(vi) $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$ (vii) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ (viii) $\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ (ix) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

53) Find the values of a and b in each of the following:-
(i) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a - 6\sqrt{3}$ (ii) $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = a\sqrt{5} - \frac{19}{11}$

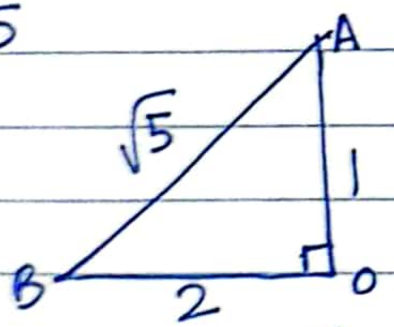
46) $-\frac{3}{2} = -1.5$

$-\frac{12}{5} = -2.4$



Thus A, B, C and D represent $7, 7.2, -\frac{3}{2}$ and $-\frac{12}{5}$ on the number line

47) (i) $\sqrt{5}$



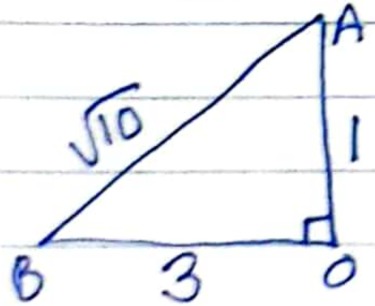
Using Pythagoras Theorem,

$AB^2 = OA^2 + OB^2$

$= 1^2 + 2^2 = 1 + 4 = 5$

$\therefore AB = \sqrt{5}$ units

(ii)

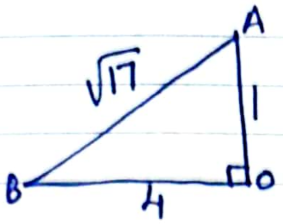


Using Pythagoras Theorem,

$AB^2 = OA^2 + OB^2 = 3^2 + 1^2 = 9 + 1 = 10$

$\therefore AB = \sqrt{10}$ units

(iii) $\sqrt{17}$



Using Pythagoras theorem,

$AB^2 = OA^2 + OB^2$

$= 1^2 + 4^2 = 1 + 16 = 17$

$\therefore AB = \sqrt{17}$ units

48) (i) $\sqrt{4.5}$

(ii) $\sqrt{5.6}$

(iii) $\sqrt{8.1}$

(iv) $\sqrt{2.3}$

49) (i) $0.2 = \frac{2}{10} = \frac{1}{5}$

(ii) Let $x = 0.888\ldots \longrightarrow (1)$

$10x = 8.888\ldots \longrightarrow (2)$

(2)−(1), $9x = 8$

$x = \frac{8}{9}$

(iii) Let $x = 5.222\ldots \longrightarrow (1)$

$10x = 52.222\ldots \longrightarrow (2)$

(2)−(1), $9x = 47$

$x = \frac{47}{9}$

(iv) Let $x = 0.001001001\ldots \longrightarrow (1)$

$1000x = 1.001001\ldots \longrightarrow (2)$

(2)−(1), $999x = 1$

$x = \frac{1}{999}$

(v) Let $x = 0.2555\ldots$

$10x = 2.5555\ldots \longrightarrow (1)$

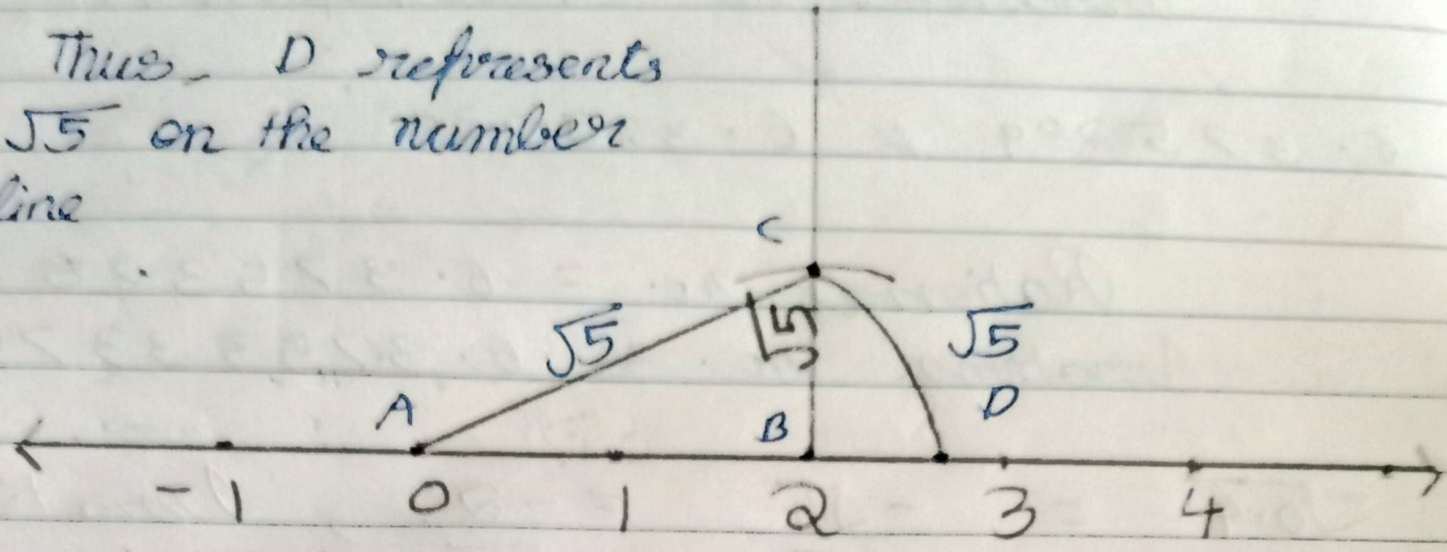
$100x = 25.555\ldots \longrightarrow (2)$

(2)−(1), $90x = 23$

$x = \frac{23}{90}$

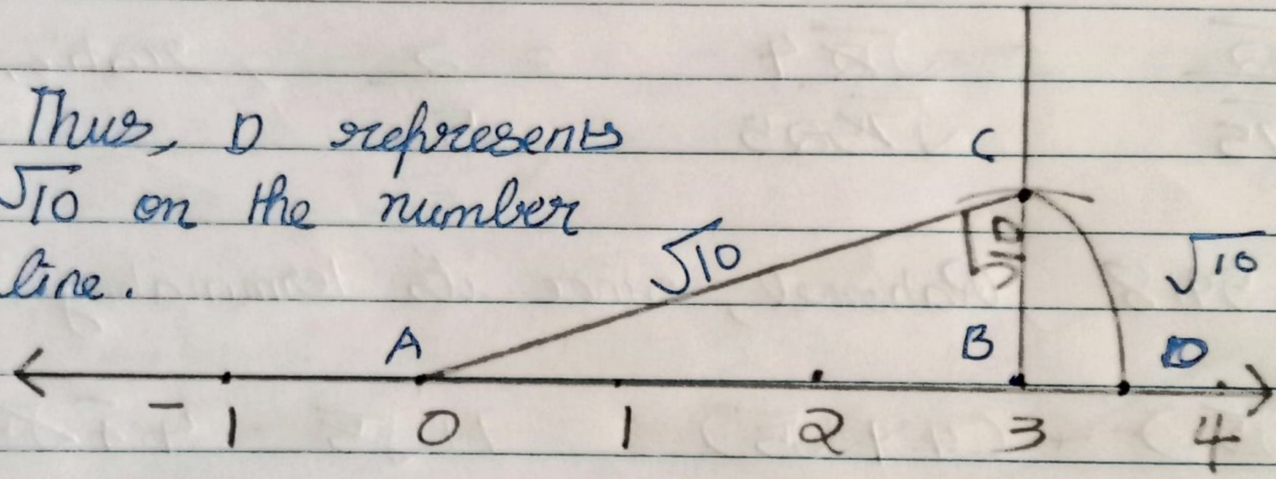
47. (i) $\sqrt{5}$

Thus, D represents $\sqrt{5}$ on the number line



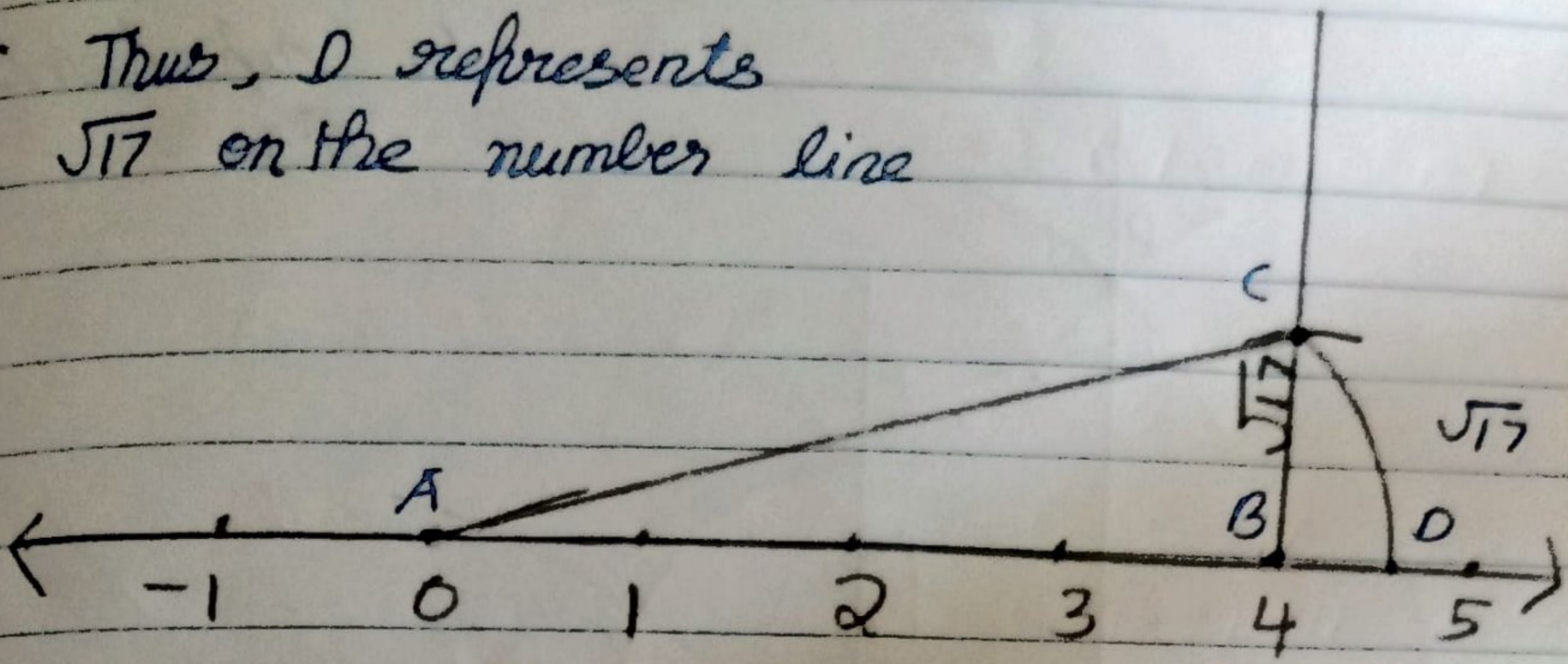
(ii) $\sqrt{10}$

Thus, D represents $\sqrt{10}$ on the number line.

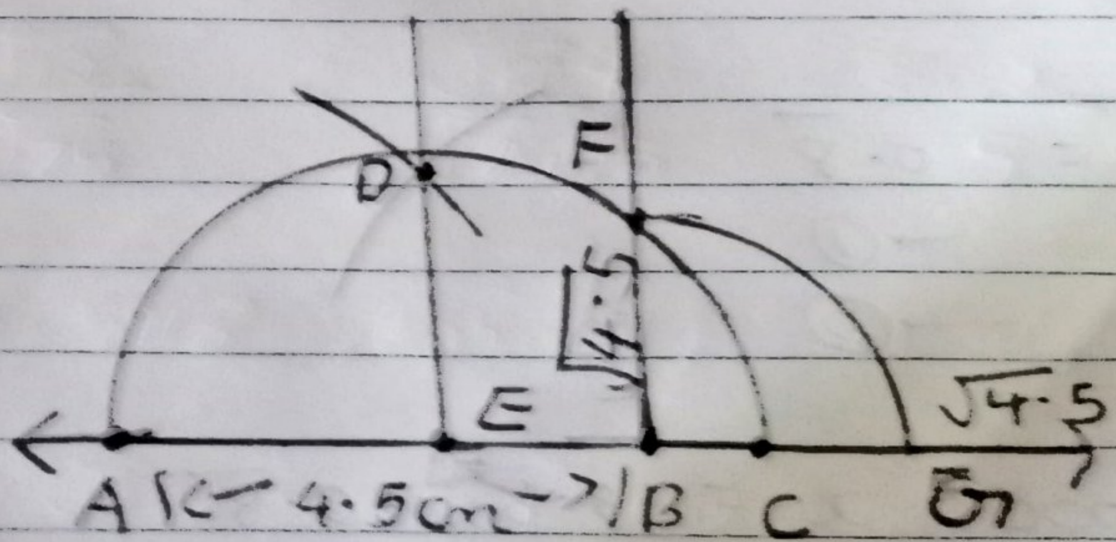


(ii) $\sqrt{17}$

- Thus, D represents $\sqrt{17}$ on the number line

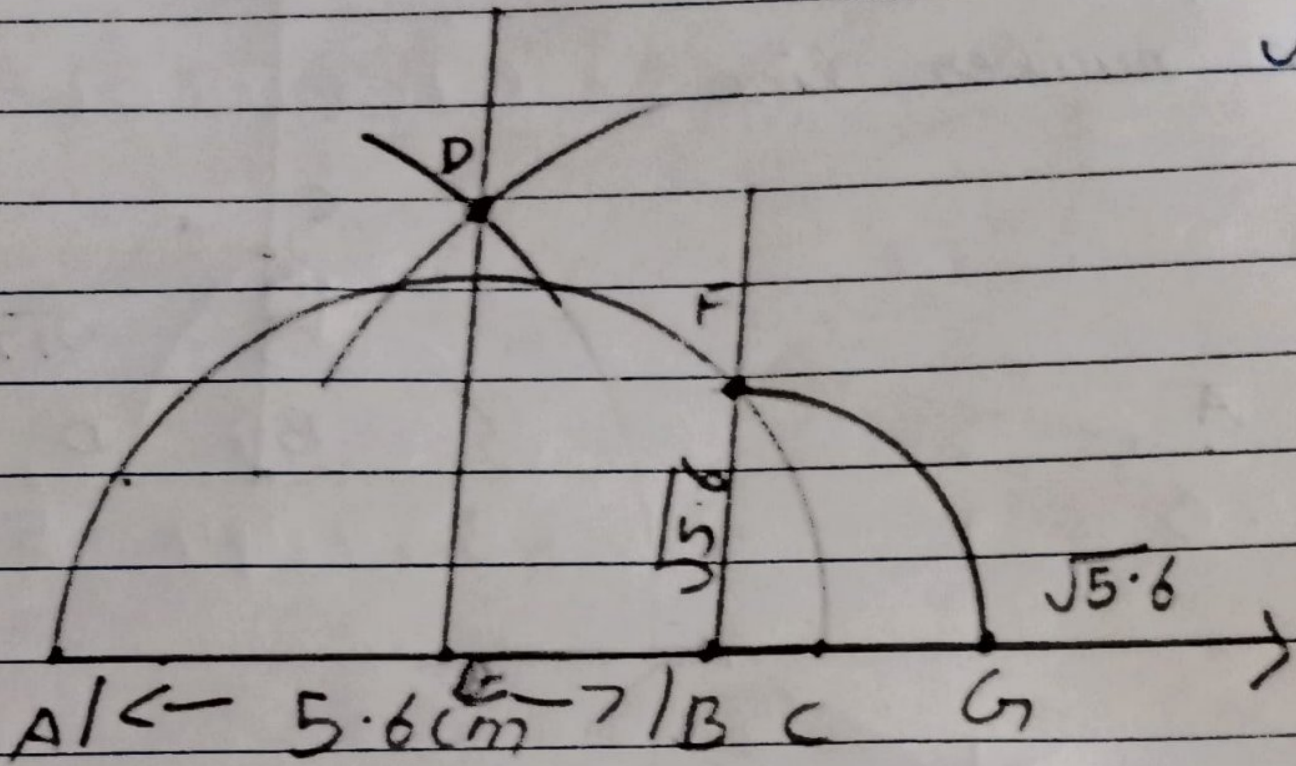


48. (i) $\sqrt{4.5}$



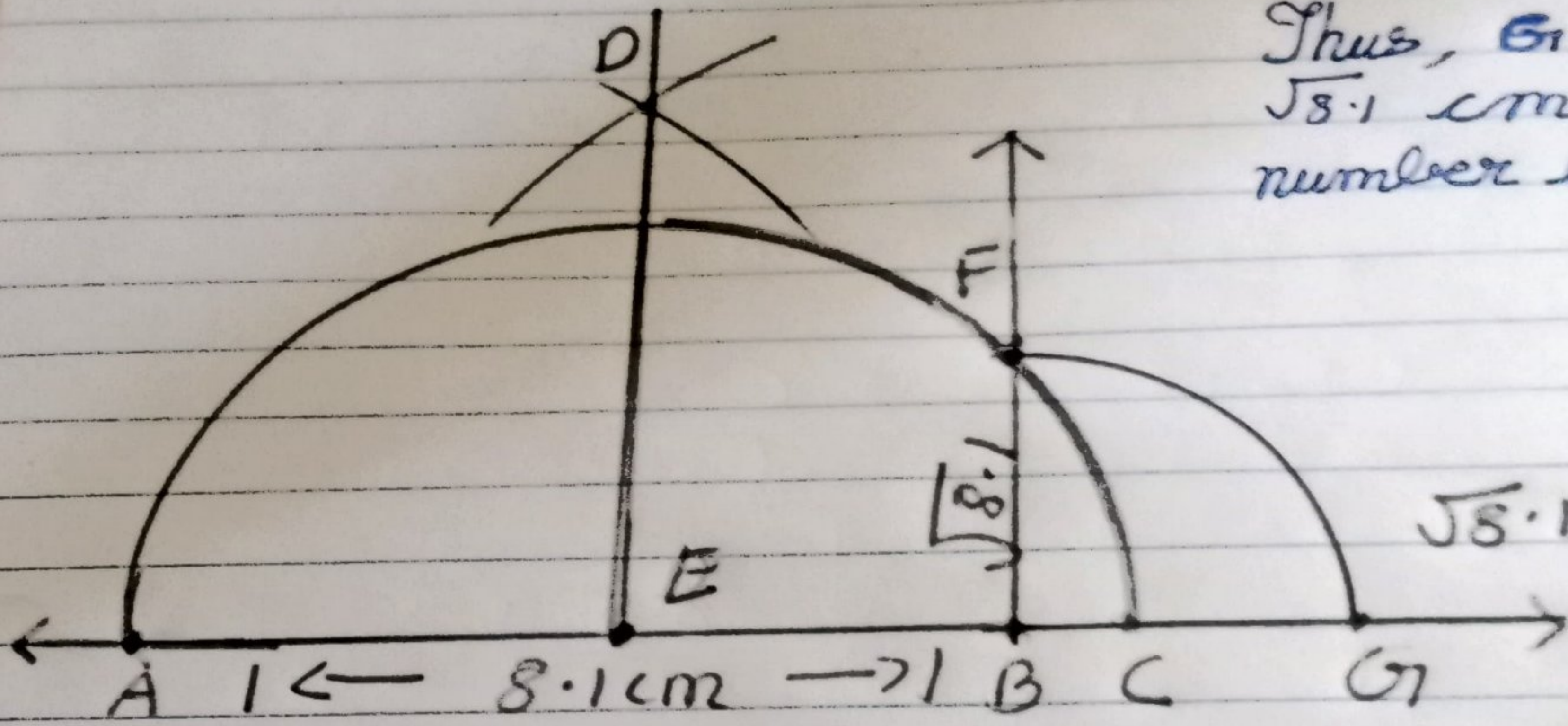
G represents $\sqrt{4.5}$.

(ii) $\sqrt{5.6}$



G represents $\sqrt{5.6}$.

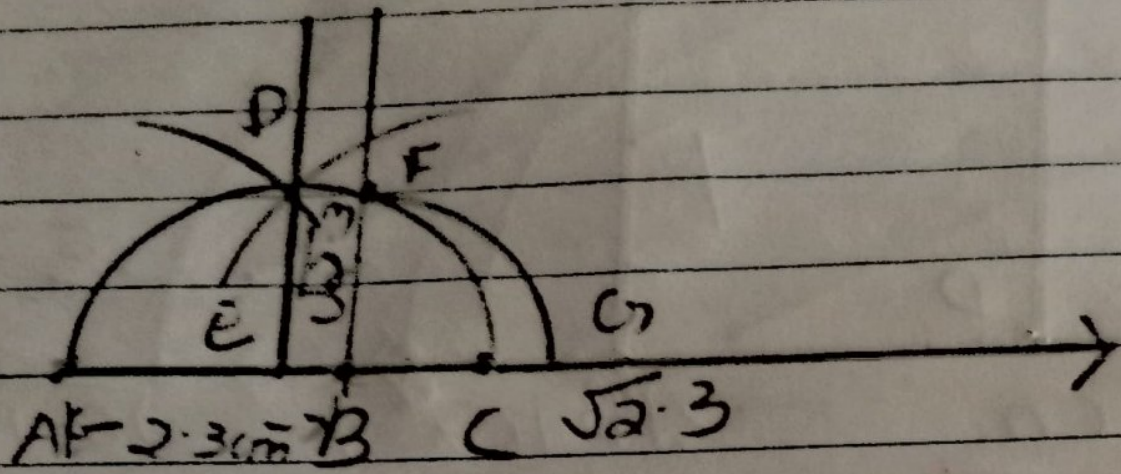
45 (iii) $\sqrt{8.1}$



Thus, G represents $\sqrt{8.1}$ cm on the number line.

(iii) $\sqrt{2.3}$

G represent $\sqrt{2.3}$.



(vi) Let $x = 0.1343434\cdots$

$10x = 1.343434\cdots \longrightarrow (1)$

$1000x = 134.3434\cdots \longrightarrow (2)$

$990x = 133$

$x = \frac{133}{990}$

(vii) Let $x = 0.00323232\cdots$

$100x = 0.323232\cdots \longrightarrow (1)$

$10000x = 32.3232\cdots \longrightarrow (2)$

(2)-(1), $9900x = 32$

$x = \frac{\cancel{32}\ \cancel{16}\ 8}{\cancel{9900}\ \cancel{4950}\ 2475} = \frac{8}{2475}$

(viii) Let $x = 0.404040\cdots \longrightarrow (1)$

$100x = 40.4040\cdots \longrightarrow (2)$

(2)-(1), $99x = 40$

$x = \frac{40}{99}$

50) Let $x = 0.142857142857\cdots \longrightarrow (1)$

$1000000x = 142857.142857\cdots \longrightarrow (2)$

(2)-(1), $999999x = 142857$

$x = \frac{142857}{999999} = \frac{1}{7}$

51) (i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

$= 3\sqrt{5} - 3\times2\sqrt{5} + 4\sqrt{5}$

$= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5}$

$= \sqrt{5}$

$5\,|\,45$, $3\,|\,9$, 3 $\qquad$ $2\,|\,20$, $2\,|\,10$, 5

(ii) $\frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$

$= \frac{\cancel{2}^{1}\sqrt{6}}{\cancel{8}\,4} + \frac{\cancel{3}^{1}\sqrt{6}}{\cancel{9}\,3}$

$= \frac{\sqrt{6}\times3}{4\times3} + \frac{\sqrt{6}\times4}{3\times4} = \frac{3\sqrt{6}+4\sqrt{6}}{12} = \frac{7\sqrt{6}}{12}$

$2\,|\,24$, $2\,|\,12$, $2\,|\,6$, 3 $\qquad$ $2\,|\,54$, $3\,|\,27$, $3\,|\,9$, 3

(iii) $\sqrt[4]{12} \times \sqrt[7]{6}$

$= 12^{\frac{1\times7}{4\times7}} \times 6^{\frac{1\times4}{7\times4}} = 12^{\frac{7}{28}} \times 6^{\frac{4}{28}} = \sqrt[28]{12^7 \times 6^4}$

$= \sqrt[28]{(2^2\times3)^7 \times (2\times3)^4} = \sqrt[28]{2^{18}\times3^{11}}$

(iv) $\left(4\sqrt{28} \div 3\sqrt{7}\right) \div \sqrt[3]{7}$

$= \left(\dfrac{4\times 2\cancel{\sqrt{7}}}{3\cancel{\sqrt{7}}}\right) \div \sqrt[3]{7} = \dfrac{8}{3} \div \sqrt[3]{7} = \dfrac{8}{3\sqrt[3]{7}}$

(v) $3\sqrt{3} + 2\sqrt{27} + \dfrac{7\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = 3\sqrt{3} + 2\times 3\sqrt{3} + \dfrac{7\sqrt{3}}{3}$

$= \dfrac{9\sqrt{3} + 18\sqrt{3} + 7\sqrt{3}}{3} = \dfrac{34\sqrt{3}}{3}$

(vi) $(\sqrt{3} - \sqrt{2})^2 = (\sqrt{3})^2 - 2\times\sqrt{3}\times\sqrt{2} + (\sqrt{2})^2$

$= 3 - 2\sqrt{6} + 2 = 5 - 2\sqrt{6}$

(vii) $\sqrt[4]{81} - 8\times\sqrt[3]{216} + 15\times\sqrt[5]{32} + \sqrt{225}$

$= 3^{\cancel{4}\times\frac{1}{\cancel{4}}} - 8\times 6^{\cancel{3}\times\frac{1}{\cancel{3}}} + 15\times 2^{\cancel{5}\times\frac{1}{\cancel{5}}} + 15$

$= 3 - 8\times 6 + 15\times 2 + 15$

$= 3 - 48 + 30 + 15 = 3 - 48 + 45 = 48 - 48 = 0$

(viii) $\dfrac{3}{\sqrt{8}} + \dfrac{1}{\sqrt{2}} = \dfrac{3}{2\sqrt{2}} + \dfrac{1\times 2}{\sqrt{2}\times 2}$

$= \dfrac{3+2}{2\sqrt{2}} = \dfrac{5\times\sqrt{2}}{2\sqrt{2}\times\sqrt{2}} = \dfrac{5\sqrt{2}}{4}$

(ix) $\dfrac{2\sqrt{3}^{\times 2}}{3\times 2} - \dfrac{\sqrt{3}}{6} = \dfrac{4\sqrt{3} - \sqrt{3}}{6} = \dfrac{3\sqrt{3}}{6} = \dfrac{\sqrt{3}}{2}$

52) (i) $\dfrac{2\times\sqrt{3}}{3\sqrt{3}\times\sqrt{3}} = \dfrac{2\sqrt{3}}{9}$

(ii) $\dfrac{\sqrt{40}\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \dfrac{\sqrt{120}}{3} = \dfrac{2\sqrt{30}}{3}$

$$\begin{array}{r|l} 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ & 5 \end{array}$$

(iii) $\dfrac{(3+\sqrt{2})\times\sqrt{2}}{4\sqrt{2}\times\sqrt{2}} = \dfrac{3\sqrt{2}+2}{8}$

(iv) $\dfrac{16\times(\sqrt{41}+5)}{(\sqrt{41}-5)(\sqrt{41}+5)} = \dfrac{16(\sqrt{41}+5)}{41-25} = \dfrac{16(\sqrt{41}+5)}{16}$

$= \sqrt{41}+5$

(v) $\dfrac{(2+\sqrt{3})^2}{(2-\sqrt{3})(2+\sqrt{3})} = \dfrac{2^2+(\sqrt{3})^2+2\times2\times\sqrt{3}}{4-3} = \dfrac{4+3+4\sqrt{3}}{1} = \underline{\underline{7+4\sqrt{3}}}$

(vi) $\dfrac{\sqrt{6}\times(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})} = \dfrac{\sqrt{12}-\sqrt{18}}{2-3} = \dfrac{2\sqrt{3}-3\sqrt{2}}{-1}$ $\quad 2\lfloor 12,\ 2\lfloor 6,\ 3 \quad 2\lfloor 18,\ 3\lfloor 9,\ 3$

$= \underline{\underline{3\sqrt{2}-2\sqrt{3}}}$

(vii) $\dfrac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} = \dfrac{(\sqrt{3})^2+(\sqrt{2})^2+2\sqrt{3}\times\sqrt{2}}{3-2} = \dfrac{3+2+2\sqrt{6}}{1} = \underline{\underline{5+2\sqrt{6}}}$

(viii) $\dfrac{(3\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} = \dfrac{3\times5+3\sqrt{15}+\sqrt{15}+3}{5-3} = \dfrac{15+4\sqrt{15}+3}{2}$

$= \dfrac{18+4\sqrt{15}}{2} = \dfrac{\cancel{2}(9+2\sqrt{15})}{\cancel{2}\,1} = \underline{\underline{9+2\sqrt{15}}}$

(ix) $\dfrac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \dfrac{(4\sqrt{3}+5\sqrt{2})(4\sqrt{3}-3\sqrt{2})}{(4\sqrt{3}+3\sqrt{2})(4\sqrt{3}-3\sqrt{2})}$ $\quad 2\lfloor 48,\ 2\lfloor 24,\ 2\lfloor 12,\ 2\lfloor 6,\ 3 \quad 2\lfloor 18,\ 3\lfloor 9,\ 3$

$= \dfrac{16\times3-12\sqrt{6}+20\sqrt{6}-15\times2}{48-18}$

$= \dfrac{48+8\sqrt{6}-30}{30} = \dfrac{18+8\sqrt{6}}{30} = \dfrac{\cancel{2}^{1}(9+4\sqrt{6})}{\cancel{30}\,15} = \underline{\underline{\dfrac{9+4\sqrt{6}}{15}}}$

53)

(i) $\dfrac{(5+2\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})} = \dfrac{35-20\sqrt{3}+14\sqrt{3}-24}{49-48} = 11-6\sqrt{3}$

$\therefore\ 11-6\sqrt{3} = a-6\sqrt{3}$

$\boxed{a=11}$

(ii) $\dfrac{(3-\sqrt{5})(3-2\sqrt{5})}{(3+2\sqrt{5})(3-2\sqrt{5})} = \dfrac{9-6\sqrt{5}-3\sqrt{5}+10}{9-20} = \dfrac{19-9\sqrt{5}}{-11} = \dfrac{9\sqrt{5}-19}{11}$

$\therefore\ \dfrac{9}{11}\sqrt{5}-\dfrac{19}{11} = a\sqrt{5}-\dfrac{19}{11}$

$\boxed{a=\dfrac{9}{11}}$

(iii) $\dfrac{(\sqrt{2}+\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})} = \dfrac{6+2\sqrt{6}+3\sqrt{6}+6}{18-12} = \dfrac{12+5\sqrt{6}}{6} = 2+\dfrac{5}{6}\sqrt{6}$

$\therefore\ 2+\dfrac{5}{6}\sqrt{6} = 2-b\sqrt{6}$

$\boxed{b=-5/6}$

(iii) $\dfrac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = 2 - b\sqrt{6}$

(iv) $\dfrac{7+\sqrt{5}}{7-\sqrt{5}} - \dfrac{7-\sqrt{5}}{7+\sqrt{5}} = a + \dfrac{7}{11}\sqrt{5}\,b$

54) If $a = 2+\sqrt{3}$, then find the value of $a - \frac{1}{a}$

55) Rationalise the denominator in each of the following and hence evaluate by taking $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$, upto three places of decimal.

(i) $\dfrac{4}{\sqrt{3}}$

(ii) $\dfrac{6}{\sqrt{6}}$

(iii) $\dfrac{\sqrt{10}-\sqrt{5}}{2}$

(iv) $\dfrac{\sqrt{2}}{2+\sqrt{2}}$

(v) $\dfrac{1}{\sqrt{3}+\sqrt{2}}$

56) Simplify:

(i) $\left(1^3+2^3+3^3\right)^{\frac{1}{2}}$

(ii) $\left(\dfrac{3}{5}\right)^4 \left(\dfrac{8}{5}\right)^{-12} \left(\dfrac{32}{5}\right)^6$

(iii) $\left(\dfrac{1}{27}\right)^{-\frac{2}{3}}$

(iv) $\left(\left((625)^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right)^2$

(v) $\dfrac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}}$

(vi) $64^{-\frac{1}{3}}\left[64^{\frac{1}{3}} - 64^{\frac{2}{3}}\right]$

(vii) $\dfrac{8^{\frac{1}{3}} \times 16^{\frac{1}{3}}}{32^{-\frac{1}{3}}}$

57) If $a = 5+2\sqrt{6}$ and $b = \frac{1}{a}$, then what will be the value of a^2+b^2?

58) Express $0.6 + 0.\overline{7} + 0.4\overline{7}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

59) Simplify: $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

60) If $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, then find the value of

$\frac{4}{3\sqrt{3}-2\sqrt{2}} + \frac{3}{3\sqrt{3}+2\sqrt{2}}$

61) If $a = \frac{3+\sqrt{5}}{2}$, then find the value of $a^2 + \frac{1}{a^2}$

62) If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then find the value of x^2+y^2

63) Simplify: $(256)^{-(4^{-3/2})}$

64) Find the value of $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$

(iii) $\dfrac{(\sqrt{2}+\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})} = \dfrac{6+2\sqrt{6}+3\sqrt{6}+6}{18-12} = \dfrac{12+5\sqrt{6}}{6} = 2+\dfrac{5}{6}\sqrt{6}$

$\therefore 2+\dfrac{5}{6}\sqrt{6} = 2-b\sqrt{6}$

$\boxed{b = -5/6}$

(iv) $\frac{(7+\sqrt{5})^2}{(7-\sqrt{5})(7+\sqrt{5})} = \frac{(7)^2+(\sqrt{5})^2+2\times7\times\sqrt{5}}{49-5} = \frac{49+5+14\sqrt{5}}{44}$

$= \frac{54+14\sqrt{5}}{44} = \frac{\cancel{2}^{1}(27+7\sqrt{5})}{\cancel{44}_{22}} = \frac{27+7\sqrt{5}}{22}$

$\frac{(7-\sqrt{5})^2}{(7+\sqrt{5})(7-\sqrt{5})} = \frac{(7)^2+(\sqrt{5})^2-2\times7\times\sqrt{5}}{49-5} = \frac{49+5-14\sqrt{5}}{44}$

$= \frac{54-14\sqrt{5}}{44} = \frac{27-7\sqrt{5}}{22}$

$\therefore \frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = \frac{27+7\sqrt{5}}{22} - \frac{27-7\sqrt{5}}{22} = \frac{\cancel{27}+7\sqrt{5}-\cancel{27}+7\sqrt{5}}{22}$

$= \frac{\cancel{14}^{7}\sqrt{5}}{\cancel{22}_{11}} = 0+\frac{7}{11}\sqrt{5}$

$\therefore 0+\frac{7}{11}\sqrt{5} = a+\frac{7}{11}\sqrt{5}\,b$

$\boxed{\begin{array}{l} a=0 \\ b=1 \end{array}}$

54) $a = 2+\sqrt{3}$

$\frac{1}{a} = \frac{1\,(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

$\therefore a-\frac{1}{a} = (2+\sqrt{3})-(2-\sqrt{3}) = \cancel{2}+\sqrt{3}-\cancel{2}+\sqrt{3} = \underline{\underline{2\sqrt{3}}}$

55)

(i) $\frac{4\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{4\sqrt{3}}{3} = \frac{4\times1.732}{3} = \frac{6.928}{3} = \underline{\underline{2.309}}.$

(ii) $\frac{6\times\sqrt{6}}{\sqrt{6}\times\sqrt{6}} = \frac{6\sqrt{6}}{6} = \sqrt{6} = \sqrt{2}\times\sqrt{3} = 1.414\times1.732 = \underline{\underline{2.449}}$

(iii) $\frac{\sqrt{10}-\sqrt{5}}{2} = \frac{\sqrt{2}\times\sqrt{5}-\sqrt{5}}{2} = \frac{\sqrt{5}(\sqrt{2}-1)}{2} = \frac{\cancel{2.236}^{1.118}(1.414-1)}{\cancel{2}_{1}}$

$= 1.118\times0.414 = \underline{\underline{0.463}}$

(iv) $\frac{\sqrt{2}(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})} = \frac{2\sqrt{2}-2}{4-2} = \frac{2(\sqrt{2}-1)}{2} = \sqrt{2}-1 = 1.414-1 = \underline{\underline{0.414}}$

(v) $\frac{1}{(\sqrt{3}+\sqrt{2})}\times\frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})} = \frac{\sqrt{3}-\sqrt{2}}{3-2} = \frac{1.732-1.414}{1} = \underline{\underline{0.318}}$

56)

(i) $(1^3+2^3+3^3)^{\frac{1}{2}} = (1+8+27)^{\frac{1}{2}} = 36^{\frac{1}{2}} = \sqrt{36} = \underline{\underline{6}}$

(ii) $\left(\frac{3}{5}\right)^4 \times \left(\frac{8}{5}\right)^{-12} \times \left(\frac{32}{5}\right)^6 = \frac{3^4}{5^4} \times \frac{5^{12}}{(2^3)^{12}} \times \frac{(2^5)^6}{5^6} = \frac{3^4 \times 5^{12} \times 2^{30}}{5^4 \times 2^{36} \times 5^6}$

$= 3^4 \times 5^{12-4-6} \times 2^{30-36}$

$= 3^4 \times 5^2 \times 2^{-6} = \frac{3^4 \times 5^2}{2^6} = \frac{81 \times 25}{64} = \underline{\underline{\frac{2025}{64}}}$

(iii) $\left(\frac{1}{27}\right)^{-\frac{2}{3}} = (27)^{\frac{2}{3}} = 3^{3 \times \frac{2}{3}} = 3^2 = \underline{\underline{9}}$

(iv) $\left[\left((625)^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right]^2 = 5^{4 \times -\frac{1}{2} \times -\frac{1}{4} \times 2} = \underline{5}$

(v) $\frac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}} = \frac{3^{\frac{2}{3}} \times 3^{-\frac{3}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}} = \frac{3^{\frac{2}{3}-\frac{3}{2}}}{3^{\frac{1}{6}-\frac{2}{3}}} = \frac{3^{\frac{4-9}{6}}}{3^{\frac{1-4}{6}}}$

$= \frac{3^{-\frac{5}{6}}}{3^{-\frac{3}{6}}} = 3^{-\frac{5}{6}+\frac{3}{6}} = 3^{-\frac{2}{6}} = 3^{-\frac{1}{3}} = \underline{\underline{\frac{1}{3^{1/3}}}}$

(vi) $64^{-\frac{1}{3}}\left(64^{\frac{1}{3}} - 64^{\frac{2}{3}}\right) = 4^{-\frac{3}{3}}\left(4^{\frac{3}{3}} - 4^{\frac{6}{3}}\right)$

$= 4^{-1}(4-4^2) = \frac{4-16}{4} = \frac{-12}{4} = \underline{\underline{-3}}$

(vii) $\frac{8^{\frac{1}{3}} \times 16^{\frac{1}{3}}}{32^{-\frac{1}{3}}} = \frac{2^{\frac{3}{3}} \times 2^{\frac{4}{3}}}{2^{-\frac{5}{3}}} = \frac{2^{\frac{7}{3}}}{2^{-\frac{5}{3}}} = 2^{\frac{7}{3}+\frac{5}{3}} = 2^{\frac{12}{3}} = 2^4 = \underline{\underline{16}}$

57) $a = 5+2\sqrt{6}$

$b = \frac{1}{a} = \frac{1 \times (5-2\sqrt{6})}{(5+2\sqrt{6})(5-2\sqrt{6})} = \frac{5-2\sqrt{6}}{25-24} = 5-2\sqrt{6}$

$\therefore a^2+b^2 = (5+2\sqrt{6})^2 + (5-2\sqrt{6})^2$

$= 25+24+\cancel{20\sqrt{6}}+25+24-\cancel{20\sqrt{6}} = \underline{\underline{98}}$

58) $0.6 = \frac{6}{10}$

Let $x = 0.7777\cdots \rightarrow (1)$

$10x = 7.7777\cdots \rightarrow (2)$

$(2)-(1)$, $9x = 7$

$x = \frac{7}{9}$

Let $y = 0.4777\cdots$

$10y = 4.7777\cdots \rightarrow (1)$

$100y = 47.7777\cdots \rightarrow (2)$

$(2)-(1)$, $90y = 43$

$y = \frac{43}{90}$

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$\therefore 0.\dot{6}+0.\dot{7}+0.4\dot{7} = \frac{6^{\times 9}}{10_{\times 9}} + \frac{7^{\times 10}}{9_{\times 10}} + \frac{43}{90} = \frac{54+70+43}{90} = \frac{167}{90}$

59) $\frac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{(\sqrt{10}+\sqrt{3})(\sqrt{10}-\sqrt{3})} - \frac{2\sqrt{5}(\sqrt{6}-\sqrt{5})}{(\sqrt{6}+\sqrt{5})(\sqrt{6}-\sqrt{5})} - \frac{3\sqrt{2}(\sqrt{15}-3\sqrt{2})}{(\sqrt{15}+3\sqrt{2})(\sqrt{15}-3\sqrt{2})}$

$= \frac{7\sqrt{30}-7\times 3}{10-3} - \frac{2\sqrt{30}-10}{6-5} - \frac{3\sqrt{30}-18}{15-18}$

$= \frac{\cancel{7}(\sqrt{30}-3)}{\cancel{7}} - 2\sqrt{30}+10 + \frac{\cancel{3}(\sqrt{30}-6)}{\cancel{+3}}$

$= \sqrt{30}-3-2\sqrt{30}+10+\sqrt{30}-6$

$= \cancel{2\sqrt{30}} - \cancel{2\sqrt{30}} + 10 - 9 = \underline{\underline{1}}$

60) $\frac{4\times(3\sqrt{3}+2\sqrt{2})}{(3\sqrt{3}-2\sqrt{2})(3\sqrt{3}+2\sqrt{2})} + \frac{3\times(3\sqrt{3}-2\sqrt{2})}{(3\sqrt{3}+2\sqrt{2})(3\sqrt{3}-2\sqrt{2})}$

$= \frac{4(3\sqrt{3}+2\sqrt{2})}{27-8} + \frac{3(3\sqrt{3}-2\sqrt{2})}{27-8}$

$= \frac{12\sqrt{3}+8\sqrt{2}+9\sqrt{3}-6\sqrt{2}}{19} = \frac{21\sqrt{3}+2\sqrt{2}}{19} = \frac{21\times 1.732+2\times 1.414}{19}$

$= \frac{36.372+2.828}{19} = \frac{39.2}{19} = \underline{\underline{2.063}}$

61) $a = \frac{3+\sqrt{5}}{2}$

$a^2 = \left(\frac{3+\sqrt{5}}{2}\right)^2 = \frac{9+5+6\sqrt{5}}{4} = \frac{14+6\sqrt{5}}{4}$

$\frac{1}{a} = \frac{2(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})} = \frac{2(3-\sqrt{5})}{9-5} = \frac{\cancel{2}(3-\sqrt{5})}{\cancel{4}2} = \frac{3-\sqrt{5}}{2}$

$\left(\frac{1}{a}\right)^2 = \left(\frac{3-\sqrt{5}}{2}\right)^2 = \frac{9+5-6\sqrt{5}}{4} = \frac{14-6\sqrt{5}}{4}$

$\therefore a^2 + \frac{1}{a^2} = \frac{14+\cancel{6\sqrt{5}}+14-\cancel{6\sqrt{5}}}{4} = \frac{28}{4} = \underline{\underline{7}}$

62) $x = \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} = \frac{3+2+2\sqrt{6}}{3-2} = 5+2\sqrt{6}$

$x^2 = (5+2\sqrt{6})^2 = 25+24+20\sqrt{6} = 49+20\sqrt{6}$

$$y = \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{3+2-2\sqrt{6}}{3-2} = 5-2\sqrt{6}$$

$$y^2 = (5-2\sqrt{6})^2 = 25+24-20\sqrt{6} = 49-20\sqrt{6}$$

$$\therefore x^2+y^2 = 49+\cancel{20\sqrt{6}}+49-\cancel{20\sqrt{6}} = \underline{\underline{98}}$$

63) $(256)^{-(4^{-\frac{3}{2}})}$

$$= (256)^{-(2^{2\times-\frac{3}{2}})} = (256)^{-(2^{-3})} = (256)^{-\frac{1}{8}}$$

$$= 2^{8\times-\frac{1}{8}} = 2^{-1} = \underline{\underline{\frac{1}{2}}}$$

64) $\dfrac{4}{6^{3\times-\frac{2}{3}}} + \dfrac{1}{4^{4\times-\frac{3}{4}}} + \dfrac{2}{3^{5\times-\frac{1}{5}}}$

$$= \frac{4}{6^{-2}} + \frac{1}{4^{-3}} + \frac{2}{3^{-1}}$$

$$= 4\times 6^2 + 4^3 + 2\times 3$$

$$= 4\times 36 + 64 + 6$$

$$= 144+70 = \underline{\underline{214}}$$