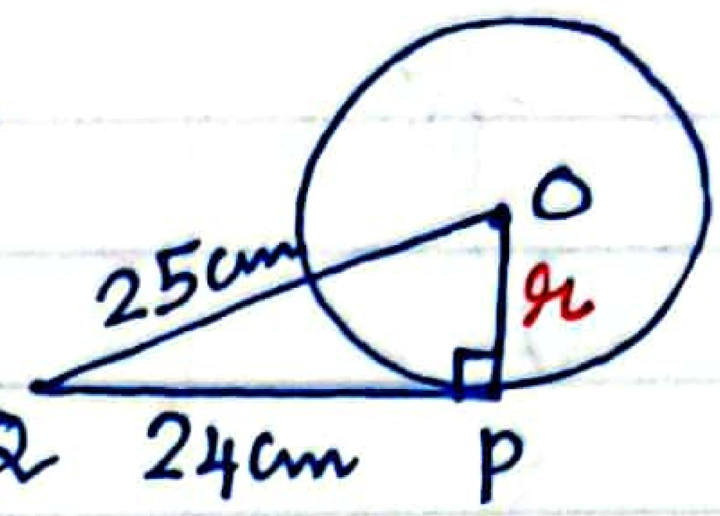
X CIRCLES

1) From a point Q, the length of the tangents to a circle is 24cm and the distance of Q from the centre is 25cm. Find the radius of the circle.

ans:- Since radius OP is perpendicular to the tangent QP at P.



$\angle OPQ = 90°$

In rt. $\triangle OPQ$, $\therefore OP^2 = OQ^2 - PQ^2$

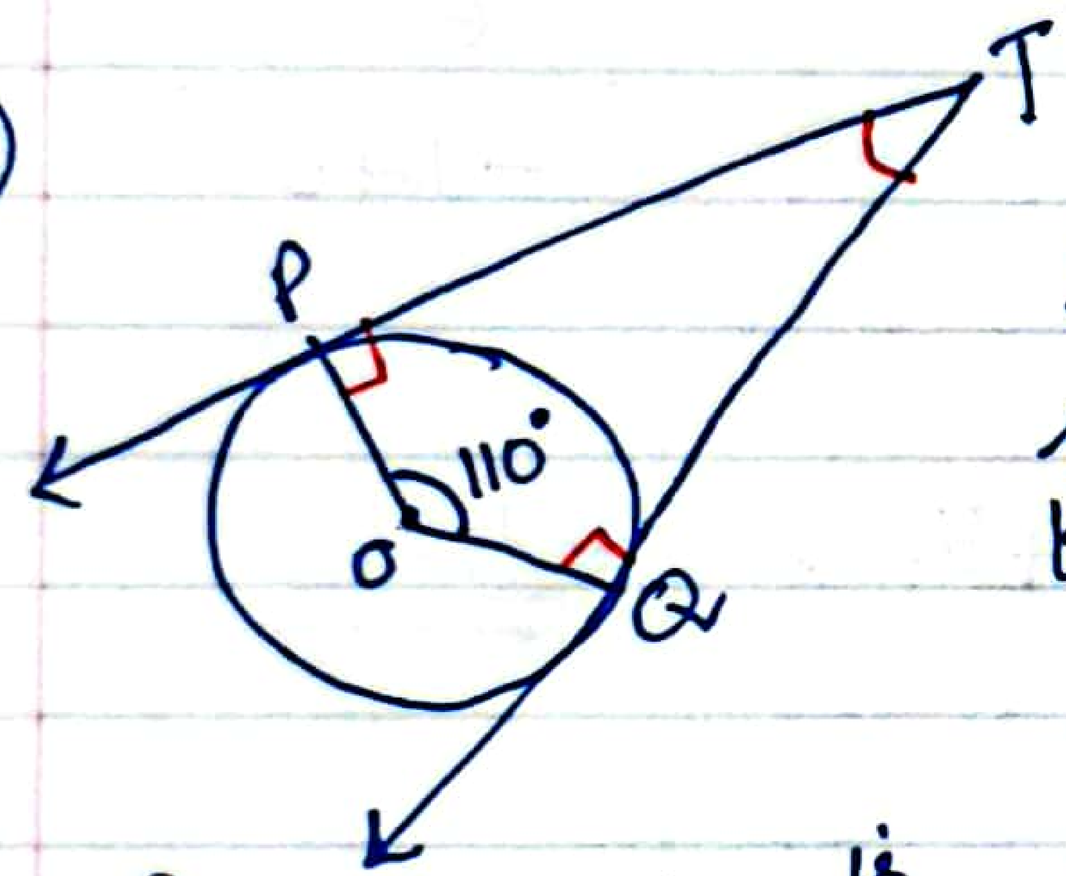
$= 25^2 - 24^2 = (25+24)(25-24)$

$= 49 \times 1$

$\therefore OP = \sqrt{49} = 7cm$

Hence, the radius of the circle = 7cm

2)



If TP and TQ are two tangents to a circle with Centre O so that $\angle POQ = 110°$, then find $\angle PTQ$.

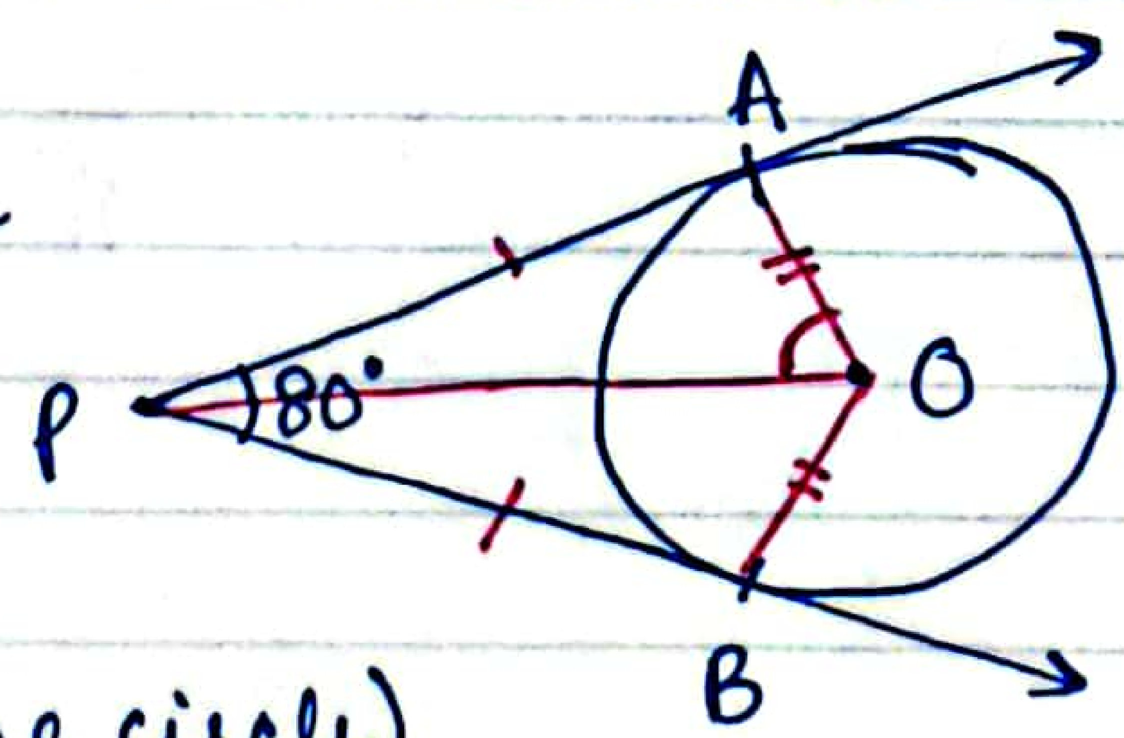
ans:- Since radius is perpendicular to the tangent through the point of contact, $\angle OPT = \angle OQT = 90°$

In quadrilateral OPTQ, $\angle PTQ = 360° - (110° + 90° + 90°)$

$= 360° - 290° = 70°$

3) If tangents PA and PB from a point P to a circle with Centre O are inclined to each other at an angle of 80°, then find $\angle POA$.

ans:- Since tangents drawn from an external point are equal in lengths,



PA = PB

OA = OB (radii of the same circle)

OP = OP (common side)

$\therefore \triangle OAP \cong \triangle OBP$ (SSS congruency)

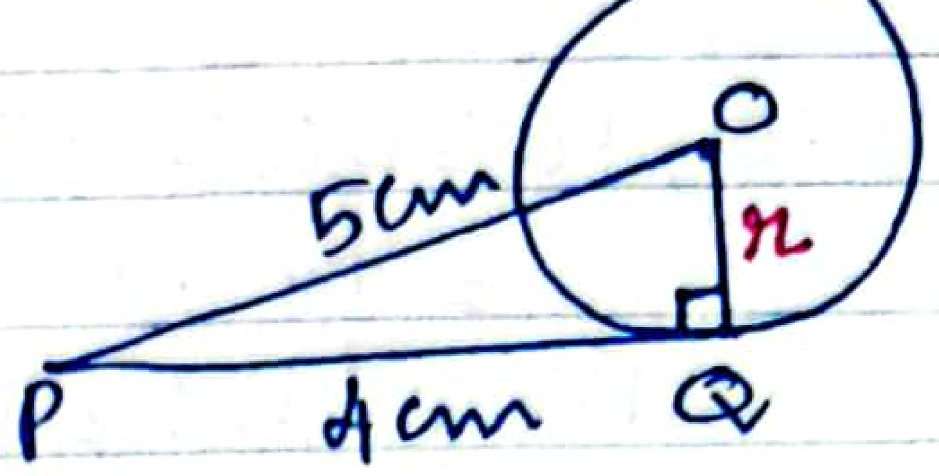
Thus $\angle APO = \angle BPO = \frac{80}{2} = 40°$ [by CPCT]

In rt. $\triangle OAP$, $\angle POA = 180° - (90° + 40°)$

$= 180° - 130° = 50°$

4) The length of a tangent from a point A at a distance of 5cm from the centre of the circle is 4cm. Find the radius of the circle.

ans:- Since radius is perpendicular to the tangent through the point of contact, $\angle OQP = 90°$.



In rt. ΔOQP, $OQ^2 = OP^2 - PQ^2$

$$= 5^2 - 4^2 = (5+4)(5-4)$$

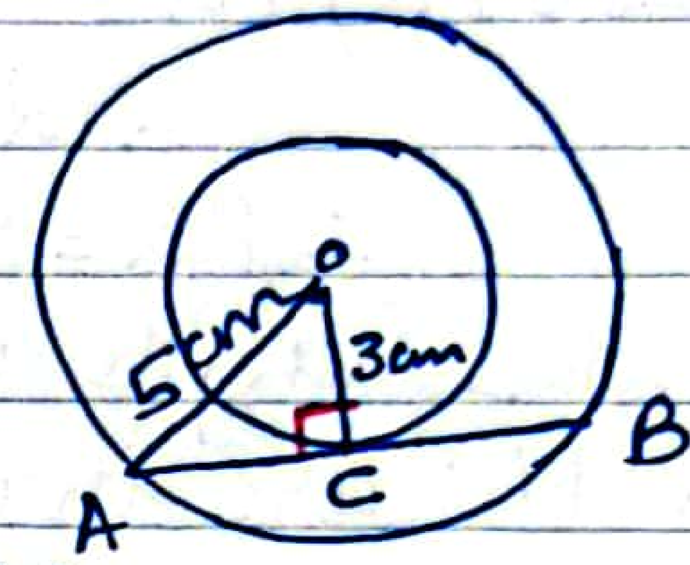
$$= 9 \times 1 = 9$$

$$\therefore OQ = \sqrt{9} = 3cm$$

Hence, the radius of the circle = 3cm.

5) If two concentric circles are of radii 3cm and 5cm, then find the length of the chord of the larger circle which touches the smaller circle.

ans:- Since radius is perpendicular to the tangent through the point of contact, $\angle OCA = 90°$.



In rt. ΔOCA, $AC^2 = OA^2 - OC^2$

$$= 5^2 - 3^2 = 25 - 9 = 16$$

$$\therefore AC = \sqrt{16} = 4cm.$$

Since ⊥ drawn from the centre of a circle to a chord bisects the chord, $AC = BC = \frac{1}{2} AB$

$$\therefore AB = 2AC = \underline{8cm}$$

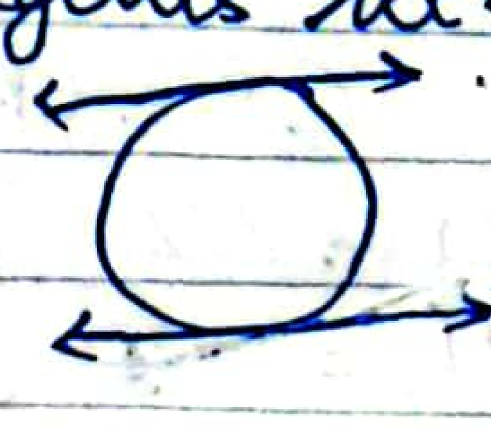
6) Atmost how many tangents a circle can have?

ans:- Infinitely many

7) A tangent to a circle intersects it in two points. Is it true?

ans:- False, a tangent to a circle touches it only at 1 point.

8) A circle can have two parallel tangents at the most. Is it true?



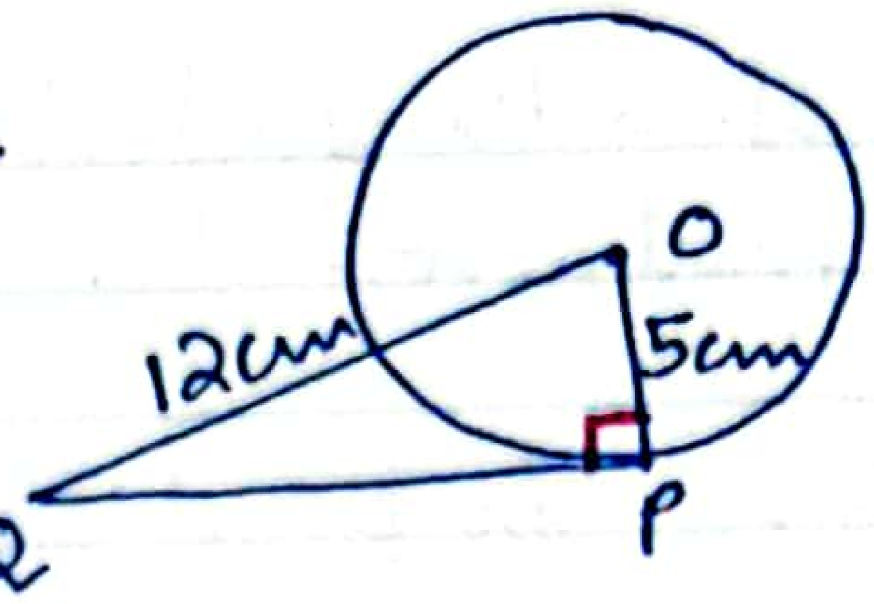
ans:- True

9) Name the common point of a tangent to a circle and the circle.

ans:- Point of contact.

10) A tangent PQ at a point P of a circle of radius 5cm meets a line through the centre O at a point Q so that OQ = 12cm. Find the length of PQ.

ans:- Since radius is perpendicular to the tangent through the point of contact, $\angle OPQ = 90°$.



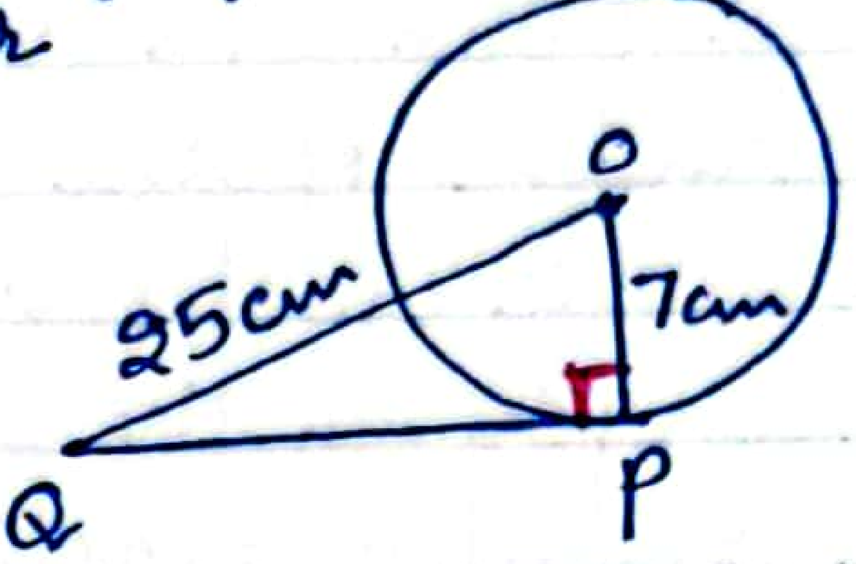
In rt. ΔOPQ, $PQ^2 = OQ^2 - OP^2$

$= 12^2 - 5^2 = 144 - 25 = 119$

$PQ = \sqrt{119}$ cm

11) Find the length of a tangent drawn to a circle of radius 7cm from a point 25cm away from the centre.

ans:- Since radius is perpendicular to the tangent through the point of contact, $\angle OPQ = 90°$.



In rt. ΔOPQ, $PQ^2 = OQ^2 - OP^2$

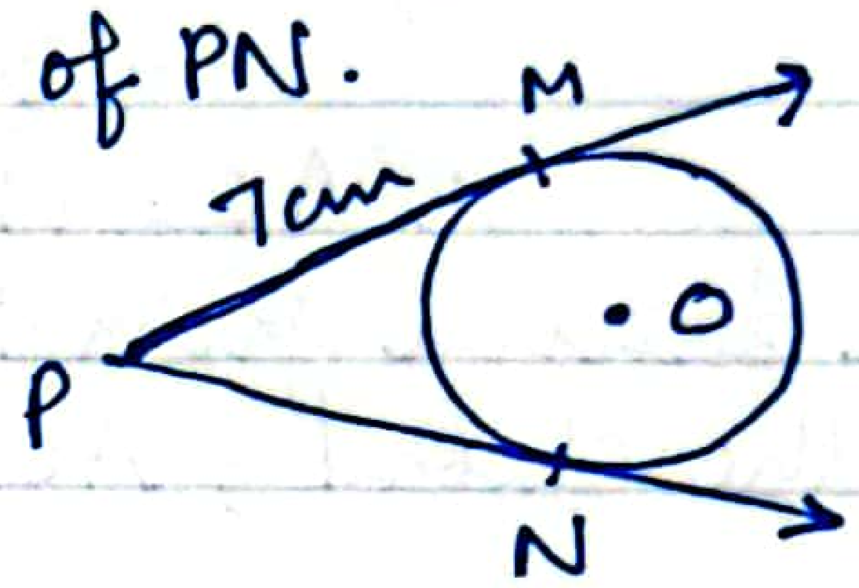
$= 25^2 - 7^2 = 625 - 49 = 576$

$\therefore PQ = \sqrt{576} = \underline{24 cm}$

Hence, the length of the tangent $= 24$ cm

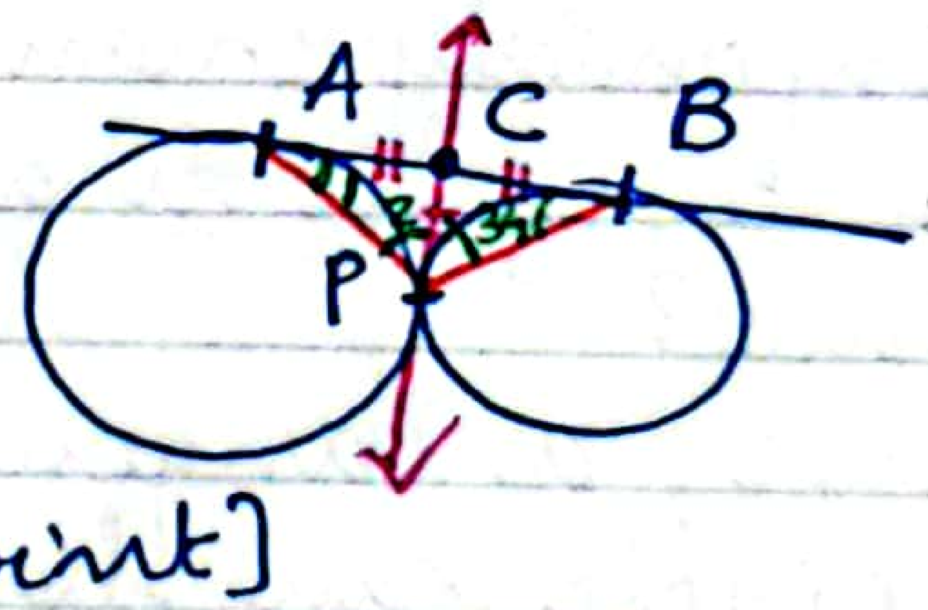
12) PM and PN are two tangents to a circle with centre O. If PM = 7cm, then find the value of PN.

ans:- Since tangents are drawn from an external point are equal in lengths, PM = PN = 7cm //



13) Two circles touch each other externally at P. AB is a common tangent to the circle touching them at A and B. Find the value of $\angle APB$.

ans:- Since tangents drawn from an external point are equal in lengths, $CA = CP \rightarrow (1)$ [$\because$ C is an external point]



and $CB = CP \rightarrow (2)$

Thus, $\left.\begin{array}{l}\angle 1 = \angle 2 \\ \angle 4 = \angle 3\end{array}\right\}$ angles opposite to equal sides

Using angle sum property in ΔAPB,

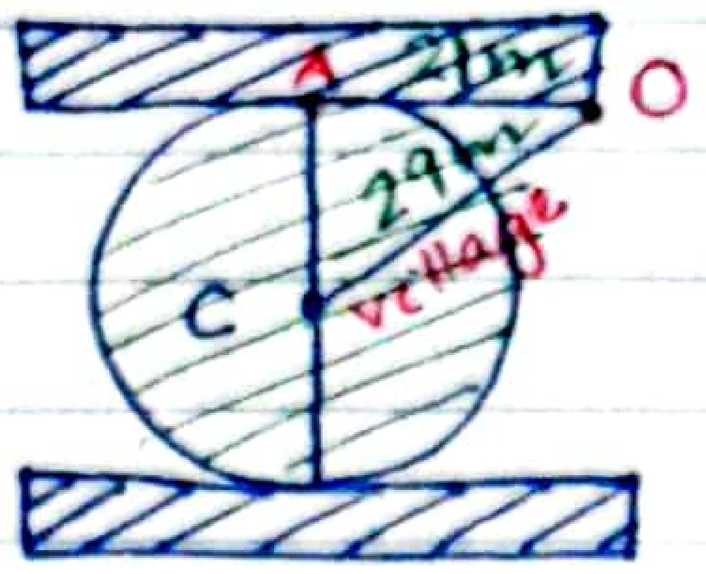
$\angle PAB + \angle PBA + \angle APB = 180°$

$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

$\Rightarrow 2\angle 2 + 2\angle 3 = 180°$

$\angle 2 + \angle 3 = 90° \Rightarrow \underline{\angle APB = 90°}$

Case-Study



The people of a small village want to construct a concrete road in their village. The road cannot pass through the village but villagers want the road should be at the shortest distance from the Centre of the village. Suppose the road starts from point O, which is outside the circular village and touch the boundary of the circular village at point A, such that OA = 21m. And also the straight distance of the point O from the Centre C of the village is 29m.

Answer the following:

(i) The shortest distance of the road from the centre of the village is: (a) 15m (b) 17m (c) 18m (d) 20m

ans:- Since radius ⊥ tangent through the point of contact, $\angle CAO = 90^\circ$

In rt. $\triangle CAO$, using Pythagoras Theorem, $AC^2 = CO^2 - OA^2$

$= 29^2 - 21^2$

$= 841 - 441 = 400$

$AC = \sqrt{400} = 20m$ (d)

(ii) Which method should be applied to find the shortest distance (a) Pythagoras Theorem (b) Concept of secant to a circle (c) Concept of tangent to a circle (d) both (a) and (c)

ans:- both (a) and (c) (d)

(iii) If a point is inside a circle, how many tangents can be drawn from that point? (a) 1 (b) 0 (c) 2 (d) Infinite

ans:- 0 (b)

(iv) The lengths of tangents drawn from an external point to a circle are:
(a) equal (b) unequal (c) both (a) and (b) (d) none of these

ans:- equal (a)

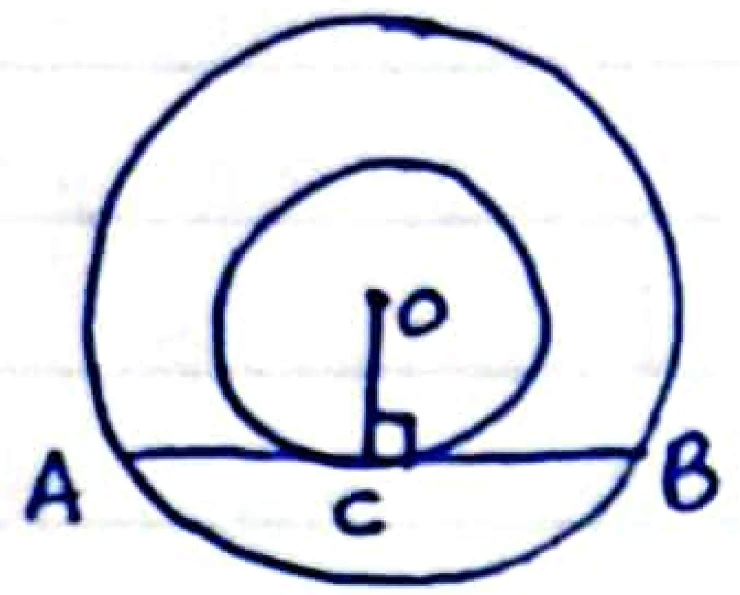
(v) If we draw two tangents roads at the end of the diameter, then these tangents roads are always
(a) coincident (b) Parallel (c) perpendicular (d) none of these

ans:- parallel (b)

15) Prove that in two concentric circles, the chord of the larger

Circle, which touches the smaller circle, is bisected at the point of contact.

ans:-



Given: O is the centre of two concentric circles. AB is the chord of larger circle touching the smaller circle at C.

To prove: $AC = BC$

Construction: Join OC.

Proof:- since radius is perpendicular through the point of contact, $OC \perp AB$.

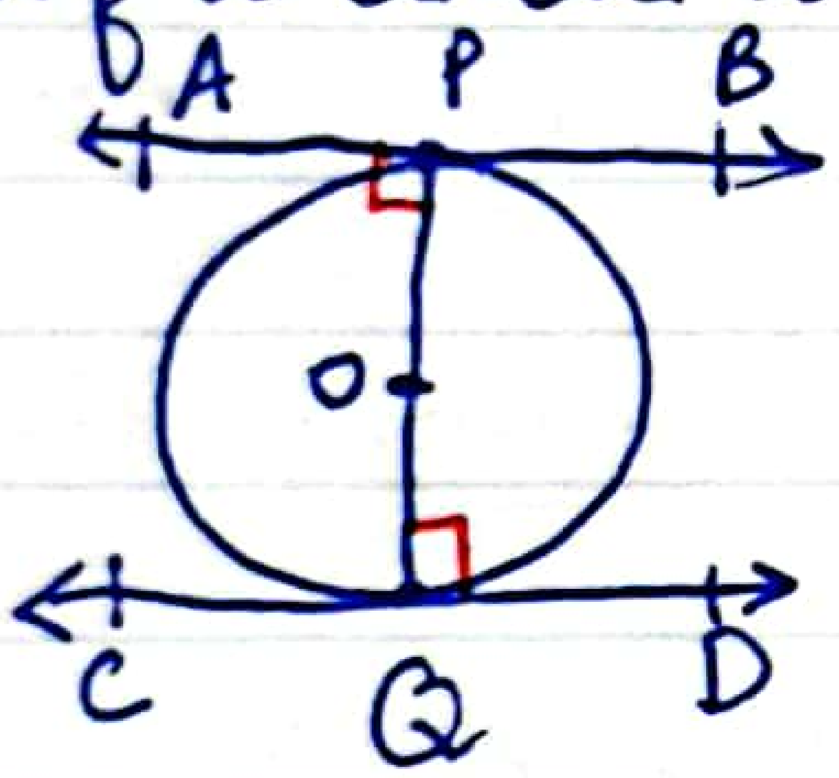
Also, we know that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

Thus, $AC = BC$.

Hence, AB is bisected at C.

16) Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

ans:-



Given: AB and CD are the two tangents at P and Q.

To prove: $AB \parallel CD$

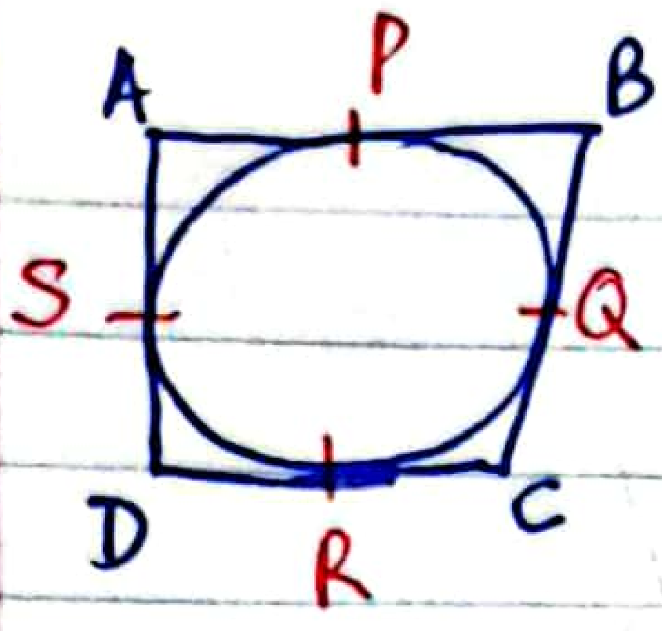
Proof:- Since the tangent at any point of a circle is perpendicular to the radius through the point of contact, $AB \perp OP$

and $CD \perp OQ$

Thus, $\angle APO = \angle OQD$.

These angles form a pair of alternate interior angles only when $AB \parallel CD$!

Hence, tangents drawn at the ends of a diameter of a circle are parallel.

17) 

A quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB+CD = AD+BC$. Hence find AD, if $AB = 6cm$, $BC = 7cm$ and $CD = 4cm$.

Given: a quadrilateral ABCD touches a circle at P, Q, R and S.

To prove: $AB+CD = AD+BC$

Proof :- We know that the tangents drawn from an external point to a circle are equal in lengths.

$AP = AS$ [$\because$ A is the external point] $\rightarrow (1)$

$BP = BQ$ [$\because$ B is the external point] $\rightarrow (2)$

$DR = DS$ [$\because$ D is the external point] $\rightarrow (3)$

$CR = CQ$ [$\because$ C is the external point] $\rightarrow (4)$

$(1)+(2)+(3)+(4)$, $(AP+BP)+(DR+CR) = (AS+DS)+(BQ+CQ)$

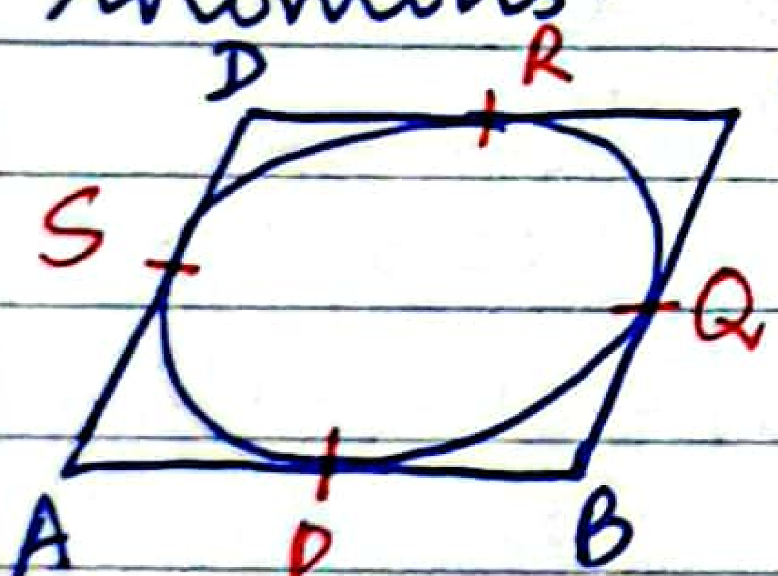
$\Rightarrow AB+CD = AD+BC$. Hence Proved.

* To find AD: $AB+CD = AD+BC$

$\Rightarrow 6+4 = AD+7$

$\Rightarrow AD = 10-7 = \underline{3cm}$

18) Prove that the parallelogram circumscribing a circle is a rhombus.



Given: in ||gm ABCD, the sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively.

To prove : ABCD is a rhombus

Proof :- We know that the tangents drawn from an external point to a circle are equal in lengths,

$AP = AS$ [$\because$ A is the external point] $\rightarrow (1)$

$BP = BQ$ [$\because$ B is the external point] $\rightarrow (2)$

$DR = DS$ [$\because$ D is the external point] $\rightarrow (3)$

$CR = CQ$ [$\because$ C is the external point] $\rightarrow (4)$

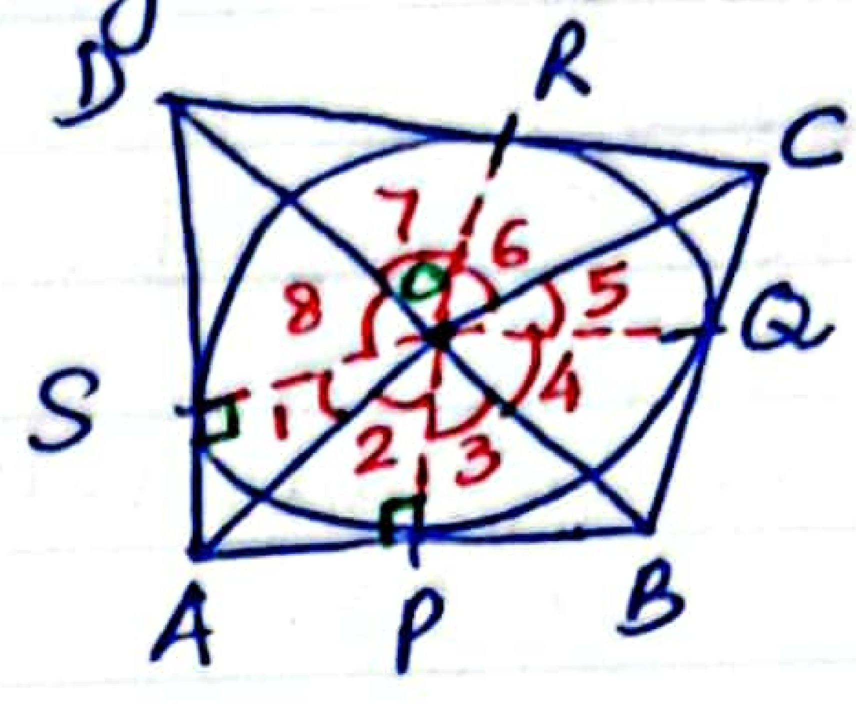
$\therefore (AP+BP)+(DR+CR) = (AS+DS)+(BQ+CQ)$

$\Rightarrow AB+CD = AD+BC$

$\Rightarrow 2AB = 2AD \Rightarrow AB = AD$

Thus parallelogram ABCD is a rhombus with adjacent sides equal. Hence proved.

19) Prove that opposite sides of a quadrilateral circumscribing a circle subtends supplementary angles at the centre of the circle.



ans:- Given:- in quadrilateral ABCD, Sides AB, BC, CD and DA touch the circle at P, Q, R and S respectively.

To prove : $\angle AOB + \angle COD = 180°$

and $\angle BOC + \angle AOD = 180°$

Construction: Join OP, OQ, OR and OS

Proof:- Since radius is perpendicular to the tangent through the point of contact, $OP \perp AB$, $OQ \perp BC$, $OR \perp CD$ and $OS \perp AD$.

In $\triangle AOS$ and $\triangle AOP$, $OS = OP$ (radii of the same circle)

$OA = OA$ (common side)

$\angle OSA = \angle OPA$ (each 90°)

$\therefore \triangle AOS \cong \triangle AOP$ (RHS Congruency)

Thus $\angle 1 = \angle 2$ (by CPCT) $\rightarrow$ (1)

Similarly, we can prove $\triangle BOP \cong \triangle BOQ$

$\Rightarrow \angle 3 = \angle 4 \rightarrow (2)$

$\triangle COQ \cong \triangle COR$

$\Rightarrow \angle 5 = \angle 6 \rightarrow (3)$

$\triangle DOR \cong \triangle DOS$

$\Rightarrow \angle 7 = \angle 8 \rightarrow (4)$

But, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$ (angles around a point)

$\Rightarrow 2\angle 2 + 2\angle 3 + 2\angle 6 + 2\angle 7 = 360°$ [from eq: (1), (2), (3) and (4)]

$\Rightarrow 2(\angle 2 + \angle 3) + 2(\angle 6 + \angle 7) = 360°$

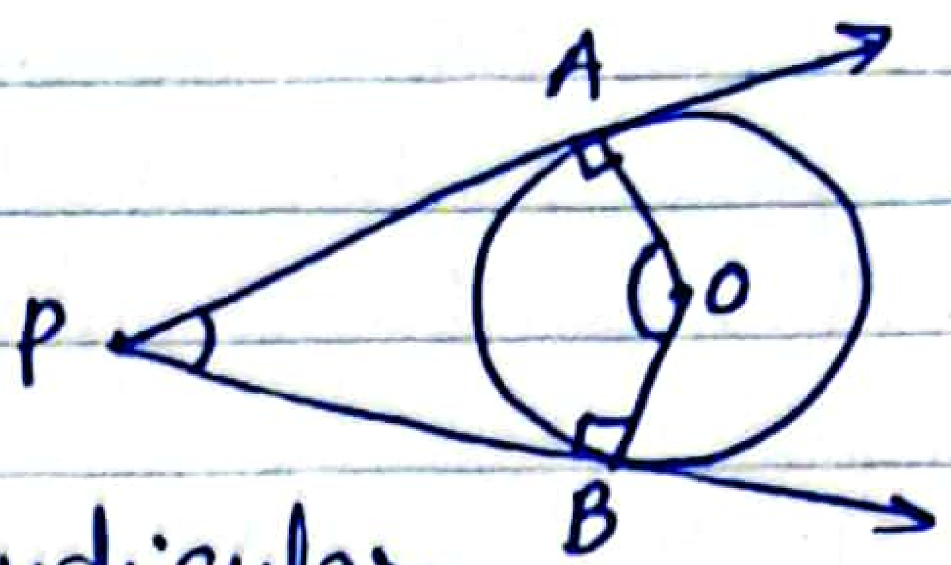
$\Rightarrow (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180°$

$\Rightarrow \angle AOB + \angle COD = 180°$

Similarly, $\angle BOC + \angle AOD = 180°$. Hence Proved.

20) Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

ans:- Given: a circle with centre O.
PA and PB are tangents
from external point P.

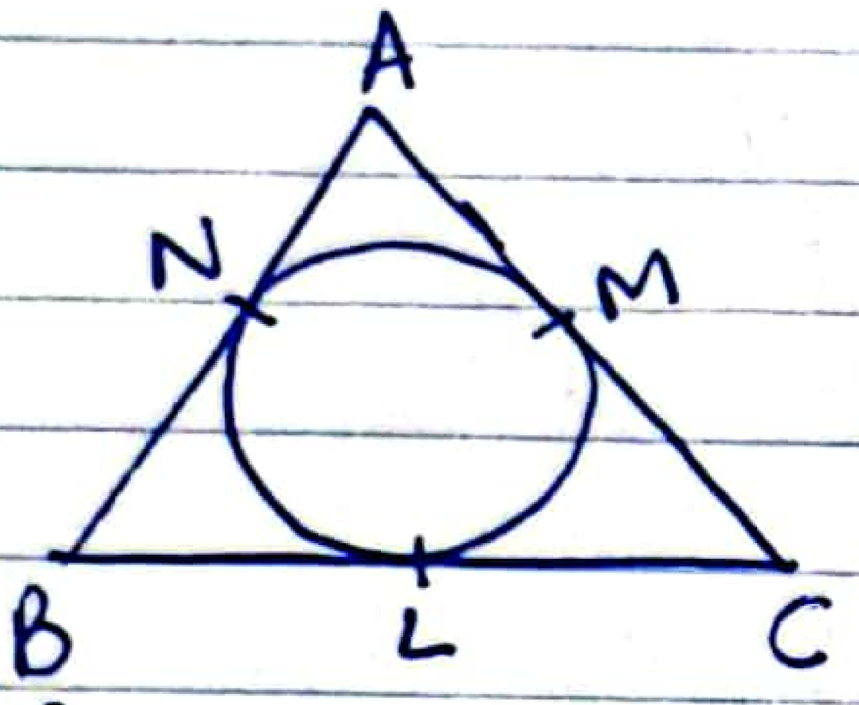


To prove: $\angle APB + \angle AOB = 180^\circ$

Proof:- Since radius is perpendicular to the tangent through the point of contact,
$\angle OAP = \angle OBP = 90^\circ$

In quadrilateral OAPB, $\angle OAP + \angle APB + \angle PBO + \angle AOB = 360^\circ$
$\Rightarrow 90^\circ + \angle APB + 90^\circ + \angle AOB = 360^\circ$
$\therefore \angle APB + \angle AOB = 360^\circ - 180^\circ = 180^\circ$

Hence Proved.



21) If $\triangle ABC$ is isosceles with AB = AC and C(O, r) is the incircle of the $\triangle ABC$ touching BC at L. Prove that L bisects BC.

ans:- Given:- In isosceles $\triangle ABC$, AB = AC. Circle touches $\triangle ABC$ at L, M and N.

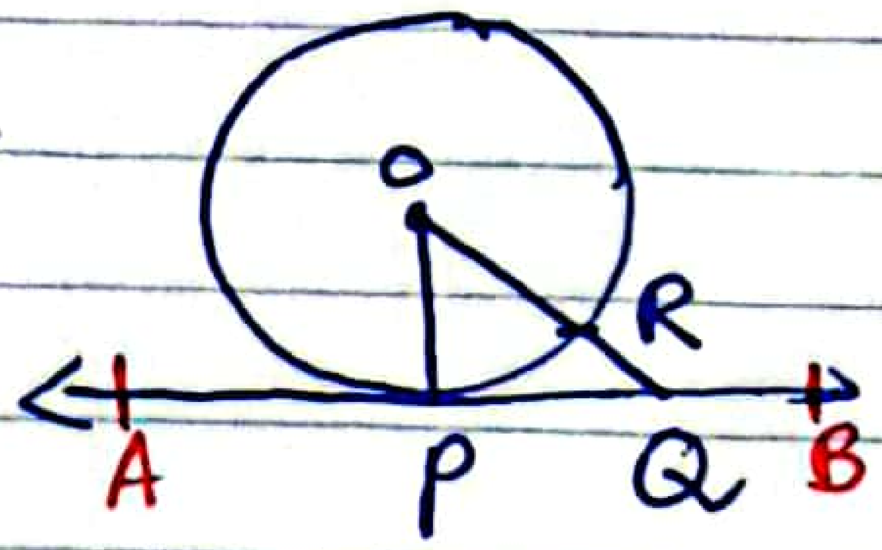
To prove :- L bisects BC.

Proof:- Since tangents drawn from an external point are equal in lengths, BN = BL [$\because$ B is the external pt.] $\rightarrow$ (1)
AN = AM [$\because$ A is the external pt.] $\rightarrow$ (2)
CL = CM [$\because$ C is the external pt.] $\rightarrow$ (3)

Given, AB = AC
$\Rightarrow$ AB − AN = AC − AM [$\because$ AN = AM]
$\Rightarrow$ BN = CM
$\Rightarrow$ BL = CL [from eq: (1) and (3)]
$\Rightarrow$ L bisects BC. Hence Proved.

22) Prove that the tangents at any point of a circle is perpendicular to the radius through the point of contact.

ans:- Given:- a circle with centre O and radius r. AB is a tangent at P.



To prove: $OP \perp AB$

Construction: take a point Q on AB, other than the point of contact. ~~who~~ Join OQ to meet the circle at R.

Proof:- OP = OR (radii of the same circle)
$OQ = OR + RQ \Rightarrow OQ > OR$
$\Rightarrow OQ > OP$ [$\because OP = OR$]
$\therefore OP < OQ$

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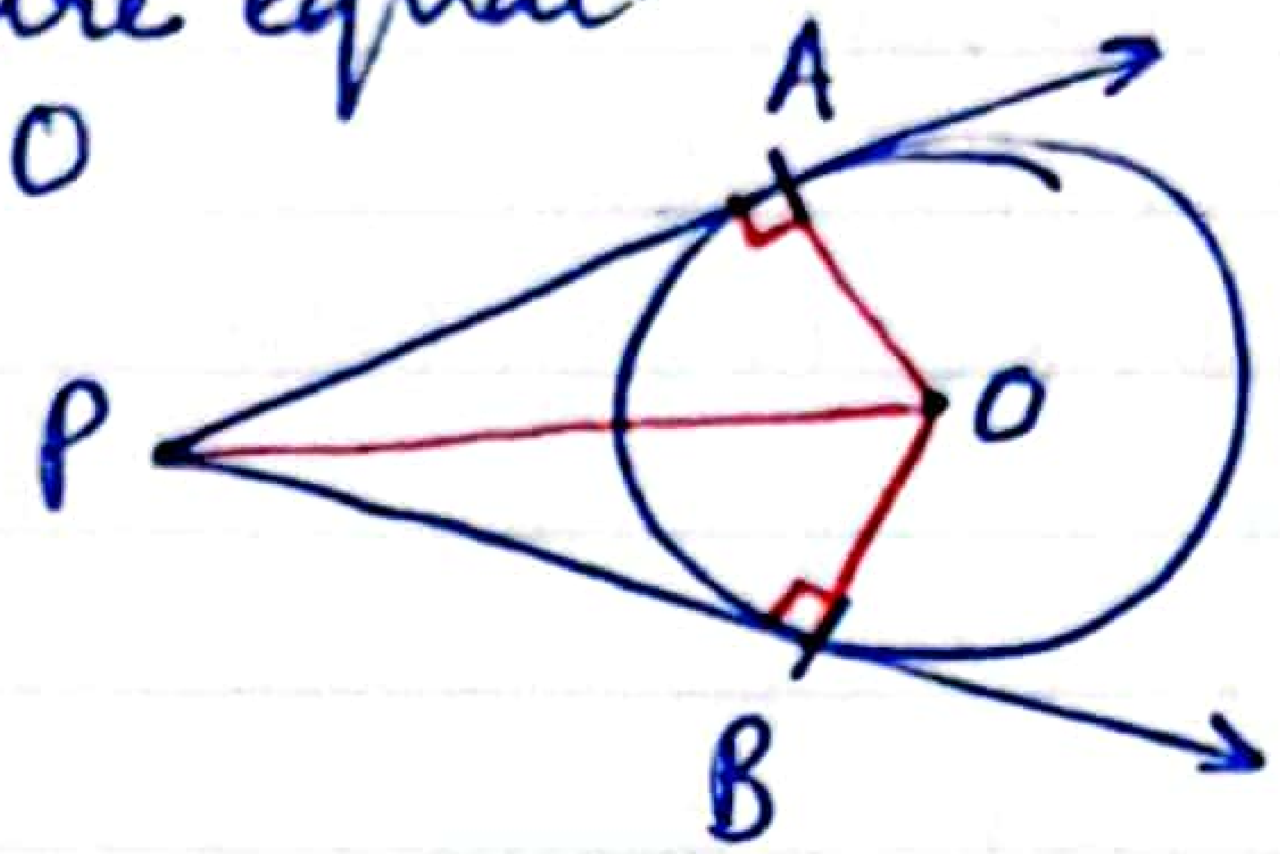
Since the shortest distance between a point and a line is the perpendicular distance, OP is the shortest distance between the centre and the point of contact and $OP \perp AB$.
Hence Proved.

23) Prove that the lengths of tangents drawn from an external point to a circle are equal.

ans:- Given: a circle with centre O and radius r.
PA and PB are the tangents from the external point P.



To prove: $PA = PB$

Proof :- In $\triangle OAP$ and $\triangle OBP$, $OA = OB$ (radii of the same circle)

$\angle OAP = \angle OBP$ (each $90°$)

$OP = OP$ (common side)

$\therefore \triangle OAP \cong \triangle OBP$ (RHS congruency)

Thus $PA = PB$ (by CPCT)

Hence Proved.
