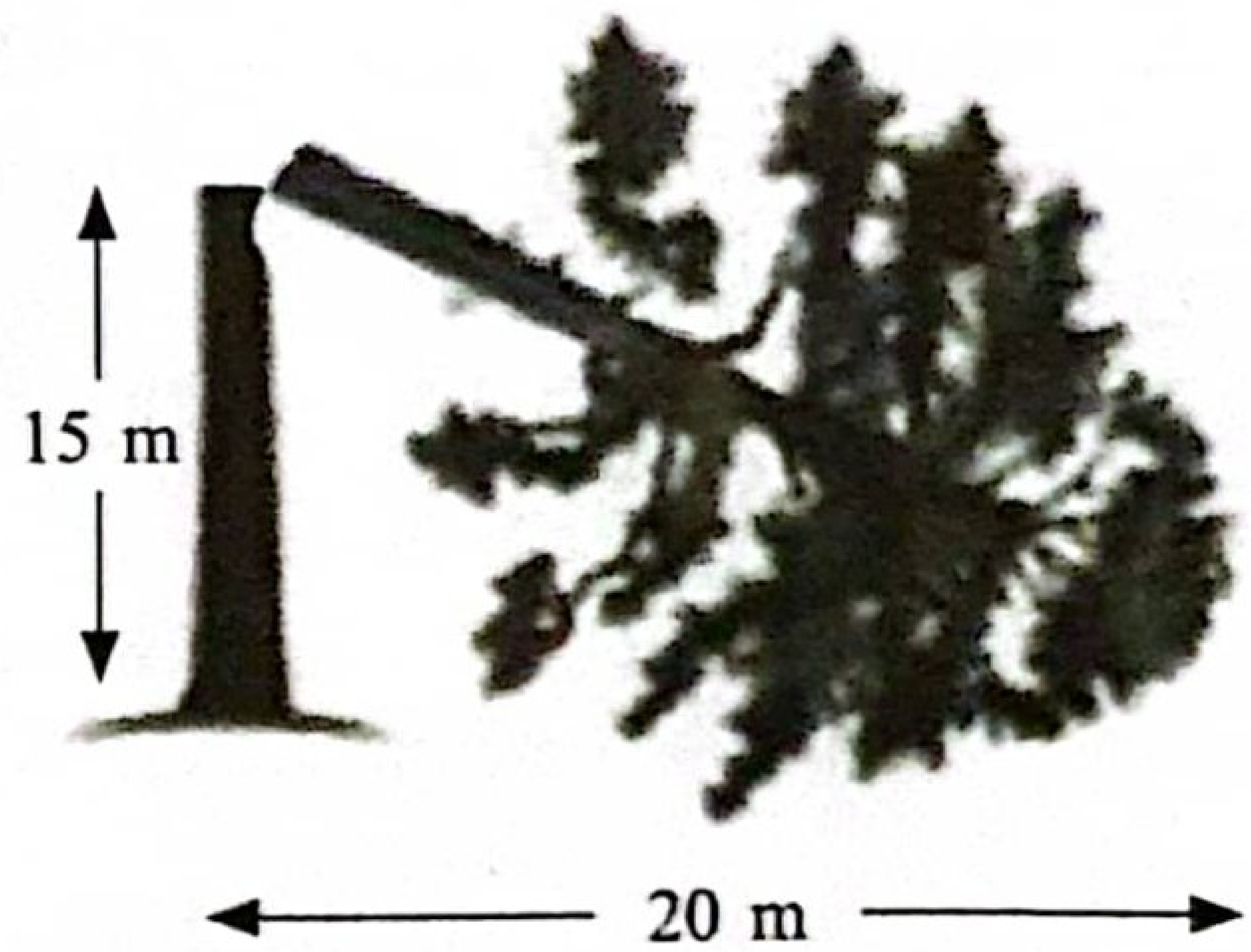
X | Test – 19th/23rd December [Draw figure for each]

1) If the altitude of the Sun is 60°, then find the height of the vertical tower that will cast a shadow of length 30m.
2) A tower is $100\sqrt{3}$ m high. find the angle of evation of its top from a point 100m away from its foot.
3) Find the angular elevation of the sun when the shadow of a 10m long pole is $10\sqrt{3}$ m
4) The angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of the tower is 30°. find the height of the tower.
5) The tops of two poles of height 20m and 14m are connected by a wire. If the wire makes an angle of 30° with horizontal, then find the length of the wire.
6) When we raise our head to look at the object, the angle formed by the line of sight with horizontal is known as angle of depression. Is it true?
7) When we lower our head to look at the object, the angle formed by the line of sight with horizontal is known as angle of elevation? Is it false?
8) When the length of the shadow of a pillar is equal to its height, find the elevation of source of sight.
9) A pole of 10m high, casts a shadow 10m long on the ground, then find the sun's elevation.
10) A ladder is 10m long. If it touches a wall of height of 5m, find the angle θ made by it with the horizontal.
11) A tower stands vertically on the ground. From a point on the ground, which is 15m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower.
12) An observer 1.5m tall is 28.5m away from chimney. The angle of elevation of the top of the chimney from her eyes is 45°. What is the height of the chimney?
13) The shadow of a tower standing on a level ground is found to be 40m longer when the sun's altitude is 30° than when it is 60°. Find the height of the tower

(Question no. 14 is a Case Study Based Question. It contains 5 subparts. Each subpart carries 1 mark.)

14. Mr. Jhadav is having a garden near Shimla. In the garden, there are different types of trees and flower plants. One day due to heavy rain and storm, one of the trees got broken as shown in the figure. The height of the unbroken part is 15 m and the broken part of the tree has fallen at 20 m away from the base of the tree.



Answer the following questions:

(i) What is the height of the broken part?

(a) 15 m (b) 20 m

(c) 25 m (d) 30 m

Scanned with CamScanner

(ii) What was the height of full tree?

(a) 40 m (b) 50 m (c) 35 m (d) 30 m

(iii) The length of the hypotenuse of the right triangle formed in the above situation is :

(a) 15 m (b) 20 m (c) 25 m (d) 30 m

(iv) Area of right angled triangle formed in the above situation is :

(a) 100 m^2 (b) 200 m^2 (c) 60 m^2 (d) 150 m^2

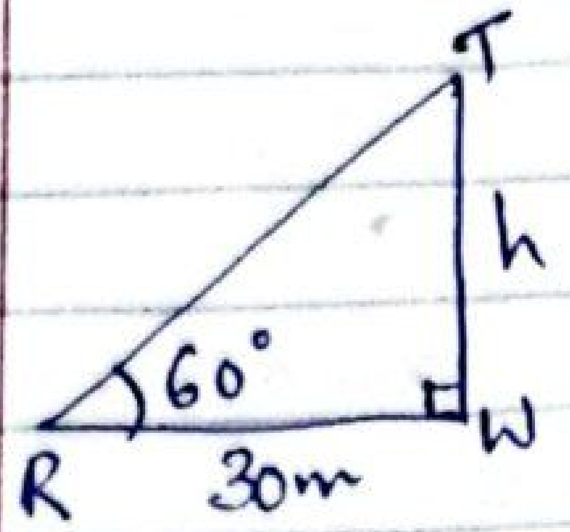
(v) The perimeter of the triangle so formed is :

(a) 60 m (b) 50 m (c) 45 m (d) 100 m

Scanned with CamScanner

X

1)



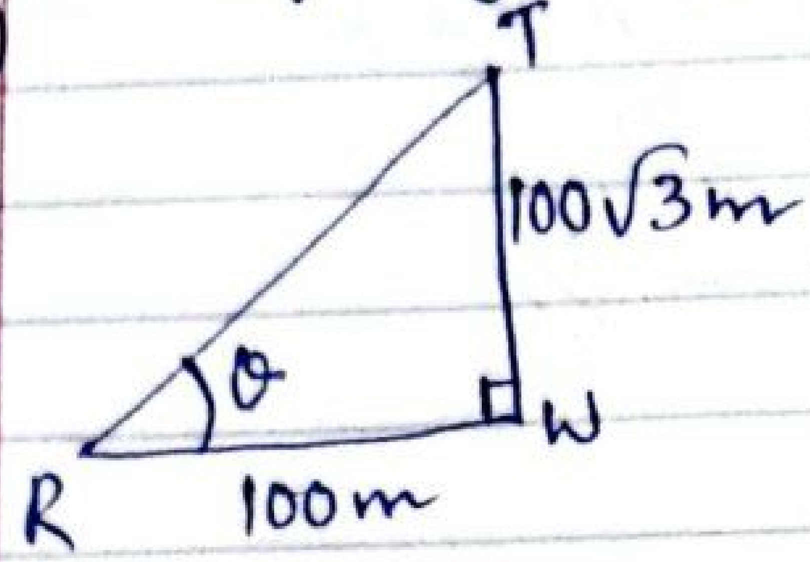
Let TW be the height of the tower

$\tan 60° = \sqrt{3} = \frac{h}{30}$

$\Rightarrow h = 30\sqrt{3}$

Hence, height of the vertical tower $= 30\sqrt{3}m$

2)

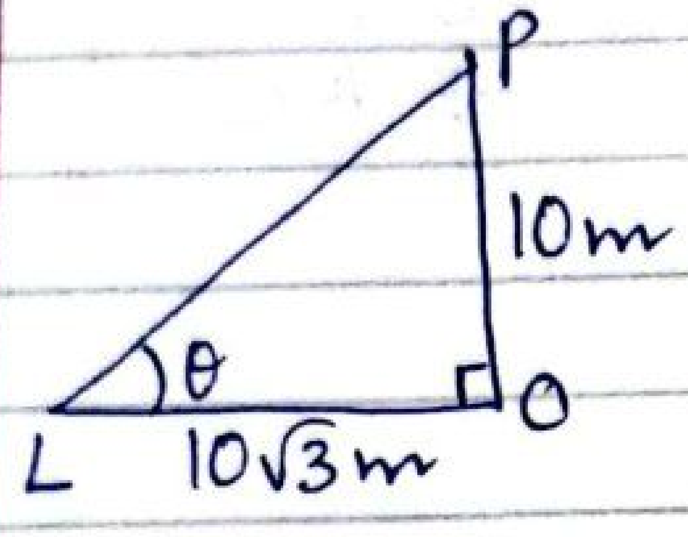


Let θ be the angle of elevation of its top from R.

$\tan\theta = \frac{TW}{RW} = \frac{100\sqrt{3}}{100}$

$\therefore \tan\theta = \sqrt{3} \Rightarrow \underline{\underline{\theta = 60°}}$

3)

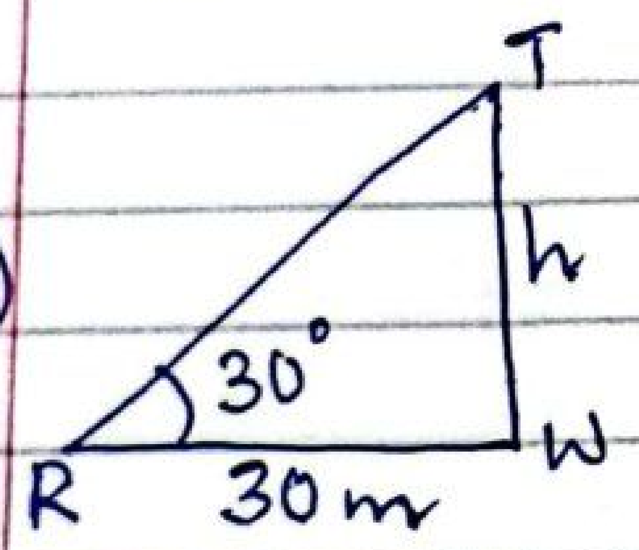


Let θ be the angular elevation of the Sun.

$\tan\theta = \frac{OP}{OL} = \frac{10}{10\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \underline{\underline{\theta = 30°}}$

4)



Let TW be the height of the tower.

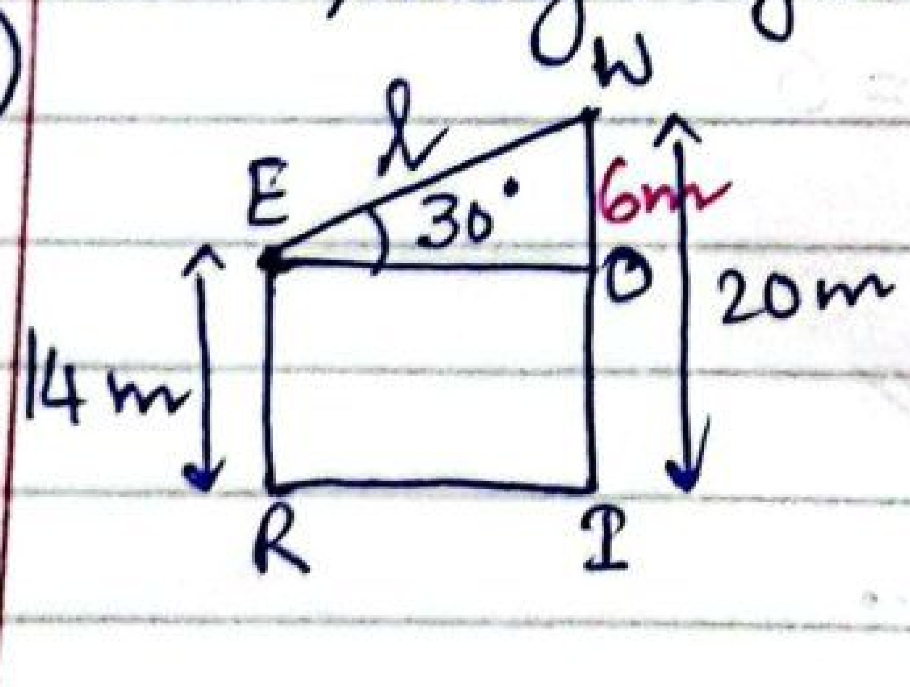
$\tan 30° = \frac{TW}{RW} = \frac{h}{30}$

$\frac{1}{\sqrt{3}} = \frac{h}{30} \Rightarrow h = \frac{30}{\sqrt{3}} = \frac{30\sqrt{3}}{3}$

$= \underline{\underline{10\sqrt{3}m}}$

Hence, height of the tower $= 10\sqrt{3}m$ //

5)



Let EW be the length of the wire.

$\sin 30° = \frac{OW}{EW} = \frac{6}{l}$

$\Rightarrow \frac{1}{2} = \frac{6}{l}$

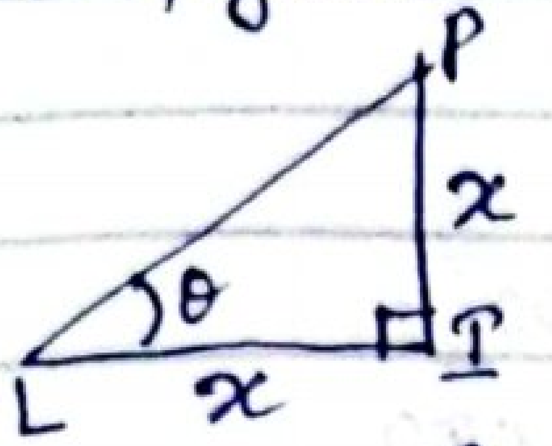
$\Rightarrow l = 12m$

Hence, length of the wire = 12m

6) False, it is angle of elevation.

7) Yes, false. It is angle of depression.

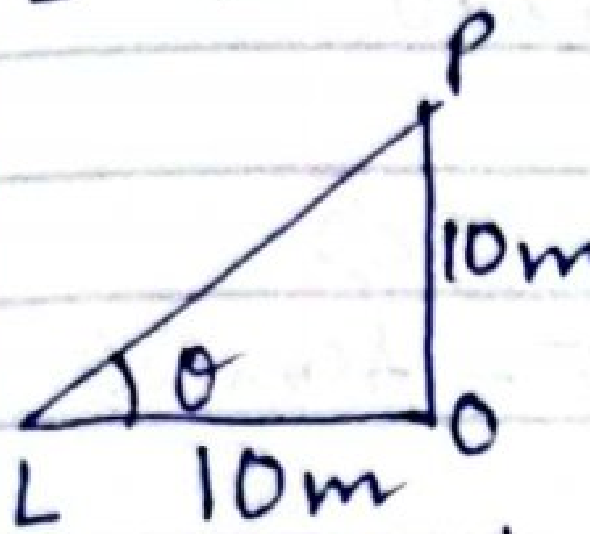
8)



$\tan\theta = \frac{PT}{LT} = \frac{x}{x} = 1$

$\therefore \theta = 45^\circ$. Hence, elevation of source of light $= 45^\circ$

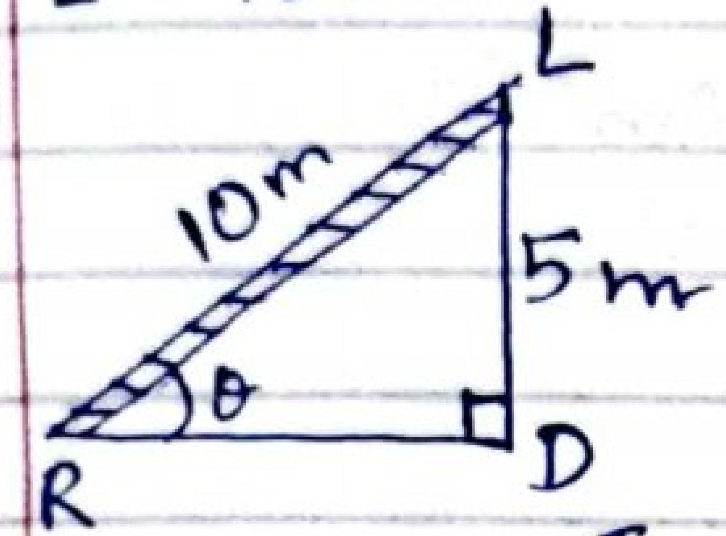
9)



$\tan\theta = \frac{OP}{OL} = \frac{10}{10} = 1$

$\therefore \theta = 45^\circ$. Hence, Sun's elevation $= 45^\circ$

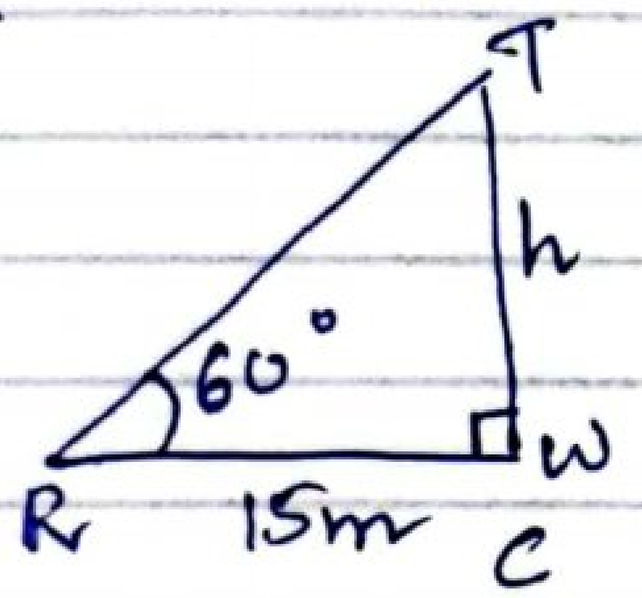
10)



$\sin\theta = \frac{LD}{DR} = \frac{5}{10} = \frac{1}{2}$

$\therefore \theta = 30^\circ$

11)

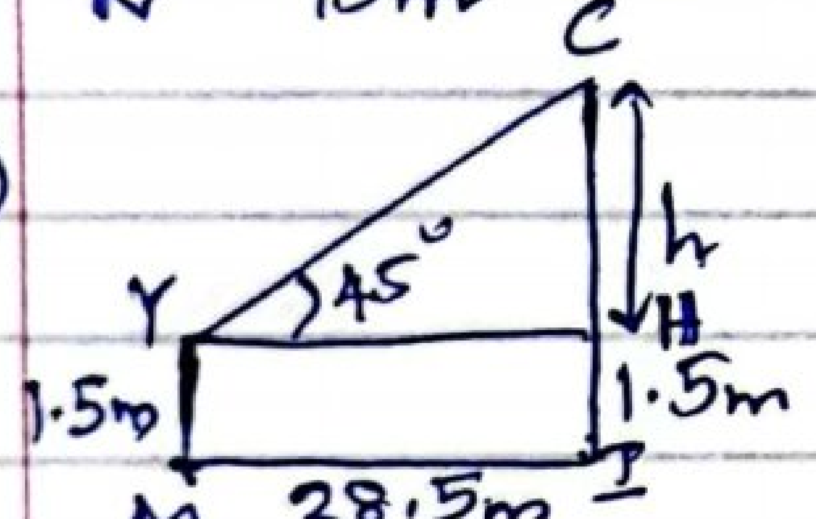


$\tan 60^\circ = \sqrt{3} = \frac{TW}{RW} = \frac{h}{15}$

$\therefore h = 15\sqrt{3}$ m

Hence, height of the tower $= 15\sqrt{3}$m

12)

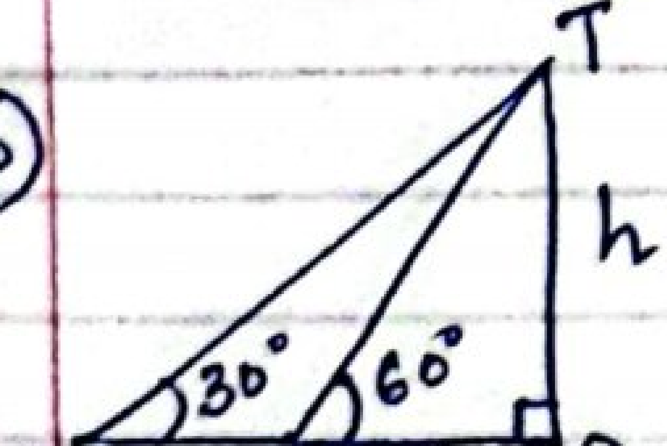


$\tan 45^\circ = \frac{CH}{YH} = \frac{h}{28.5}$

$1 = \frac{h}{28.5} \Rightarrow h = 28.5m$

Hence, height of the chimney $= h + 1.5 = 28.5 + 1.5 = \underline{30m}$

13)



$\tan 60^\circ = \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$

$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{h}{x+40} \Rightarrow x + 40 = h\sqrt{3}$

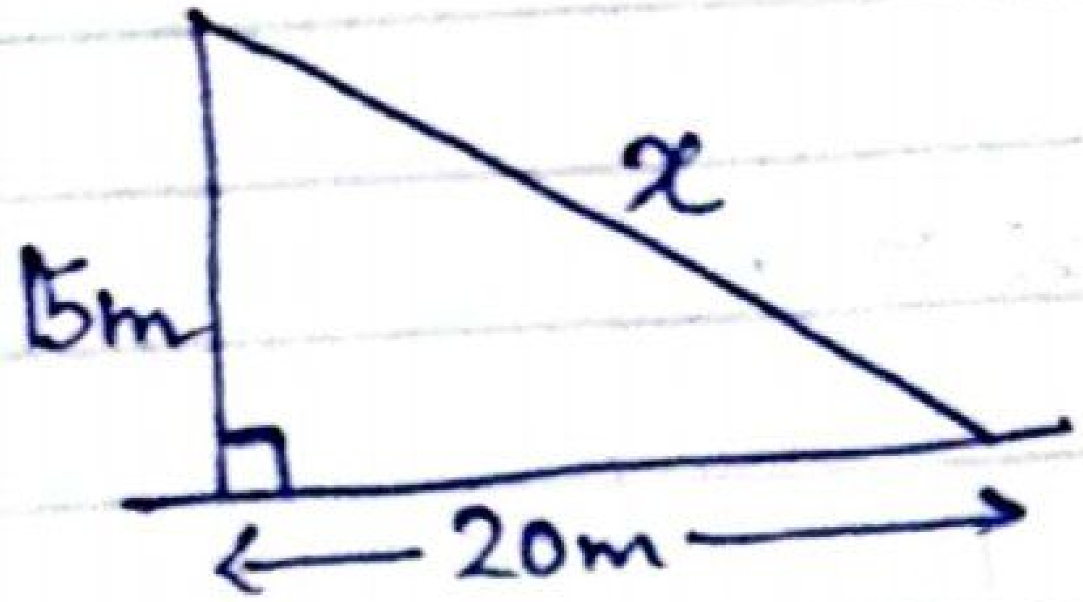
$\Rightarrow \frac{h}{\sqrt{3}} + 40 = h\sqrt{3} \Rightarrow h + 40\sqrt{3} = 3h$

$\Rightarrow 2h = 40\sqrt{3}$

$\Rightarrow h = 20\sqrt{3}m$

Hence, height of the tower $= 20\sqrt{3}$m

Scanned with CamScanner

14) 

(i) using Pythagoras Theorem, $x^2 = 15^2 + 20^2$

$= 225 + 400$

$= 625$

$x = 25m$ (c)

(ii) Height of full tree $= x + 15 = 25 + 15 = 40m$ (a)

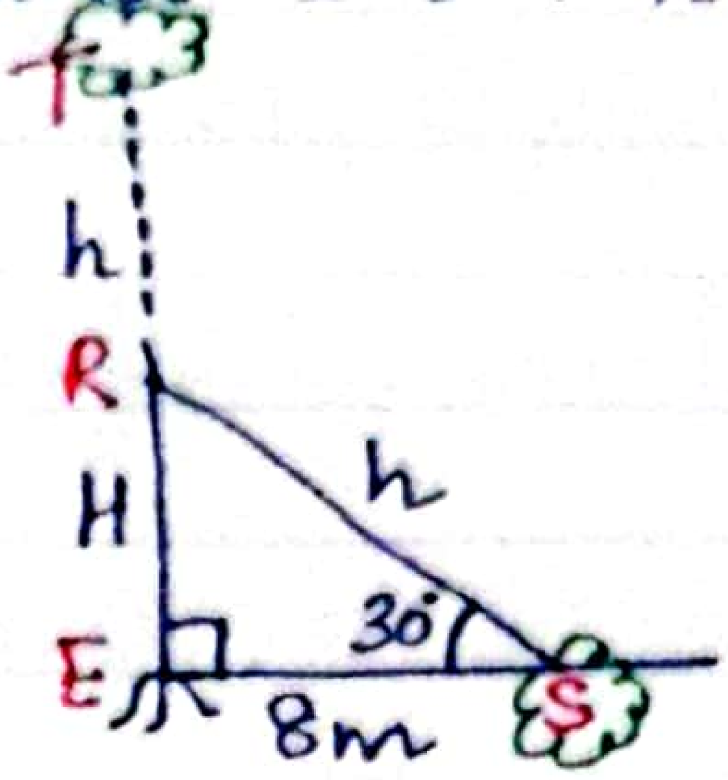
(iii) 25m (c)

(iv) area $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 20 \times 15 = 150m^2$ (d)

(v) Perimeter $= 15 + 20 + 25 = 60m$ (a)

PART – B

15) A tree breaks due to storm and broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8m. Find the height of the tree.



ans:- Let TE be the height of the tree.

In rt. ΔRES, $\tan 30^\circ = \frac{RE}{ES}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{H}{8}$$

$$\therefore H = \frac{8}{\sqrt{3}} m$$

Also, $\cos 30^\circ = \frac{ES}{RS}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{h}$$

$$\therefore h = \frac{16}{\sqrt{3}} m$$

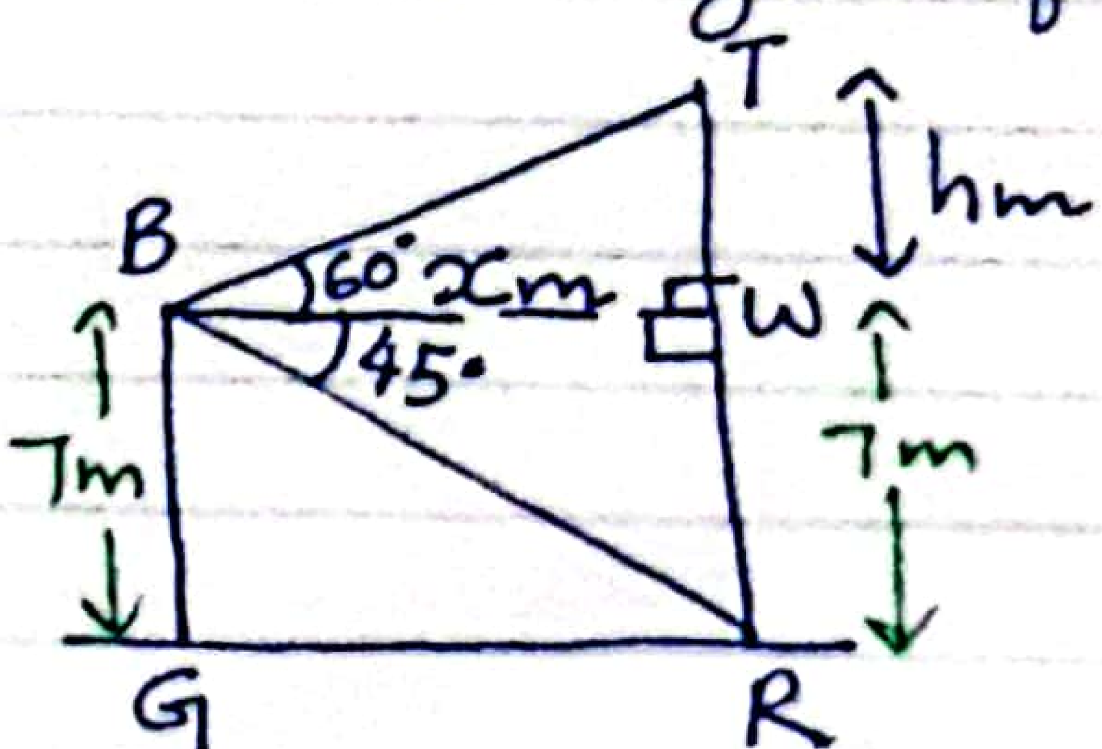
Hence, the height of the tree $= h + H$

$$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}}$$

$$= \frac{24}{\sqrt{3}} = \frac{24\sqrt{3}}{3}$$

$$= 8\sqrt{3} m$$

16) From the top of a 7m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.



ans:- Let TR be the height of the tower.

In rt. ΔTWB, $\tan 60^\circ = \frac{TW}{BW}$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} \longrightarrow (1)$$

In rt. ΔBWR, $\tan 45^\circ = \frac{WR}{BW}$

$$\Rightarrow 1 = \frac{7}{x}$$

$$\Rightarrow x = 7 \longrightarrow (2)$$

From (1) and (2),

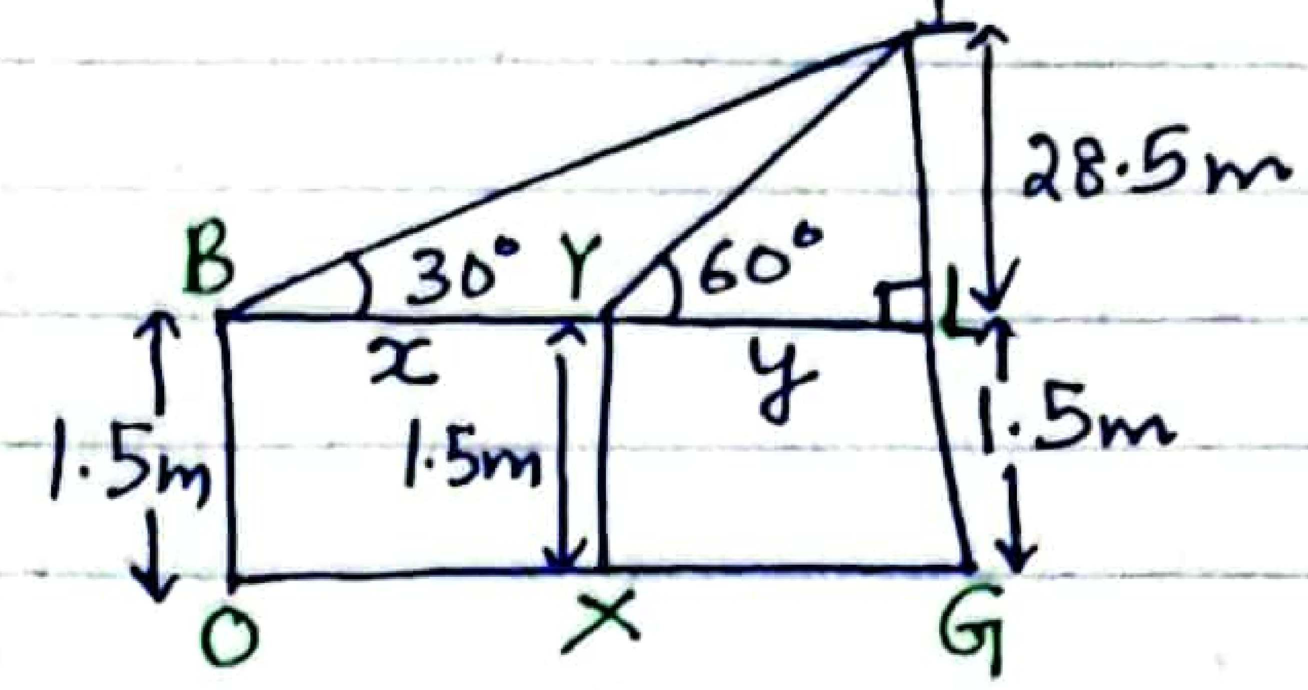
$$\frac{h}{\sqrt{3}} = 7$$

$$\therefore h = 7\sqrt{3}$$

Hence the height of the tower $= h + 7$

$= 7\sqrt{3} + 7 = 7(\sqrt{3}+1) m$

17) A 1.5m tall boy is standing at some distance from a 30m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60°, as he walks towards the building. Find the distance he walked towards the building.



ans:- Let IG be the height of the building.

In rt. $\triangle ILB$, $\tan 30° = \frac{28.5}{x+y}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{x+y}$$

$$\Rightarrow x+y = 28.5\sqrt{3}$$

$$\Rightarrow y = 28.5\sqrt{3} - x \rightarrow (1)$$

In rt. $\triangle ILY$, $\tan 60° = \frac{28.5}{y}$

$$\Rightarrow \sqrt{3} = \frac{28.5}{y}$$

$$\Rightarrow y = \frac{28.5}{\sqrt{3}} \rightarrow (2)$$

From (1) and (2), $28.5\sqrt{3} - x = \frac{28.5}{\sqrt{3}}$

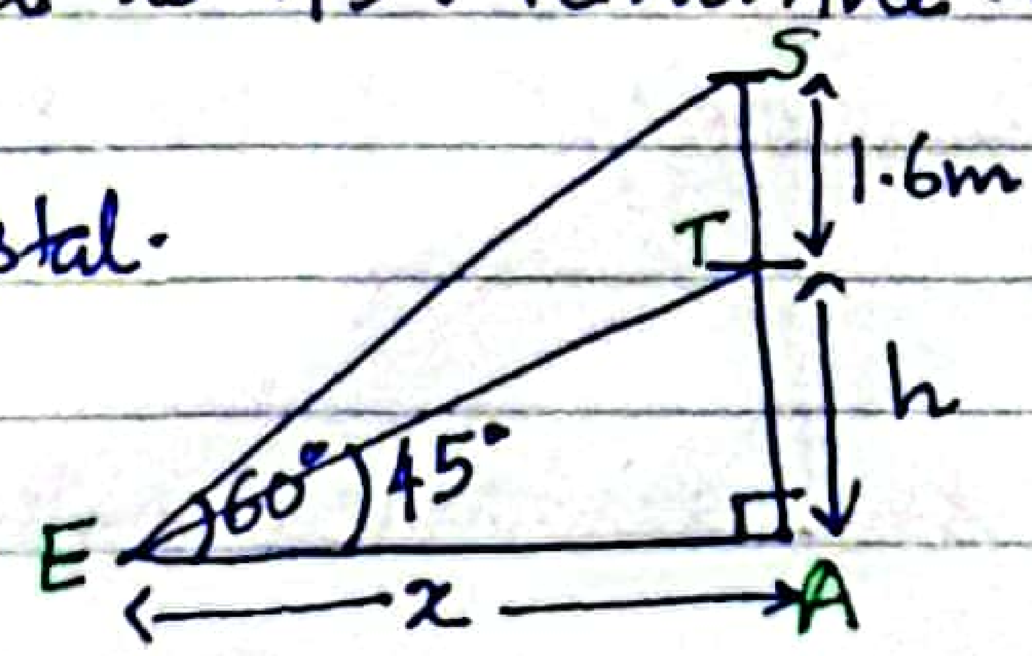
$$85.5 - x\sqrt{3} = 28.5$$

$$x\sqrt{3} = 85.5 - 28.5 = 57$$

$$\therefore x = \frac{57}{\sqrt{3}} = \frac{57\sqrt{3}}{3} = \underline{\underline{19\sqrt{3}m}}$$

Hence, the distance he walked towards the building $= \underline{\underline{19\sqrt{3}m}}$

18) A Statue 1.6m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.



ans:- Let TA be the height of the pedestal.

In rt. $\triangle TEA$, $\tan 45° = \frac{TA}{EA} \Rightarrow 1 = \frac{h}{x}$

$$\therefore x = h \rightarrow (1)$$

In rt. $\triangle SEA$, $\tan 60° = \sqrt{3} = \frac{SA}{EA} \Rightarrow \sqrt{3} = \frac{1.6+h}{x} \Rightarrow x = \frac{1.6+h}{\sqrt{3}} \rightarrow (2)$

From (1) and (2), $h = \frac{1.6+h}{\sqrt{3}}$

$\Rightarrow h\sqrt{3} = 1.6+h$

$\Rightarrow h\sqrt{3} - h = 1.6$

$\Rightarrow h(\sqrt{3}-1) = 1.6$

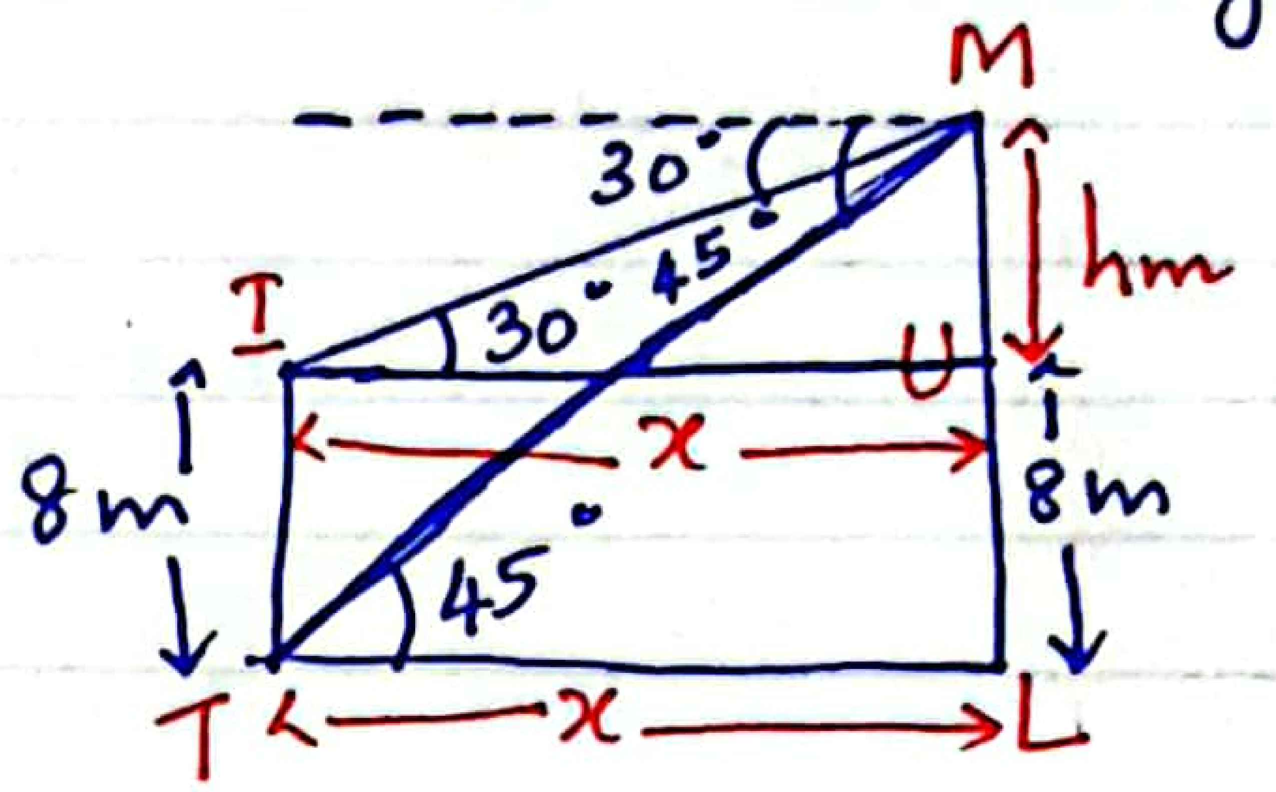
$\therefore h = \frac{1.6}{\sqrt{3}-1} = \frac{1.6(\sqrt{3}+1)}{3-1} = \frac{1.6(\sqrt{3}+1)}{2}$

$= \underline{0.8(\sqrt{3}+1)\,m}$

Hence, the height of the pedestal $= 0.8(\sqrt{3}+1)m$.

19) The angles of depression of the top and bottom of an 8m tall building from the top of a multi-storeyed building are 30° and 45° respectively. find the height of the multi-storeyed building and the distance between the two buildings.

ans:- Let ML be the height of the multi-storeyed building.



In rt. ΔMUI, $\tan 30° = \frac{MU}{IU}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow x = h\sqrt{3} \rightarrow (1)$

In rt. ΔMLT, $\tan 45° = \frac{ML}{TL}$

$\Rightarrow 1 = \frac{h+8}{x} \Rightarrow x = h+8 \rightarrow (2)$

From (1) and (2), $h\sqrt{3} = h+8$

$h(\sqrt{3}-1) = 8$

$h = \frac{8}{\sqrt{3}-1} = \frac{8(\sqrt{3}+1)}{3-1} = \frac{8(\sqrt{3}+1)}{2}$

$= 4(\sqrt{3}+1)m$

Then, $x = h\sqrt{3} = 4\sqrt{3}(\sqrt{3}+1) = 12+4\sqrt{3}$

$= 4(3+\sqrt{3})m$

Hence, height of the multi-storeyed building

$= h+8 = 4\sqrt{3}+4+8 = 4\sqrt{3}+12$

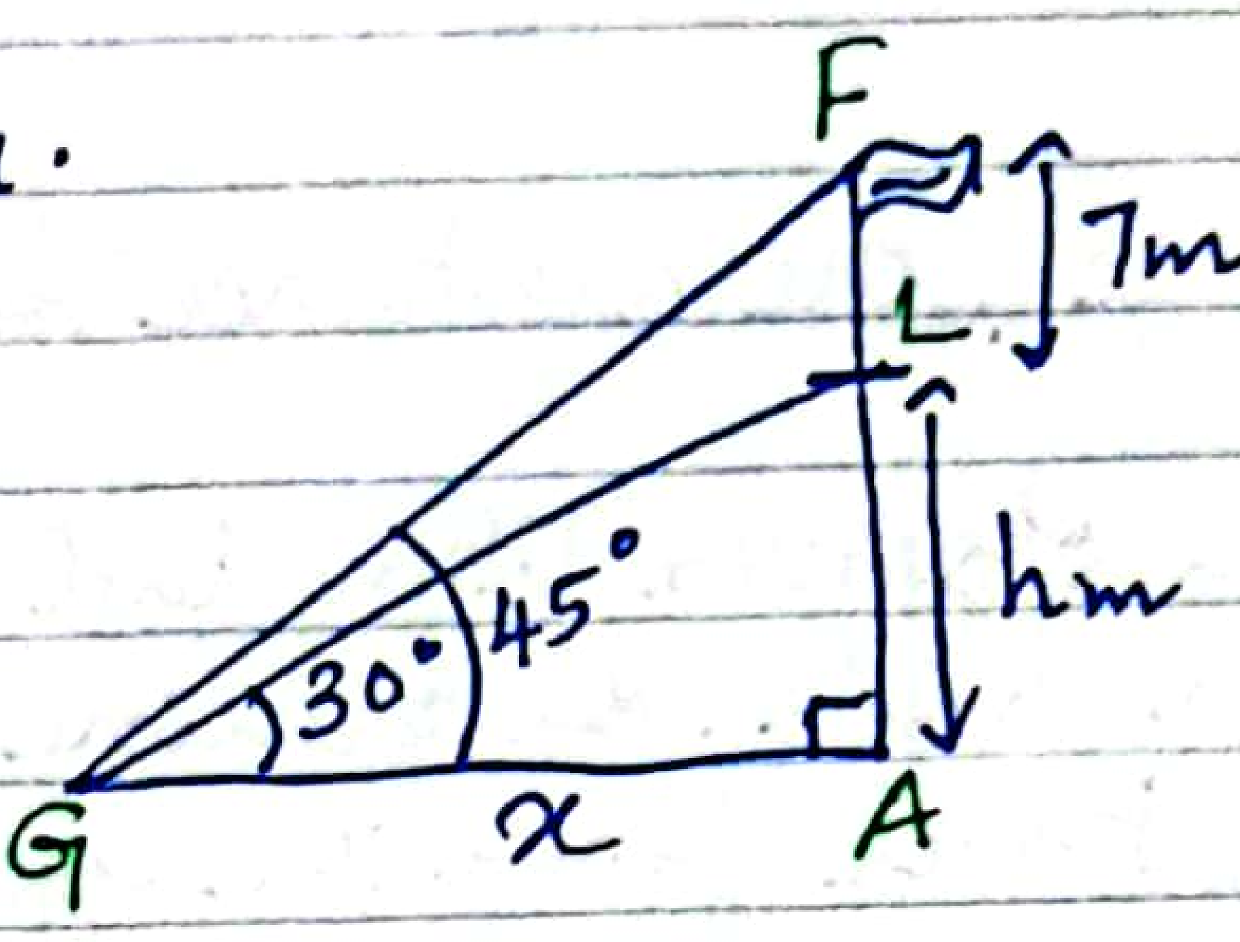
$= 4(\sqrt{3}+3)m$ //

and the distance between the two buildings

$= 4(3+\sqrt{3})m$ //

20) A vertical tower stands on a horizontal plane and is surmounted by a flagstaff of height 7m. From a point on the plane, the angle of elevation of the bottom of the flagstaff is 30° and that of the top of the flag-staff is 45°. Find the height of the tower.

Ans:- Let LA be the height of the tower.



In rt. ΔLAG, $\tan 30° = \dfrac{LA}{GA}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow x = h\sqrt{3} \rightarrow (1)$$

In rt. ΔFAG, $\tan 45° = \dfrac{FA}{GA}$

$$\Rightarrow 1 = \frac{7+h}{x} \Rightarrow x = 7+h \rightarrow (2)$$

From eq: (1) and (2), $h\sqrt{3} = 7+h$

$$h\sqrt{3} - h = 7$$

$$h(\sqrt{3}-1) = 7$$

$$\therefore h = \frac{7}{\sqrt{3}-1} = \frac{7(\sqrt{3}+1)}{3-1} = \frac{7(\sqrt{3}+1)}{2}$$

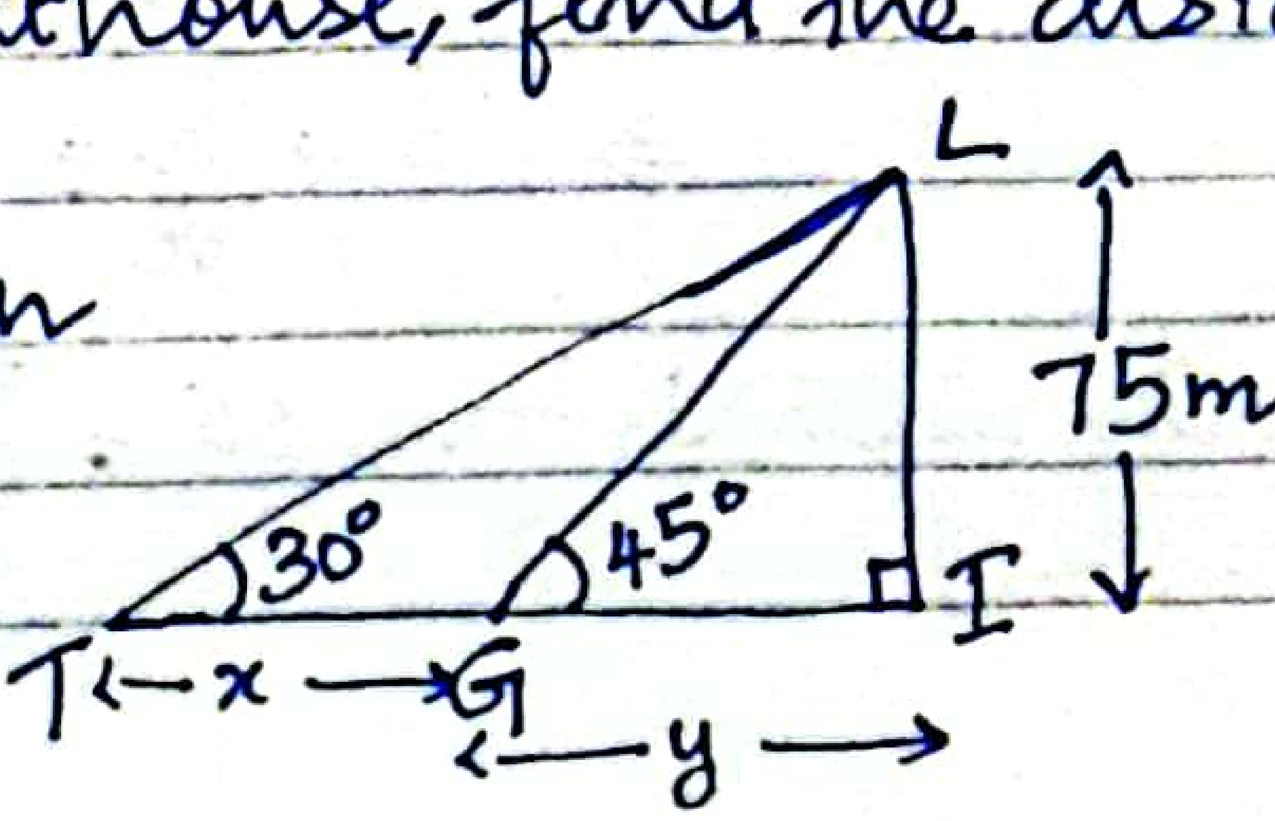
$$= 3.5(\sqrt{3}+1) = 3.5(1.732+1)$$

$$= 3.5 \times 2.732 = 9.56\,m$$

Hence, the height of the tower = 9.56m

21) As observed from the top of a 75m high lighthouse from the sea-level, the angle of depression of two ships are 30° and 45°. If one ship is exactly behind the other and are on the same side of the lighthouse, find the distance between two ships.

Ans:- Let TG be the distance between two ships.



In rt. ΔLIG, $\tan 45° = 1 = \dfrac{LI}{GI}$

$\Rightarrow 1 = \frac{75}{y}$

$\Rightarrow y = 75m$

In rt. ΔLIT, $\tan 30° = \frac{LI}{TI}$ $\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{x+y}$

$\Rightarrow x+y = 75\sqrt{3}$

$\Rightarrow x+75 = 75\sqrt{3}$

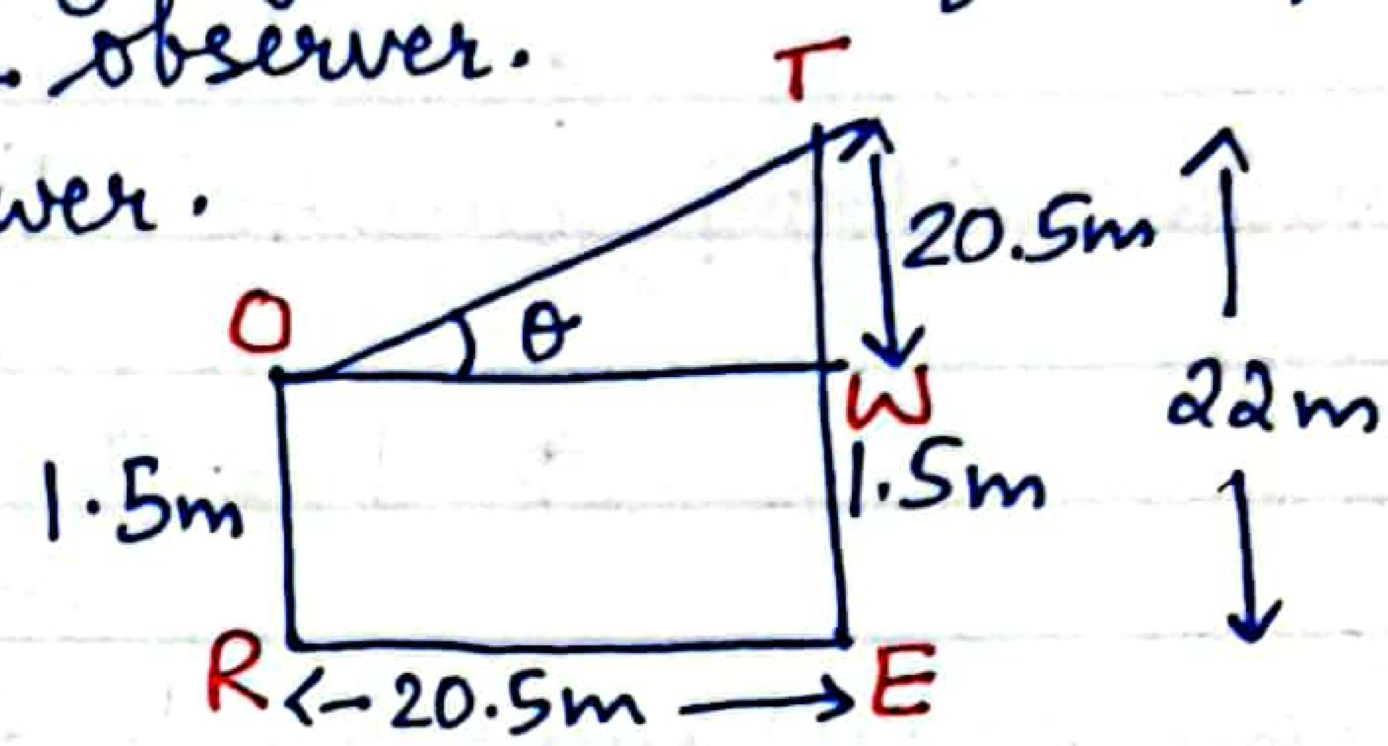
$\therefore x = 75\sqrt{3} - 75$

$x = 75(\sqrt{3}-1)m$

Hence, the distance between the two ships $= 75(\sqrt{3}-1)m$

22) An observer 1.5m tall is 20.5m away from a tower 22m high. Determine the angle of elevation of the top of the tower from the eye of the observer.

ans:- Let TE be the height of the tower.



$TW = TE - WE$

$= TE - OR = 22 - 1.5 = 20.5m$

In rt. ΔTWO, $\tan\theta = \frac{TW}{OW}$

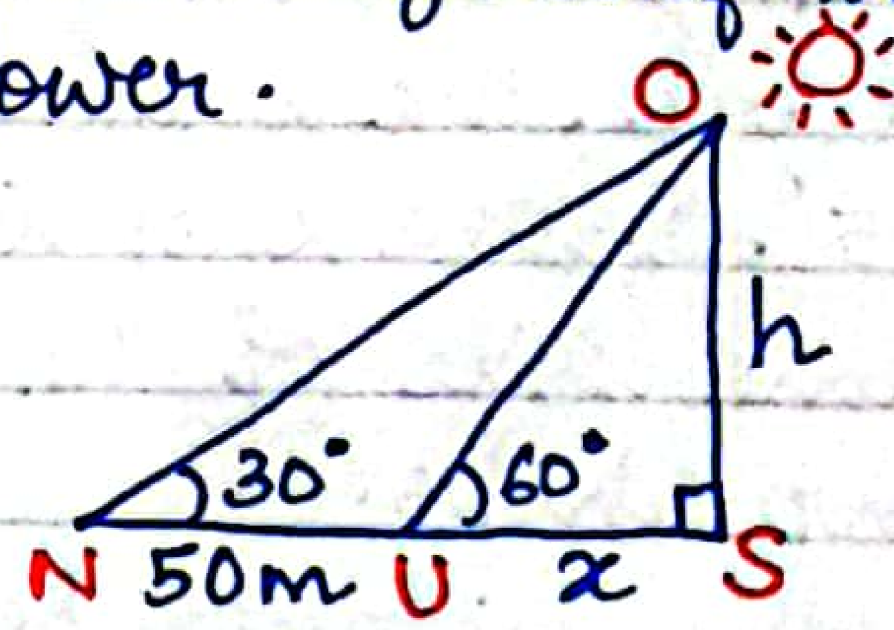
$\Rightarrow \tan\theta = \frac{\cancel{20.5}^{1}}{\cancel{20.5}_{1}} = 1$

$\therefore \theta = 45°$ $[\because \tan 45° = 1]$

Hence, the angle of elevation $= 45°$

23) The shadow of a tower standing on a level ground is found to be 50m longer when Sun's elevation is 30° than when it is 60°. Find the height of the tower.

ans:- Let OS be the height of the tower.



In rt. ΔOSU, $\tan 60° = \frac{OS}{US}$

$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$

In rt. ΔOSN, $\tan 30° = \frac{OS}{NS}$ $\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{50+x}$ $\Rightarrow 50+x = h\sqrt{3}$

$\Rightarrow 50 + \frac{h}{\sqrt{3}} = h\sqrt{3}$

$\Rightarrow 50\sqrt{3} + h = 3h$

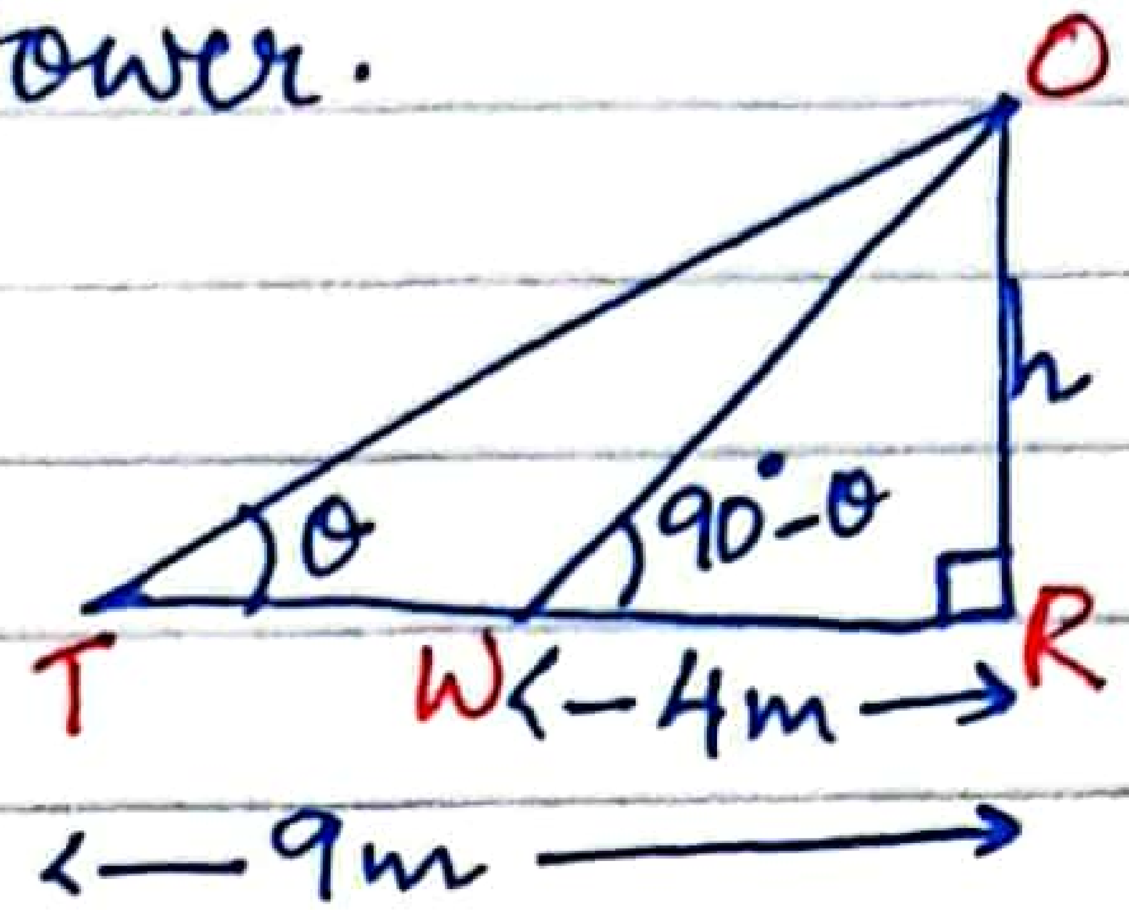
$\Rightarrow \quad 2h = 50\sqrt{3}$

$\therefore h = 25\sqrt{3}$ m

Hence, the height of the tower = $\underline{\underline{25\sqrt{3}\,m}}$

24) The angle of elevation of the top of a tower from two points at a distance of 4m and 9m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6m.

Ans:- Let OR be the height of the tower.



In rt. ΔORW, $\tan(90-\theta) = \frac{OR}{RW}$

$\Rightarrow \cot\theta = \frac{h}{4}$

$\Rightarrow \tan\theta = \frac{4}{h} \rightarrow (1)$

In rt. ΔORT, $\tan\theta = \frac{OR}{TR}$

$\Rightarrow \tan\theta = \frac{h}{9} \rightarrow (2)$

From (1) and (2),

$\frac{h}{9} = \frac{4}{h} \Rightarrow h^2 = 36$

$\therefore h = 6m$ //

Hence, height of the tower is 6m