

VIII Factorisation

1) Factorise the following :-

$$(a) a^2 - 2ab + b^2 - c^2$$
$$= (a^2 - 2ab + b^2) - c^2$$
$$= (a-b)^2 - c^2$$

$$x^2 - y^2 = (x+y)(x-y)$$
$$= (a-b+c)(a-b-c) //$$

$$(b) 64x^2 - 225$$
$$(8x)^2 - (15)^2$$
$$a^2 - b^2 = (a+b)(a-b)$$
$$= (8x+15)(8x-15) //$$

$$(c) \frac{p^2}{4} - \frac{q^2}{16}$$
$$a^2 - b^2 = (a+b)(a-b)$$
$$\left(\frac{p}{2}\right)^2 - \left(\frac{q}{4}\right)^2 = \left(\frac{p}{2} + \frac{q}{4}\right) \left(\frac{p}{2} - \frac{q}{4}\right) //$$

$$(d) x^4 y^4 - 1$$
$$a^2 - b^2 = (a+b)(a-b)$$
$$(x^2 y^2)^2 - (1)^2$$
$$= (x^2 y^2 + 1)(x^2 y^2 - 1) = (x^2 y^2 + 1)((xy)^2 - (1)^2)$$
$$= (x^2 y^2 + 1)(xy + 1)(xy - 1) //$$

$$(e) 15xy - 6x + 5y - 2$$
$$(15xy - 6x) + (5y - 2)$$
$$= 3x(5y - 2) + (5y - 2)$$
$$= (5y - 2)(3x + 1) //$$

$$(f) 49p^2 - 36$$
$$(7p)^2 - (6)^2$$
$$a^2 - b^2 = (a+b)(a-b)$$
$$= (7p+6)(7p-6) //$$

$$(g) x^4 - (x-z)^4$$
$$(x^2)^2 - ((x-z)^2)^2$$
$$a^2 - b^2 = (a+b)(a-b)$$
$$= (x^2 + (x-z)^2)(x^2 - (x-z)^2)$$
$$= (x^2 + x^2 + z^2 - 2xz)(x^2 - x^2 - z^2 + 2xz)$$
$$= (2x^2 + z^2 - 2xz)(2xz - z^2)$$
$$= z(2x^2 + z^2 - 2xz)(2x - z) //$$

$$(h) 14pq + 35pq^2 \\ = 7pq(2 + 5q) //$$

$$(i) 15pq + 15 + 9q + 25p$$

$$(15pq + 9q) + (15 + 25p) \\ = 3q(5p + 3) + 5(3 + 5p) \\ = (3q + 5)(5p + 3) //$$

$$(j) -36y^3 \div 9y^2 \\ \frac{-36y^3}{9y^2} = -4y //$$

$$(k) q^2 - 10q + 21$$

$$q^2 - 7q - 3q + 21 \\ = q(q - 7) - 3(q - 7) \\ = \underline{(q - 7)(q - 3)}$$

| | | |
|-----|----|--|
| S | P | |
| -10 | 21 | < $\begin{matrix} -7 \\ -3 \end{matrix}$ |

$$(l) p^2 + 6p - 16$$

$$p^2 + 8p - 2p - 16 \\ = p(p + 8) - 2(p + 8) \\ = \underline{(p + 8)(p - 2)}$$

| | | |
|---|-----|---|
| S | P | |
| 6 | -16 | < $\begin{matrix} 8 \\ -2 \end{matrix}$ |

$$(m) x^2 - 6x - 7$$

$$x^2 - 7x + x - 7 \\ = x(x - 7) + (x - 7) \\ = \underline{(x - 7)(x + 1)}$$

| | | |
|----|----|---|
| S | P | |
| -6 | -7 | < $\begin{matrix} -7 \\ 1 \end{matrix}$ |

$$(n) (l + m)^2 - (l - m)^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$(\cancel{l + m} + \cancel{l - m})(\cancel{l + m} - \cancel{l - m}) \\ = 2l \times 2m = \underline{4lm}$$

$$(o) a^4 - b^4$$

$$(a^2)^2 - (b^2)^2$$

$$x^2 - y^2 = (x + y)(x - y)$$

$$= (a^2 + b^2)(a^2 - b^2)$$

$$= \underline{\underline{(a^2 + b^2)(a + b)(a - b)}}$$

(p) $x^2 + 5x + 6$

$$x^2 + 2x + 3x + 6$$

$$= x(x + 2) + 3(x + 2)$$

$$= \underline{\underline{(x + 2)(x + 3)}}$$

| | | |
|---|---|-----|
| S | P | |
| 5 | 6 | < 2 |
| | | 3 |

(q) $15xy - 6x + 5y - 2$

$$3x(5y - 2) + (5y - 2)$$

$$= \underline{\underline{(3x + 1)(5y - 2)}}$$

(r) $81x^2 - \frac{64}{49}$

$$(9x)^2 - \left(\frac{8}{7}\right)^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$= \underline{\underline{\left(9x + \frac{8}{7}\right)\left(9x - \frac{8}{7}\right)}}$$

(s) $ax + by - ay - by$

$$= ax - ay$$

$$= \underline{\underline{a(x - y)}}$$

(t) $10x^2 - 18x^4 + 14x^3$

$$2x^2(5 - 9x^2 + 7x)$$

(u) $x^2 + xy + 8x + 8y$

$$x(x + y) + 8(x + y)$$

$$= \underline{\underline{(x + y)(x + 8)}}$$

(v) $z^3 - z^2 - 9z + 9$

$$z^2(z - 1) - 9(z - 1)$$

$$(z - 1)(z^2 - 9)$$

$$\underline{\underline{(z - 1)(z + 3)(z - 3)}}$$

$$[\because a^2 - b^2 = (a + b)(a - b)]$$

2) Find the common factors of
 (i) $6abc$, $24ab^2$ and $12a^2b$

Common factor is $6ab$

(ii) $2x$, $3x^2$ and $4xy$

Common factor is x

(iii) $24x^2y^2$, $42xy^2$, $64x^3y^3$

Common factor is $2xy^2$

3) Divide the following :-

(i) $10y(6y+21) \div 5(2y+7)$

$$\frac{10y(6y+21)}{5(2y+7)} = \frac{2 \cancel{10y} \times 3 \cancel{(2y+7)}}{\cancel{5} \cancel{(2y+7)}}$$

$$= \underline{6y}$$

(ii) $9x^2y^2(3z-24) \div 27xy(z-8)$

$$\frac{9 \cancel{x^2} y^2 (3 \cancel{z} - 24)}{27 \cancel{xy} (z - 8)} = \frac{3 \cancel{xy} (3 \cancel{z} - 8)}{\cancel{3} (z - 8)}$$

$$= \underline{xy}$$

(iii) $(10x-25) \div 5$

$$\frac{10x-25}{5} = \frac{5(2x-5)}{5} = \underline{2x-5}$$

(iv) $(10x-25) \div (2x-5)$

$$\frac{10x-25}{2x-5} = \frac{5(2x-5)}{2x-5} = \underline{5}$$

(v) $5(2x+1)(3x+5) \div (2x+1)$

$$\frac{5 \cancel{(2x+1)} (3x+5)}{\cancel{2x+1}} = \underline{5(3x+5)}$$

(vi) $26xy(x+5)(y-4) \div 13x(y-4)$

$$\frac{2 \cancel{26xy} (x+5) \cancel{(y-4)}}{\cancel{13x} \cancel{(y-4)}} = \underline{2y(x+5)}$$

H) Simplify: $\frac{4x^5 - 25x^3}{2x-5}$

$$\frac{x^3(4x^2-25)}{2x-5}$$

$$= \frac{x^3((2x)^2 - (5)^2)}{2x-5} = \frac{x^3(2x+5)(2x-5)}{2x-5}$$

$$= \underline{\underline{x^3(2x+5)}}$$

5) Divide: (i) $55(x^2 - 5x - 24)$ by $11x(x-8)$

$$\frac{55x^2(x^2 - 5x - 24)}{11x(x-8)} = \frac{5x(x^2 - 5x - 24)}{x-8}$$

S P
-5 -24
-8 > 3

$$= \frac{5x(x^2 - 8x + 3x - 24)}{x-8} = \frac{5x(x(x-8) + 3(x-8))}{x-8}$$

$$= \frac{5x(\cancel{x-8})(x+3)}{\cancel{x-8}} = \underline{\underline{5x(x+3)}}$$

(ii) $20(y+4)(y^2+4y+3) \div 5(y+4)$

$$\frac{20(y+4)(y^2+4y+3)}{5(y+4)} = 4(y^2+4y+3)$$

P S
4 3 < 3

$$= 4(y^2+y+3y+3)$$

$$= 4[y(y+1) + 3(y+1)]$$

$$= \underline{\underline{4(y+1)(y+3)}}$$

(iii) $44(x^2 - 5x - 24) \div 11x(x-8)$

$$\frac{44x^2(x^2 - 5x - 24)}{11x(x-8)} = \frac{4x(x^2 - 5x - 24)}{x-8}$$

S P
-5 -24 < -8
3

$$= \frac{4x(x^2 - 8x + 3x - 24)}{x-8} = \frac{4x[x(x-8) + 3(x-8)]}{x-8}$$

$$= \frac{4x(\cancel{x-8})(x+3)}{\cancel{x-8}} = \underline{\underline{4x(x+3)}}$$

$$(iv) (4x^2 - 9) \text{ by } (2x - 3)$$

$$\frac{(2x)^2 - (3)^2}{2x - 3} = \frac{(2x+3)(2x-3)}{2x-3} = \underline{\underline{2x+3}}$$

$$(v) -\frac{3}{5}xy^2q^3 \text{ by } \frac{3}{25}xyq$$

$$\frac{-\frac{3}{5}xy^2q^3}{\frac{3}{25}xyq} = \frac{-\frac{3}{5}xy^2q^3 \times 25}{3xyq} = \underline{\underline{-5yq^2}}$$

$$(vi) 52pqr(p+q)(q+r)(r+p) \div 104pq(q+r)(r+p)$$

$$\frac{52pqr(p+q)(q+r)(r+p)}{104pq(q+r)(r+p)} = \frac{r(p+q)}{2}$$

6) Divide the given polynomial by the given monomial:

$$(i) (15x^3 - 6x) \div 3x$$

$$\frac{15x^3 - 6x}{3x} = \frac{3x(5x^2 - 2)}{3x} = \underline{\underline{5x^2 - 2}}$$

$$(ii) (3y^8 - 4y^6 + 5y^4) \div y^4$$

$$y^4(3y^4 - 4y^2 + 5) \div y^4 = \underline{\underline{3y^4 - 4y^2 + 5}}$$

$$(iii) 63a^2b^4c^6 \div 7a^2b^2c^3$$

$$\frac{9 \cancel{63}a^{\cancel{2}}b^{\cancel{4}2}c^{\cancel{6}3}}{\cancel{7}a^{\cancel{2}}b^{\cancel{2}}c^{\cancel{3}}} = \underline{\underline{9b^2c^3}}$$

7) Divide as directed:

$$(i) 4yz(z^2 + 6z - 16) \div 2y(z+8)$$

$$\frac{4yz(z^2 + 6z - 16)}{2y(z+8)}$$

$$\begin{array}{r} 8 \quad 16 \\ 6 \quad -16 < 8 \\ \quad \quad -2 \end{array}$$

$$\frac{2z(z^2 + 8z - 2z - 16)}{(z+8)} = \frac{2z(z(z+8) - 2(z+8))}{z+8}$$

$$= \frac{2z(z+8)(z-2)}{z+8} = \underline{\underline{2z(z-2)}}$$

(ii) $5pq(p^2 - q^2) \div 2p(p+q)$

$$\frac{5pq(p^2 - q^2)}{2p(p+q)} = \frac{5pq(p+q)(p-q)}{2p(p+q)} = \underline{\underline{5q(p-q)}}$$

(iii) $x(x+1)(x+2)(x+3) \div x(x+1)$

$$\frac{x(x+1)(x+2)(x+3)}{x(x+1)}$$

$$= \underline{\underline{(x+2)(x+3)}}$$

(iv) $(y^2 + 7y + 10) \div (y+5)$

$$\frac{y^2 + 5y + 2y + 10}{y+5}$$

$$\begin{array}{cc} S & P \\ 7 & 10 < \frac{5}{2} \end{array}$$

$$= \frac{y(y+5) + 2(y+5)}{y+5} = \frac{(y+5)(y+2)}{y+5} = \underline{\underline{y+2}}$$

8) HCF of (i) $4a^3b^2, 12a^3b^2c, 16a^5bc$

$$\text{H.C.F} = 4a^3b$$

(ii) $6x^2y^3, 9xy^3, 3x^4y^4$

$$\text{H.C.F} = 3xy^3$$

(iii) $25p^5q^4, 75p^6q^3$

$$\text{H.C.F} = 25p^5q^3$$