VIII ALGEBRAIC EXPRESSIONS AND IDENTITIES

Q:-) Identify the coefficient of each term in the expression $x^2y^2 - 10x^2y + 5xy^2 - 20$

Soln:-

Terms	Coefficients
x^2y^2	1
$-10x^2y$	-10
$5xy^2$	5
$-20x^0y^0$	-20

Q:-) Classify the following polynomials as monomials, binomials, trinomials.

$-z+5$, $x+y+z$, $y+z+100$, $ab-ac$, 17

Soln:-

Monomial	binomials	trinomials
17	$-z+5$	$x+y+z$
	$ab-ac$	$y+z+100$

Q:-) Construct (a) 3 binomials with only x as a variable
(b) 3 binomials with x and y as variables
(c) 3 monomials with x and y as variables
(d) 2 polynomials with 4 or more terms

Soln:- (a) $3x^2+5$, x^3-9, $x+7$
(b) $x+y$, x^2+y^2, x^2-y^2
(c) xy, x^2y^2, x^3y^3
(d) $x+y+z+w$, $a+b+c-9+d$

Q:-) Write two terms which are like.
(i) $7xy$ (ii) $4mn^2$ (iii) $2l$

Soln:- (i) $11xy$, $-\frac{7}{2}xy$

(ii) $5mn^2$, $-12mn^2$

(iii) $4l$, $-9l$

Q:-) Add : $7xy + 5yz - 3zx$; $4yz + 9zx - 4y$; $-3xz + 5x - 2xy$

Soln:-

$$\begin{array}{r} 7xy + 5yz - 3zx + 0y + 0x \\ 0xy + 4yz + 9zx - 4y + 0x \\ (+)\ -2xy + 0yz - 3xz + 0y + 5x \\ \hline 5xy + 9yz + 3xz - 4y + 5x \end{array}$$

Q:-) Subtract $5x^2 - 4y^2 + 6y - 3$ from $7x^2 - 4xy + 8y^2 + 5x - 3y$

Soln:-

$$7x^2 - 4xy + 8y^2 + 5x - 3y + 0$$
$$(+)\quad \overset{(-)}{5x^2} \overset{(+)}{-} 0xy \overset{(-)}{+} 4y^2 \overset{(+)}{-} 0x \overset{(-)}{+} 6y \overset{(+)}{-} 3$$
$$2x^2 - 4xy + 12y^2 + 5x - 9y + 3$$

EXERCISE 9.1

1) Identify the terms, their coefficients for each of the following expressions :-

(i) $5xyz^2 - 3zy$ (ii) $1 + x + x^2$ (iii) $4x^2y^2 - 4x^2y^2z^2 + z^2$

(iv) $3 - pq + qr - rp$ (v) $\frac{x}{2} + \frac{y}{2} - xy$ (vi) $0.3a - 0.6ab + 0.5b$

Soln:-

	Terms	Coefficients
(i)	$5xyz^2$	5
	$-3zy$	-3
(ii)	$1x^0$	1
	$1x$	1
	$1x^2$	1
(iii)	$4x^2y^2$	4
	$-4x^2y^2z^2$	-4
	$1z^2$	1
(iv)	3	3
	$-1pq$	-1
	$+1qr$	1
	$-1rp$	-1
(v)	$\frac{x}{2}$	$\frac{1}{2}$
	$\frac{y}{2}$	$\frac{1}{2}$
	$-1xy$	-1
(vi)	$0.3a$	0.3
	$-0.6ab$	-0.6
	$0.5b$	0.5

2) Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories?

$x+y$; 1000 ; $x+x^2+x^3+x^4$; $7+y+5x$; $2y-3y^2$; $2y-3y^2+4y^3$; $5x-4y+3xy$; $4z-15z^2$; $ab+bc+cd+da$; pqr ; p^2q+pq^2 ; $2p+2q$

Soln:-

Monomials	Binomials	Trinomials	Others
1000	$x+y$	$7+y+5x$	$x+x^2+x^3+x^4$
pqr	$2y-3y^2$	$2y-3y^2+4y^3$	$ab+bc+cd+da$
	$4z-15z^2$	$5x-4y+3xy$	
	p^2q+pq^2		
	$2p+2q$		

3) Add the following:-

(i) $ab-bc$; $bc-ca$; $ca-ab$

Soln:- $ab-bc+bc-ca+ca-ab = 0$

(ii) $a-b+ab$; $b-c+bc$; $c-a+ac$

Soln:- $a-b+ab+b-c+bc+c-a+ac = ab+bc+ac$

(iii) $2p^2q^2-3pq+4$; $5+7pq-3p^2q^2$

Soln:-

$$\begin{array}{r} 2p^2q^2-3pq+4 \\ (+)\ -3p^2q^2+7pq+5 \\ \hline -p^2q^2+4pq+9 \end{array}$$

(iv) l^2+m^2 ; m^2+n^2 ; n^2+l^2 ; $2lm+2mn+2nl$

Soln:- $l^2+m^2+m^2+n^2+n^2+l^2+2lm+2mn+2nl$

$= (l^2+l^2)+(m^2+m^2)+(n^2+n^2)+2lm+2mn+2nl$

$= 2l^2+2m^2+2n^2+2lm+2mn+2nl$ //

4) (a) Subtract $4a-7ab+3b+12$ from $12a-9ab+5b-3$

Soln:-

$$\begin{array}{r} 12a-9ab+5b-3 \\ (+)\ \ (-)4a\ (+)7ab\ (-)3b\ (-)12 \\ \hline 8a-2ab+2b-15 \end{array}$$ //

(b) Subtract $3xy + 5yz - 7zx$ from $5xy - 2yz - 2zx + 10xyz$

Soln:-

$$\begin{array}{r} 5xy - 2yz - 2zx + 10xyz \\ (+)\ (-)3xy\ (-)7yz\ (+)7zx\ (-)0xyz \\ \hline 2xy - 7yz + 5zx + 10xyz \end{array}$$

(c) Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$

Soln:-

$$\begin{array}{r} 18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q \\ (+)\ (+)10\ (+)8p\ (-)7q\ (+)3pq\ (-)5pq^2\ (-)4p^2q \\ \hline 28 + 5p - 18q + 8pq - 7pq^2 + p^2q \end{array}$$

Q:-) Find the product :-

(i) $x, 3y$

Soln:- $x \times 3y = 3xy$ //

(ii) $5x, 3y$

Soln:- $5x \times 3y = 15xy$ //

(iii) $5x, -3y$

Soln:- $5x \times -3y = -15xy$ //

(iv) $5x, 4x^2$

Soln:- $5x \times 4x^2 = 20x^3$ //

(v) $5x, -4xyz$

Soln:- $5x \times -4xyz = -20x^2yz$ //

(vi) $2x, 5y, 7z$

Soln:- $2x \times 5y \times 7z = 70xyz$ //

(vii) $4xy, 5x^2y^2, 6x^3y^3$

Soln:- $4xy \times 5x^2y^2 \times 6x^3y^3 = 120x^6y^6$ //

Q:-) Complete the table for area of a rectangle with given length and breadth.

length	breadth	area
$3x$	$5y$	$3x \times 5y = 15xy$
$9y$	$4y^2$	$9y \times 4y^2 = 36y^3$
$4ab$	$5bc$	$4ab \times 5bc = 20ab^2c$
$2l^2m$	$3lm^2$	$2l^2m \times 3lm^2 = 6l^3m^3$

Q:-) Find the volume of each rectangular box with given length, breadth and height.

	length	breadth	height	volume
(i)	$2ax$	$3by$	$5cz$	$30abcxyz$
(ii)	m^2n	n^2p	p^2m	$n^3n^3p^3$
(iii)	$2q$	$4q^2$	$8q^3$	$64q^6$

EXERCISE 9.2

1) Find the product of the following pairs of monomials
(i) $4, 7p$ (ii) $-4p, 7p$ (iii) $-4p, 7pq$ (iv) $4p^3, -3p$
(v) $4p, 0$

Soln:- (i) $4 \times 7p = 28p$ //

(ii) $-4p \times 7p = -28p^2$ //

(iii) $-4p \times 7pq = -28p^2q$ //

(iv) $4p^3 \times -3p = -12p^4$ //

(v) $4p \times 0 = 0$ //

2) Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively:
(p, q) ; $(10m, 5n)$; $(20x^2, 5y^2)$; $(4x, 3x^2)$; $(3mn, 4np)$

Soln:-

length	breadth	area = $l \times b$
p	q	pq
$10m$	$5n$	$50mn$
$20x^2$	$5y^2$	$100x^2y^2$
$4x$	$3x^2$	$12x^3$
$3mn$	$4np$	$12mn^2p$

3) Complete the table of products :-

First monomial → / Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25y^2$	$-15x^2y$	$20xy^2$	$35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-63x^4y^4$	$81x^4y^4$

4) Obtain the volume of rectangular boxes with the following length, breadth and height respectively :
(i) $5a, 3a^2, 7a^4$ (ii) $2p, 4q, 8r$ (iii) $xy, 2x^2y, 2xy^2$ (iv) $a, 2b, 3c$

Soln :-

	length	breadth	height	volume = l × b × h
(i)	$5a$	$3a^2$	$7a^4$	$105a^7$
(ii)	$2p$	$4q$	$8r$	$64pqr$
(iii)	xy	$2x^2y$	$2xy^2$	$4x^4y^4$
(iv)	a	$2b$	$3c$	$6abc$

5) Obtain the product of
(i) xy, yz, zx
Soln :- $xy \times yz \times zx = x^2y^2z^2$ //
(ii) $a, -a^2, a^3$
Soln :- $a \times -a^2 \times a^3 = -a^6$ //
(iii) $2, 4y, 8y^2, 16y^3$
Soln :- $2 \times 4y \times 8y^2 \times 16y^3 = 1024y^6$ //
(iv) $a, 2b, 3c, 6abc$
Soln :- $a \times 2b \times 3c \times 6abc = 36a^2b^2c^2$ //
(v) $m, -mn, mnp$
Soln :- $m \times -mn \times mnp = -m^3n^2p$ //

Q :) Find the product :
(i) $2x(3x+5y)$
Soln :- $2x(3x+5y) = (2x \times 3x) + (2x \times 5y)$
$= 6x^2 + 10xy$ //

(ii) $a^2(2ab-5c)$

Soln:- $(a^2 \times 2ab) - (a^2 \times 5c)$

$= 2a^3b - 5a^2c$ //

Q:-) Multiply (i) $3x(13x+12z-2y)$

Soln:- $(3x \times 13x) + (3x \times 12z) - (3x \times 2y)$

$= 39x^2 + 36xz - 6xy$ //

(ii) $6y(12y+13yz-3x)$

Soln:- $(6y \times 12y) + (6y \times 13yz) - (6y \times 3x)$

$= 72y^2 + 78y^2z - 18xy$ //

* Product of a monomial and a binomial is a binomial

Q:-) Find the product :

(i) $2x(3x+5xy)$

Soln:- $(2x \times 3x) + (2x \times 5xy) = 6x^2 + 10x^2y$ //

(ii) $a^2(2ab-5c)$

Soln:- $(a^2 \times 2ab) - (a^2 \times 5c) = 2a^3b - 5a^2c$ //

(iii) $(4p^2+5p+7) \times 3p$

Soln:- $(4p^2 \times 3p) + (5p \times 3p) + (7 \times 3p)$

$= 12p^3 + 15p^2 + 21p$ //

Q:-) Simplify the expressions and evaluate them.

(i) $x(x-3)+2$ for $x=1$

Soln:- $x(x-3)+2 = x^2 - 3x + 2$

When $x=1$, $x^2 - 3x + 2 = (1)^2 - 3 \times 1 + 2$.

$= 1 - 3 + 2 = 3 - 3 = 0$ //

(ii) $3y(2y-7) - 3(y-4) - 63$ for $y = -2$

Soln:- $3y(2y-7) - 3(y-4) - 63 = 6y^2 - 21y - 3y + 12 - 63$

$= 6y^2 - 24y - 51$

when $x = -2$, $6y^2 - 24y - 51 = 6(-2)^2 - 24(-2) - 51$

$= 6 \times 4 + 24 \times 2 - 51$

$= 24 + 48 - 51 = 72 - 51 = 21$ //

Q:-) Add:

(i) $5m(3-m)$ and $6m^2-13m$

Soln:- $15m-5m^2+6m^2-13m$

$=(15m-13m)+(-5m^2+6m^2)$

$=2m+m^2$ //

(ii) $4y(3y^2+5y-7)$ and $2(y^3-4y^2+5)$

Soln:- $12y^3+20y^2-28y+2y^3-8y^2+10$

$=(12y^3+2y^3)+(20y^2-8y^2)+(10)+(-28y)$

$=14y^3+12y^2-28y+10$ //

Q:-) Subtract $3pq(p-q)$ from $2pq(p+q)$

Soln:- $2pq(p+q)-3pq(p-q)$

$=2p^2q+2pq^2-3p^2q+3pq^2$

$=-p^2q+5pq^2$ //

EXERCISE 9.3

1) Carry out the multiplication of the expressions in each of the following pairs.

(i) $4p, q+r$

Soln:- $4p\times(q+r)=(4p\times q)+(4p\times r)=4pq+4pr$ //

(ii) $ab, a-b$

Soln:- $ab(a-b)=(ab\times a)-(ab\times b)=a^2b-ab^2$ //

(iii) $a+b, 7a^2b^2$

$(a+b)7a^2b^2=(a\times 7a^2b^2)+(b\times 7a^2b^2)$

$=7a^3b^2+7a^2b^3$

(iv) $a^2-9, 4a$

$(a^2-9)4a=(a^2\times 4a)-(9\times 4a)$

$=4a^3-36a$ //

(v) $pq+qr+rp, 0$

$(pq+qr+rp)\times 0=0$ //

2. Complete the table.

First expression	Second expression	Product
a	$b+c+d$	$ab+ac+ad$
$x+y-5$	$5xy$	$5x^2y+5xy^2-25xy$
p	$6p^2-7p+5$	$6p^3-7p^2+5p$
$4p^2q^2$	p^2-q^2	$4p^4q^2-4p^2q^4$
$a+b+c$	abc	$a^2bc+ab^2c+abc^2$

3) Find the product

(i) $a^2 \times 2a^{22} \times 4a^{26}$

Soln:- $8a^2 \times 2a^{22} \times 4a^{26} = 8 \times a^{2+22+26}$

$= 8 \times a^{50}$

$= \underline{\underline{8a^{50}}}$

(ii) $\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right)$

Soln:- $-\frac{\cancel{2}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{9}^{3}}{\cancel{10}_{5}} x \times x^2 \times y \times y^2 = \underline{\underline{-\frac{3}{5}x^3y^3}}$

(iii) $\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right)$

Soln:- $-\frac{\cancel{10}^{2}}{\cancel{3}_{1}} \times \frac{\cancel{6}^{2}}{\cancel{5}_{1}} \times p \times p^3 \times q^3 \times q = \underline{\underline{-4p^4q^4}}$

(iv) $x \times x^2 \times x^3 \times x^4$

Soln:- $x^{1+2+3+4} = \underline{\underline{x^{10}}}$

4) (a) Simplify $3x(4x-5)+3$ and find its values for (i) $x=3$ (ii) $x=\frac{1}{2}$

Soln:- $3x(4x-5)+3 = 12x^2-15x+3$

When $x=3$, $12x^2-15x+3 = 12(3)^2-15\times3+3$

$= 12\times9-45+3$

$= 108-42$

$= \underline{\underline{66}}$

When $x = \frac{1}{2}$, $12x^2 - 15x + 3 = 12 \times \left(\frac{1}{2}\right)^2 - 15 \times \frac{1}{2} + 3$

$= \frac{\cancel{12}^{3} \times 1}{\cancel{4}_{1}} - \frac{15}{2} + 3$

$= 3 + 3 - \frac{15}{2} = 6 - \frac{15}{2}$

$= \frac{12 - 15}{2} = \underline{\underline{\frac{-3}{2}}}$

(b) Simplify $a(a^2+a+1)+5$ and find its value for
(i) $a = 0$ (ii) $a = 1$ (iii) $a = -1$

Soln:-

$a(a^2+a+1)+5 = a^3+a^2+a+5$

(i) when $a = 0$, $a^3+a^2+a+5 = 0^3+0^2+0+5 = 5$//

(ii) when $a = 1$, $a^3+a^2+a+5 = 1^3+1^2+1+5 = 1+1+1+5 = 8$//

(iii) when $a = -1$, $a^3+a^2+a+5 = (-1)^3+(-1)^2+(-1)+5$

$= -1+1-1+5 = 4$//

5) (a) Add : $p(p-q)$, $q(q-r)$ and $r(r-p)$

Soln:- $p(p-q)+q(q-r)+r(r-p) = p^2-pq+q^2-qr+r^2-rp$

$= p^2+q^2+r^2-pq-qr-pr$//

(b) Add : $2x(z-x-y)$ and $2y(z-y-x)$

Soln:- $(2xz-2x^2-2xy)+(2yz-2y^2-2yx)$

$= -2x^2-2y^2-4xy+2yz+2xz$//

(c) Subtract : $3l(l-4m+5n)$ from $4l(10n-3m+2l)$

Soln:- $4l(10n-3m+2l)-3l(l-4m+5n)$

$= 40ln-12lm+8l^2-3l^2+12lm-15ln$

$= (40ln-15ln)-12\cancel{lm}+12\cancel{lm}+(8l^2-3l^2)$

$= 25ln+5l^2$//

(a) Subtract : $3a(a+b+c)-2b(a-b+c)$ from $4c(-a+b+c)$

Soln:- $3a(a+b+c)-2b(a-b+c) = 3a^2+3ab+3ac-2ab+2b^2-2bc$

$4c(-a+b+c) = -4ac+4bc+4c^2$

$\therefore -4ac+4bc+4c^2-(3a^2+ab+2b^2+3ac-2bc)$

$= -4ac+4bc+4c^2-3a^2-ab-2b^2-3ac+2bc$

$= -3a^2-2b^2+4c^2-ab-7ac+6bc$//

Q:) Multiply :-

(i) $(x-4)$ and $(2x+3)$

ans:- $(x-4)(2x+3)$

$= x(2x+3) - 4(2x+3)$

$= 2x^2 + 3x - 8x - 12$

$= 2x^2 - 5x - 12$ //

(ii) $(x-y)$ and $(3x+5y)$

ans:- $(x-y)(3x+5y)$

$= x(3x+5y) - y(3x+5y)$

$= 3x^2 + 5xy - 3xy - 5y^2$

$= 3x^2 + 2xy - 5y^2$ //

(iii) $(a+7)$ and $(b-5)$

ans:- $(a+7)(b-5)$

$= a(b-5) + 7(b-5)$

$= ab - 5a + 7b - 35$ //

(iv) (a^2+2b^2) and $(5a-3b)$

ans:- $(a^2+2b^2)(5a-3b)$

$= a^2(5a-3b) + 2b^2(5a-3b)$

$= 5a^3 - 3a^2b + 10ab^2 - 6b^3$ //

(v) $(a+7)$ and (a^2+3a+5)

ans:- $(a+7)(a^2+3a+5)$

$= a(a^2+3a+5) + 7(a^2+3a+5)$

$= a^3 + 3a^2 + 5a + 7a^2 + 21a + 35$

$= a^3 + 10a^2 + 26a + 35$ //

Q:) Simplify : $(a+b)(2a-3b+c) - (2a-3b)c$

ans:- $a(2a-3b+c) + b(2a-3b+c) - c(2a-3b)$

$= 2a^2 - 3ab + ac + 2ab - 3b^2 + bc - 2ac + 3bc$

$= 2a^2 - 3b^2 - ab + 4bc - ac$ //

EXERCISE 9.4

1) Multiply the binomials :-

(i) $(2x+5)$ and $(4x-3)$

ans:- $(2x+5)(4x-3) = 2x(4x-3)+5(4x-3)$
$= 8x^2 - 6x + 20x - 15$
$= 8x^2 + 14x - 15$ //

(ii) $(y-8)$ and $(3y-4)$

ans:- $(y-8)(3y-4) = y(3y-4) - 8(3y-4)$
$= 3y^2 - 4y - 24y + 32$
$= 3y^2 - 28y + 32$ //

(iii) $(2.5l - 0.5m)$ and $(2.5l + 0.5m)$

ans:- $(2.5l-0.5m)(2.5l+0.5m) = 2.5l(2.5l+0.5m) - 0.5m(2.5l+0.5m)$
$= 6.25l^2 + 1.25lm - 1.25lm - 0.25m^2$
$= 6.25l^2 - 0.25m^2$ //

(iv) $(a+3b)$ and $(x+5)$

ans:- $(a+3b)(x+5) = a(x+5) + 3b(x+5)$
$= ax + 5a + 3bx + 15b$ //

(v) $(2pq+3q^2)$ and $(3pq-2q^2)$

ans:- $(2pq+3q^2)(3pq-2q^2) = 2pq(3pq-2q^2) + 3q^2(3pq-2q^2)$
$= 6p^2q^2 - 4pq^3 + 9pq^3 - 6q$
$= 6p^2q^2 + 5pq^3 - 6q^4$ //

(vi) $\left(\frac{3}{4}a^2+3b^2\right)$ and $4\left(a^2-\frac{2}{3}b^2\right)$

ans:- $\left(\frac{3}{4}a^2+3b^2\right) \times 4\left(a^2-\frac{2}{3}b^2\right) = \left(\frac{3a^2}{4}+3b^2\right)\left(4a^2-\frac{8}{3}b^2\right)$

$= \frac{3}{4}a^2\left(4a^2-\frac{8}{3}b^2\right) + 3b^2\left(4a^2-\frac{8}{3}b^2\right)$

$= \frac{3}{4_1}a^2 \times \cancel{4}^{1}a^2 - \frac{\cancel{3}^{1} \times \cancel{8}^{2}}{\cancel{4}_1 \times \cancel{3}_1}a^2b^2 + 12a^2b^2 - \cancel{8}^{1}b^2 \times \frac{8}{\cancel{3}_1}b^2$

$= 3a^4 - 2a^2b^2 + 12a^2b^2 - 8b^4$

$= 3a^4 + 10a^2b^2 - 8b^4$ //

2) Find the product

(i) $(5-2x)(3+x)$

ans:- $5(3+x)-2x(3+x)$

$= 15+5x-6x-2x^2$

$= 15-x-2x^2$ //

(ii) $(x+7y)(7x-y)$

ans:- $x(7x-y)+7y(7x-y)$

$= 7x^2-xy+49xy-7y^2$

$= 7x^2+48xy-7y^2$ //

(iii) $(a^2+b)(a+b^2)$

ans:- $a^2(a+b^2)+b(a+b^2)$

$= a^3+a^2b^2+ab+b^3$ //

(iv) $(p^2-q^2)(2p+q)$

ans:- $p^2(2p+q)-q^2(2p+q)$

$= 2p^3+p^2q-2pq^2-q^3$ //

3) Simplify:

(i) $(x^2-5)(x+5)+25$

ans:- $x^2(x+5)-5(x+5)+25$

$= x^3+5x^2-5x-25+25$

$= x^3+5x^2-5x$ //

(ii) $(t+s^2)(t^2-s)$

ans:- $t(t^2-s)+s^2(t^2-s)$

$= t^3-st+s^2t^2-s^3$ //

(iii) $(a^2+5)(b^3+3)+5$

ans:- $a^2(b^3+3)+5(b^3+3)+5$

$= a^2b^3+3a^2+5b^3+15+5$

$= a^2b^3+3a^2+5b^3+20$ //

(iv) $(a+b)(c-d)+(a-b)(c+d)+2(ac+bd)$

ans:- $a(c-d)+b(c-d)+a(c+d)-b(c+d)+2ac+2bd$

$= ac-ad+bc-bd+ac+ad-bc-bd+2ac+2bd$

$= 4ac-2bd+2bd$

$= 4ac$

(v) $(x+y)(2x+y)+(x+2y)(x-y)$

ans:- $x(2x+y)+y(2x+y)+x(x-y)+2y(x-y)$

$= 2x^2+xy+2xy+y^2+x^2-xy+2xy-2y^2$

$= 3x^2+4xy-y^2$

(vi) $(x+y)(x^2-xy+y^2)$

ans:- $x(x^2-xy+y^2)+y(x^2-xy+y^2)$

$= x^3-x^2y+xy^2+x^2y-xy^2+y^3$

$= x^3+y^3$

(vii) $(1.5x-4y)(1.5x+4y+3)-4.5x+12y$

ans:- $1.5x(1.5x+4y+3)-4y(1.5x+4y+3)-4.5x+12y$

$= 2.25x^2+6xy+4.5x-6xy-16y^2-12y-4.5x+12y$

$= 2.25x^2-16y^2$

(viii) $(a+b+c)(a+b-c)$

ans:- $a(a+b-c)+b(a+b-c)+c(a+b-c)$

$= a^2+ab-ac+ab+b^2-bc+ac+bc-c^2$

$= a^2+2ab+b^2-c^2$

$(a+b)^2 = a^2+2ab+b^2$

$(a-b)^2 = a^2-2ab+b^2$

$(a+b)(a-b) = a^2-b^2$

$(x+a)(x+b) = x^2+(a+b)x+ab$

Q:-) Using appropriate identity find (i) $(2x+3y)^2$ (ii) 103^2

ans:- (i) $(2x+3y)^2 = (2x)^2+2\times 2x\times 3y+(3y)^2$ [$\because (a+b)^2 = a^2+2ab+b^2$]

$= 4x^2+12xy+9y^2$

(ii) $(103)^2 = (100+3)^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$(100+3)^2 = 100^2 + 2\times100\times3 + 3^2$

$= 10000 + 600 + 9 = \underline{\underline{10609}}$

Q:-) Using Identity find (i) $(4p-3q)^2$

ans:- $(a-b)^2 = a^2 - 2ab + b^2$

$(4p-3q)^2 = (4p)^2 - 2\times4p\times3q + (3q)^2$

$= 16p^2 - 24pq + 9q^2$ //

(ii) $(4.9)^2$

$(a-b)^2 = a^2 - 2ab + b^2$

ans:- $(4.9)^2 = (5-0.1)^2$

$= 5^2 - 2\times5\times0.1 + (0.1)^2$

$= 25 - 1 + 0.01 = 24.01$ //

Q:-) Using suitable identity, find

(i) $\left(\frac{3}{2}m + \frac{2}{3}n\right)\left(\frac{3}{2}m - \frac{2}{3}n\right)$

ans:- $(a+b)(a-b) = a^2 - b^2$

$\left(\frac{3}{2}m + \frac{2}{3}n\right)\left(\frac{3}{2}m - \frac{2}{3}n\right) = \left(\frac{3}{2}m\right)^2 - \left(\frac{2}{3}n\right)^2$

$= \frac{9}{4}m^2 - \frac{4}{9}n^2$ //

(ii) $983^2 - 17^2$

ans:- $a^2 - b^2 = (a+b)(a-b)$

$983^2 - 17^2 = (983+17)(983-17)$

$= 1000 \times 966 = 966000$ //

(iii) 194×206

ans:- $(a-b)(a+b) = a^2 - b^2$

$194 \times 206 = (200-6)(200+6)$

$= 200^2 - 6^2$

$= 40000 - 36$

$= 39964$ //

Q:-) Use Suitable Identity to find:

(i) 501×502

ans:- $(x+a)(x+b) = x^2+(a+b)x+ab$

$501 \times 502 = (500+1)(500+2)$

$= (500)^2 + (1+2) \times 500 + 1 \times 2$

$= 250000 + 3 \times 500 + 2$

$= 250000 + 1500 + 2 = 251502$ //

(ii) 95×103

ans:- $(x+a)(x+b) = x^2+(a+b)x+ab$

$95 \times 103 = (100+3)(100-5)$

$= 100^2 + (3-5) \times 100 + 3 \times (-5)$

$= 10000 + (-2) \times 100 - 15$

$= 10000 - 200 - 15 = 9785$ //

EXERCISE 9.5

1) Use a suitable identity to get each of the following products:

(i) $(x+3)(x+3)$

ans:- $(a+b)^2 = a^2+2ab+b^2$

$(x+3)^2 = x^2 + 2 \times x \times 3 + 3^2 = x^2 + 6x + 9$ //

(ii) $(2y+5)(2y+5)$

ans:- $(a+b)^2 = a^2+2ab+b^2$

$(2y+5)^2 = (2y)^2 + 2 \times 2y \times 5 + 5^2 = 4y^2 + 20y + 25$ //

(iii) $(2a-7)(2a-7)$

ans:- $(a-b)^2 = a^2 - 2ab + b^2$

$(2a-7)^2 = (2a)^2 - 2 \times 2a \times 7 + 7^2 = 4a^2 - 28a + 49$ //

(iv) $\left(3a-\frac{1}{2}\right)\left(3a-\frac{1}{2}\right)$

ans:- $(a-b)^2 = a^2 - 2ab + b^2$

$\left(3a-\frac{1}{2}\right)^2 = (3a)^2 - 2 \times 3a \times \frac{1}{2} + \left(\frac{1}{2}\right)^2 = 9a^2 - 3a + \frac{1}{4}$ //

(v) $(1.1m-0.4)(1.1m+0.4)$

ans:- $(a-b)(a+b) = a^2 - b^2$

$(1.1m-0.4)(1.1m+0.4) = (1.1m)^2 - (0.4)^2$

$= 1.21m^2 - 0.16$ //

(vi) $(a^2+b^2)(-a^2+b^2)$

ans:- $(a+b)(a-b) = a^2-b^2$

$(b^2+a^2)(b^2-a^2) = (b^2)^2-(a^2)^2 = b^4-a^4$ //

(vii) $(6x-7)(6x+7)$

ans:- $(a+b)(a-b) = a^2-b^2$

$(6x+7)(6x-7) = (6x)^2-7^2 = 36x^2-49$ //

(viii) $(-a+c)(-a+c)$

ans:- $(a-b)^2 = a^2-2ab+b^2$

$(c-a)(c-a) = (-a)^2 = c^2-2\times c\times a+a^2$

$= c^2-2ac+a^2$ //

(ix) $\left(\frac{x}{2}+\frac{3y}{4}\right)\left(\frac{x}{2}+\frac{3y}{4}\right)$

ans:- $(a+b)^2 = a^2+2ab+b^2$

$\left(\frac{x}{2}+\frac{3y}{4}\right)^2 = \left(\frac{x}{2}\right)^2+2\times\frac{x}{2}\times\frac{3y}{4}+\left(\frac{3y}{4}\right)^2$

$= \frac{x^2}{4}+\frac{3}{4}xy+\frac{9y^2}{16}$ //

(x) $(7a-9b)(7a-9b)$

ans:- $(a-b)^2 = a^2-2ab+b^2$

$(7a-9b)^2 = (7a)^2-2\times 7a\times 9b+(9b)^2$

$= 49a^2-126ab+81b^2$ //

2) Use suitable identity to find the products:-

(i) $(x+3)(x+7)$

ans:- $(x+a)(x+b) = x^2+(a+b)x+ab$

$(x+3)(x+7) = x^2+(3+7)x+3\times 7$

$= x^2+10x+21$ //

(ii) $(4x+5)(4x+1)$

ans:- $(x+a)(x+b) = x^2+(a+b)x+ab$

$(4x+5)(4x+1) = (4x)^2+(5+1)\times 4x+5\times 1$

$= 16x^2+6\times 4x+5$

$= 16x^2+24x+5$ //

(iii) $(4x-5)(4x-1)$

ans:- $(x+a)(x+b) = x^2+(a+b)x+ab$

$(4x-5)(4x-1) = (4x)^2+(-5-1)\times 4x+(-5)\times(-1)$

$= 16x^2+(-6)\times 4x+5$

$= 16x^2-24x+5$ //

(iv) $(4x+5)(4x-1)$

ans:- $(x+a)(x+b) = x^2+(a+b)x+ab$

$(4x+5)(4x-1) = (4x)^2+(5-1)\times 4x+5\times(-1)$

$= 16x^2+4\times 4x-5$

$= 16x^2+16x-5$ //

(v) $(2x+5y)(2x+3y)$

ans:- $(x+a)(x+b) = x^2+(a+b)x+ab$

$(2x+5y)(2x+3y) = (2x)^2+(5y+3y)\times 2x+5y\times 3y$

$= 4x^2+8y\times 2x+15y^2$

$= 4x^2+16xy+15y^2$ //

(vi) $(2a^2+9)(2a^2+5)$

ans:- $(x+a)(x+b) = x^2+(a+b)x+ab$

$(2a^2+9)(2a^2+5) = (2a^2)^2+(9+5)\times 2a^2+9\times 5$

$= 4a^4+14\times 2a^2+45$

$= 4a^4+28a^2+45$ //

(vii) $(xyz-4)(xyz-2)$

ans:- $(x+a)(x+b) = x^2+(a+b)x+ab$

$(xyz-4)(xyz-2) = (xyz)^2+(-4-2)\times xyz+(-4)\times(-2)$

$= x^2y^2z^2-6xyz+8$ //

3) Find the following squares by using the identities:-

(i) $(b-7)^2$

$(a-b)^2 = a^2-2ab+b^2$

$(b-7)^2 = b^2-2\times b\times 7+7^2 = b^2-14b+49$

(ii) $(xy+3z)^2$

$(a+b)^2 = a^2+2ab+b^2$

$(xy+3z)^2 = (xy)^2 + 2\times xy \times 3z + (3z)^2$

$= x^2y^2 + 6xyz + 9z^2$ //

(iii) $(6x^2-5y)^2$

$(a-b)^2 = a^2-2ab+b^2$

$(6x^2-5y)^2 = (6x^2)^2 - 2\times 6x^2 \times 5y + (5y)^2$

$= 36x^4 - 60x^2y + 25y^2$ //

(iv) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2$

$(a+b)^2 = a^2+2ab+b^2$

$\left(\frac{2}{3}m+\frac{3}{2}n\right)^2 = \left(\frac{2}{3}m\right)^2 + 2\times\frac{2}{3_1}m\times\frac{3}{2_1}n + \left(\frac{3}{2}n\right)^2$

$= \frac{4}{9}m^2 + 2mn + \frac{9}{4}n^2$ //

(v) $(0.4p-0.5q)^2$

$(a-b)^2 = a^2-2ab+b^2$

$(0.4p-0.5q)^2 = (0.4p)^2 - 2\times 0.4p\times 0.5q + (0.5q)^2$

$= 0.16p^2 - 0.4pq + 0.25q^2$ //

(vi) $(2xy+5y)^2$

$(a+b)^2 = a^2+2ab+b^2$

$(2xy+5y)^2 = (2xy)^2 + 2\times 2xy\times 5y + (5y)^2$

$= 4x^2y^2 + 20xy^2 + 25y^2$ //

Simplify:-

(i) $(a^2-b^2)^2$

$(a-b)^2 = a^2-2ab+b^2$

$(a^2-b^2)^2 = (a^2)^2 - 2\times a^2\times b^2 + (b^2)^2$

$= a^4 - 2a^2b^2 + b^4$ //

(ii) $(2x+5)^2 - (2x-5)^2$

$a^2-b^2 = (a+b)(a-b)$

$(2x+5)^2 - (2x-5)^2 = (2x+5+2x-5)(2x+5-2x+5)$

$= 4x \times 10 = 40x$ //

(iii) $(7m-8n)^2+(7m+8n)^2$

$(a-b)^2 = a^2-2ab+b^2$

$(a+b)^2 = a^2+2ab+b^2$

$(7m-8n)^2+(7m+8n)^2$

$=[(7m)^2-2\times 7m\times 8n+(8n)^2]+[(7m)^2+2\times 7m\times 8n+(8n)^2]$

$=49m^2-112mn+64n^2+49m^2+112mn+64n^2$

$=98m^2+128n^2$ //

(iv) $(4m+5n)^2+(5m+4n)^2$

$(a+b)^2 = a^2+2ab+b^2$

$(4m+5n)^2+(5m+4n)^2$

$=[(4m)^2+2\times 4m\times 5n+(5n)^2]+[(5m)^2+2\times 5m\times 4n+(4n)^2]$

$=16m^2+40mn+25n^2+25m^2+40mn+16n^2$

$=41m^2+80mn+41n^2$ //

(v) $(2.5p-1.5q)^2-(1.5p-2.5q)^2$

$a^2-b^2=(a+b)(a-b)$

$(2.5p-1.5q)^2-(1.5p-2.5q)^2$

$=(2.5p-1.5q+1.5p-2.5q)(2.5p-1.5q-1.5p+2.5q)$

$=(4p-4q)(p+q)$

$=4(p-q)(p+q)$

$=4(p^2-q^2)$ //

(vi) $(ab+bc)^2-2ab^2c$

$(a+b)^2=a^2+2ab+b^2$

$(ab+bc)^2-2ab^2c=(ab)^2+2\times ab\times bc+(bc)^2-2ab^2c$

$=a^2b^2+2ab^2c+b^2c^2-2ab^2c$

$=a^2b^2+b^2c^2$ //

(vii) $(m^2-n^2m)^2+2m^3n^2$

$(a-b)^2=a^2-2ab+b^2$

$(m^2-n^2m)^2+2m^3n^2=(m^2)^2-2\times m^2\times n^2m+(n^2m)^2+2m^3n^2$

$=m^4-2m^3n^2+n^4m^2+2m^3n^2$

$=m^4+n^4m^2$ //

5) Show that :

(i) $(3x+7)^2 - 84x = (3x-7)^2$

LHS, $(3x+7)^2 - 84x$

$= (3x)^2 + 2\times 3x \times 7 + (7)^2 - 84x$ $[\because (a+b)^2 = a^2 + 2ab + b^2]$

$= 9x^2 + 42x + 49 - 84x$

$= 9x^2 - 42x + 49$

$= (3x)^2 - 2\times 3x \times 7 + (7)^2$

$= (3x-7)^2$, RHS. $[\because a^2 - 2ab + b^2 = (a-b)^2]$

$\therefore$ LHS = RHS

(ii) $(9p-5q)^2 + 180pq = (9p+5q)^2$

LHS, $(9p)^2 - 2\times 9p \times 5q + (5q)^2 + 180pq$

$= 81p^2 - 90pq + 25q^2 + 180pq$ $[\because (a-b)^2 = a^2 - 2ab + b^2]$

$= 81p^2 + 90pq + 25q^2$

$= (9p)^2 + 2\times 9p \times 5q + (5q)^2$

$= (9p+5q)^2$, RHS $[\because a^2 + 2ab + b^2 = (a+b)^2]$

(iii) $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$

LHS, $\left(\frac{4}{3}m\right)^2 - 2\times \cancel{\frac{4}{3_1}m \times \frac{3}{4_1}n} + \left(\frac{3}{4}n\right)^2 + 2mn$

$= \frac{16m^2}{9} - \cancel{2mn} + \frac{9n^2}{16} + \cancel{2mn}$ $[\because (a-b)^2 = a^2 - 2ab + b^2]$

$= \frac{16m^2}{9} + \frac{9}{16}n^2$, RHS

(iv) $(4pq+3q)^2 - (4pq-3q)^2 = 48pq^2$

LHS, $(4pq+\cancel{3q}+4pq-\cancel{3q})(\cancel{4pq}+3q-\cancel{4pq}+3q)$

$[\because a^2 - b^2 = (a+b)(a-b)]$

$= 8pq \times 6q = 48pq^2$, RHS

(v) $(a-b)(a+b) + (b-c)(b+c) + (c-a)(c+a) = 0$

LHS, $\cancel{a^2} - \cancel{b^2} + \cancel{b^2} - \cancel{c^2} + \cancel{c^2} - \cancel{a^2}$

$= 0$, RHS $[\because (x+y)(x-y) = x^2 - y^2]$

6) Using identities, evaluate

(i) $71^2 = (70+1)^2$

$= (70)^2 + 2\times70\times1 + (1)^2$ $[(a+b)^2 = a^2+2ab+b^2]$

$= 4900 + 140 + 1$

$= 5041$ //

(ii) $99^2 = (100-1)^2$

$= 100^2 - 2\times100\times1 + 1^2$ $[\because (a-b)^2 = a^2-2ab+b^2]$

$= 10000 - 200 + 1 = 9801$ //

(iii) $102^2 = (100+2)^2$

$= 100^2 + 2\times100\times2 + 2^2$ $[\because (a+b)^2 = a^2+2ab+b^2]$

$= 10000 + 400 + 4 = 10404$ //

(iv) $998^2 = (1000-2)^2$

$= 1000^2 - 2\times1000\times2 + 2^2$

$= 1000000 - 4000 + 4$ $[\because (a-b)^2 = a^2-2ab+b^2]$

$= 996004$ //

(v) $5.2^2 = (5+0.2)^2$

$= 5^2 + 2\times5\times0.2 + (0.2)^2$ $[\because (a+b)^2 = a^2+2ab+b^2]$

$= 25 + 2 + 0.04 = 27.04$ //

(vi) $297\times303 = (300-3)(300+3)$

$= 300^2 - 3^2$ $[\because (a+b)(a-b) = a^2-b^2]$

$= 90000 - 9 = 89991$ //

(vii) $78\times82 = (80-2)(80+2)$

$= 80^2 - 2^2$ $[\because (a+b)(a-b) = a^2-b^2]$

$= 6400 - 4 = 6396$ //

(viii) $8.9^2 = (9-0.1)^2$

$= 9^2 - 2\times9\times0.1 + (0.1)^2$ $[\because (a-b)^2 = a^2-2ab+b^2]$

$= 81 - 1.8 + 0.01 = 79.21$ //

(ix) $1.05 \times 9.5 = (1+0.05)(10-0.5)$

$= 1(10-0.5) + 0.05(10-0.5)$

$= 10 - 0.5 + 0.5 - 0.025$

$= \underline{\underline{9.975}}$

7) Using suitable identity, find

(i) $51^2 - 49^2 = (51+49)(51-49)$

$= 100 \times 2$ $[(a+b)(a-b) = a^2-b^2]$

$= 200$ //

(ii) $(1.02)^2 - (0.98)^2 = (1.02+0.98)(1.02-0.98)$

$= 2 \times 0.04$ $[a^2-b^2 = (a+b)(a-b)]$

$= 0.08$ //

(iii) $153^2 - 147^2 = (153+147)(153-147)$

$= 300 \times 6$ $[a^2-b^2 = (a+b)(a-b)]$

$= 1800$ //

(iv) $12.1^2 - 7.9^2 = (12.1+7.9)(12.1-7.9)$

$= 20 \times 4.2$ $[a^2-b^2 = (a+b)(a-b)]$

$= 84$ //

8) Using suitable identity, find

(i) $103 \times 104 = (100+3)(100+4)$

$= 100^2 + (3+4) \times 100 + 3 \times 4$

$= 10000 + 700 + 12$ $[(x+a)(x+b) = x^2 + (a+b)x + ab]$

$= 10712$ //

(ii) $5.1 \times 5.2 = (5+0.1)(5+0.2)$

$= 5^2 + (0.1+0.2) \times 5 + 0.1 \times 0.2$

$= 25 + 1.5 + 0.02$ $[(x+a)(x+b) = x^2 + (a+b)x + ab]$

$= 26.52$ //

(iii) $103 \times 98 = (100+3)(100-2)$

$= 100^2 + (3-2) \times 100 + 3 \times (-2)$

$= 10000 + 100 - 6$ $[(x+a)(x+b) = x^2 + (a+b)x + ab]$

$= 10094$ //

(iv) 9.7×9.8 $= (10 - 0.3)(10 - 0.2)$

$= 10^2 + (-0.3 - 0.2) \times 10 + (-0.3) \times (-0.2)$

$= 100 - 5 + 0.06$ $[(x+a)(x+b) = x^2 + (a+b)x + ab]$

$= 95.06$ //
