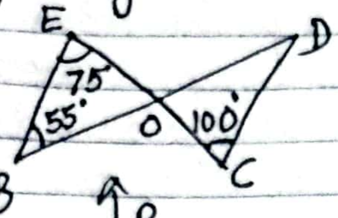
IX

Homework - 18 (Sample Paper)

1) If $3^x + 64 = 2^6 + (\sqrt{3})^8$ (a) 4 (b) 2 (c) 3 (d) 1

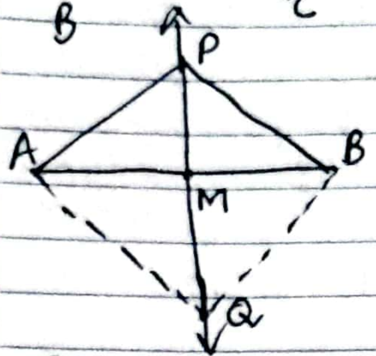
2) If $x+y = 8$ and $xy = 15$, then x^2+y^2 (a) 34 (b) 1 (c) 32 (d) 36

3)



$\angle OEB = 75°$, $\angle OBE = 55°$, $\angle OCD = 100°$, then $\angle ODC =$
(a) 30° (b) 25° (c) 35° (d) 20°

4)



If PQ is the perpendicular bisector of line segment AB, then $\Delta PBM \cong \Delta PAM$ by which congruence criterion?
(a) SSS (b) AAS (c) SAS (d) RHS

5) If both $(x+2)$ and $(2x+1)$ are factors of ax^2+2x+b, then the value of $a-b$ is (a) -1 (b) 2 (c) 1 (d) 0

6) $(27)^{-\frac{2}{3}} =$ ____

7) $x = 4$ is the equation of a line parallel to ____

8) $y+7=0$ is the equation of a line parallel to ____

9) The y-coordinate is also called the ____

10) If $x = 2+\sqrt{3}$, then find the value of $x^2 + \frac{1}{x^2}$

11) Factorise : x^4+4

12) Find the coordinates where the equation $2x+3y=6$ interesects x-axis.

13) P.T $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}} = 2$

14) Find the value of a of the following equation for $x=1, y=1$ as a solution of $5x+3y=a$

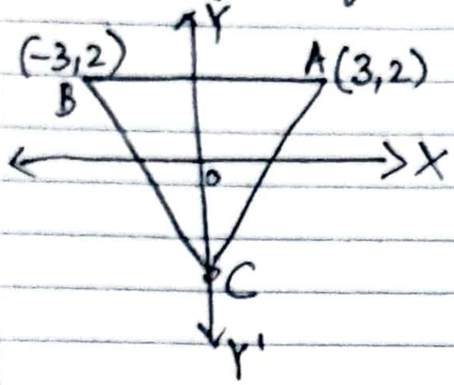
15) Evaluate the following using suitable identities : $(998)^3$

16) Factorise : $2(x+y)^2 - 9(x+y) - 5$

17) The class marks of a distribution are 47, 52, 57, 62, 67, 72, 77, 82. Determine (i) class size
(ii) class limits
(iii) true class limits

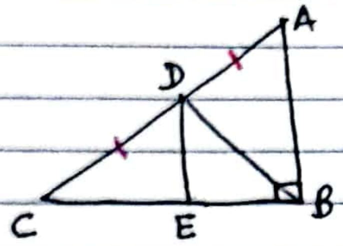
18) Rationalize the denominator of the following: $\frac{3+\sqrt{2}}{3-\sqrt{2}}$

19) Simplify the following: $\frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2-\sqrt{5}}$

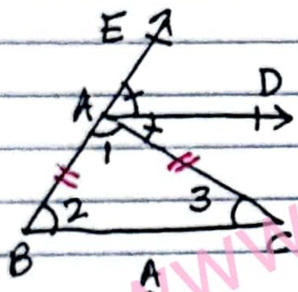
20) 

Find the coordinates of vertex C.

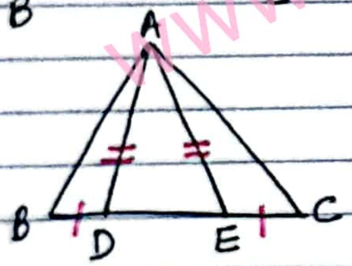
21) Write two solutions for the following equation: $3x+4y=7$

22) 

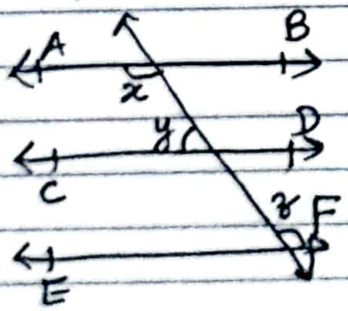
$\angle B$ is a right angle in $\triangle ABC$ and D is the mid-point of AC. Also $DE \parallel AB$ and DE intersects BC at E. Show that
(i) E is the mid-point of BC
(ii) $DE \perp BC$ (iii) $BD = AD$

23) 

$\triangle ABC$ is an isosceles $\triangle$ with $AB = AC$. AD bisects the exterior $\angle A$. Prove that $AD \parallel BC$.

24) 

D and E are points on side BC of a $\triangle ABC$ such that $BD = CE$ and $AD = AE$. Show that $\triangle ABD \cong \triangle ACE$.

25) 

If $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3:7$, find x.

26) The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 13m, 14m and 15m. The advertisements yield an earning of Rs 2000/m² a year. A Company hired one of its walls for 6 months. How much rent did it pay?

IX | Homework - 18 (Answers)

1) $3^x + \cancel{64} = \cancel{64} + (\sqrt{3})^8$

$\Rightarrow 3^x = 3^{\frac{8}{2}}$

$\Rightarrow 3^x = 3^4$

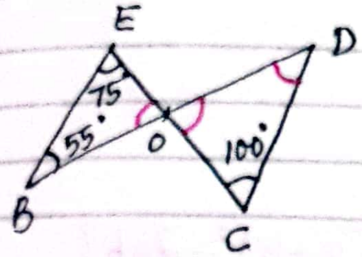
$\therefore x = 4$ (a)

2) $x^2+y^2 = (x+y)^2 - 2xy$

$= (8)^2 - 2\times 15$

$= 64 - 30 = 34$ (a)

3)



Using angle sum property in $\triangle EOB$,

$\angle EOB = 180° - (75° + 55°)$

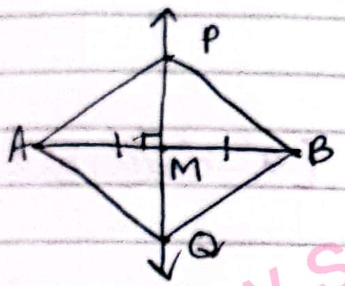
$= 180° - 130° = 50°$

$\angle EOB = \angle DOC = 50°$ (VOA)

$\therefore \angle ODC = 180° - (50° + 100°) = 180° - 150° = 30°$

(a)

4)



In $\triangle PBM$ and $\triangle PAM$,

$AM = BM$ ($\because$ PQ bisects AB)

$\angle PMB = \angle PMA$ (each 90°)

$PM = PM$ (Common side)

$\therefore \triangle PBM \cong \triangle PAM$ (SAS Congruency)

(C)

5) $p(-2) = 0 \Rightarrow 4a - 4 + b = 0$

$b = 4 - 4a \rightarrow (1)$

$p\left(-\frac{1}{2}\right) = 0 \Rightarrow \frac{a}{4} - \frac{2}{2} + b = 0$

$\Rightarrow a - 4 + 4b = 0$

$\Rightarrow a + 4b = 4 \Rightarrow a + 4(4 - 4a) = 4$ [from eq. (1)]

$\Rightarrow a + 16 - 16a = 4 \Rightarrow -15a = -12$

$a = \frac{12}{15} = \frac{4}{5}$

From eq:(1), $b = 4 - 4\times\frac{4}{5} = \frac{20-16}{5} = \frac{4}{5}$

$\therefore a - b = \frac{4}{5} - \frac{4}{5} = 0$ (d)

6) $(27)^{-\frac{2}{3}} = 3^{3\times -\frac{2}{3}} = 3^{-2} = \frac{1}{3^2} = \underline{\frac{1}{9}}$

7) y-axis

8) $y+7=0$

$y=-7$, parallel to x-axis

9) ordinate

10) $x = 2+\sqrt{3}$

$\frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

$x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2$ $[a^2+b^2=(a+b)^2-2ab]$

$= (2+\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}})^2-2$

$= 4^2-2 = 16-2 = \underline{\underline{14}}$

11) $x^4+4 = (x^2)^2+(2)^2$ $[a^2+b^2=(a+b)^2-2ab]$

$= (x^2+2)^2 - 2\times x^2\times 2$

$= (x^2+2)^2 - 4x^2$

$= (x^2+2)^2 - (2x)^2$

$= \underline{\underline{(x^2+2+2x)(x^2+2-2x)}}$ $[a^2-b^2=(a+b)(a-b)]$

12) Put $y=0$, $2x=6$

$x=3$

$\therefore$ The coordinates of the point $= (3,0)$

13) $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \frac{1}{\sqrt{5}+\sqrt{4}} + \frac{1}{\sqrt{6}+\sqrt{5}} + \frac{1}{\sqrt{7}+\sqrt{6}} + \frac{1}{\sqrt{8}+\sqrt{7}} + \frac{1}{\sqrt{9}+\sqrt{8}}$

$= \frac{\sqrt{2}-1}{(\sqrt{2})^2-1^2} + \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} + \frac{\sqrt{4}-\sqrt{3}}{(\sqrt{4})^2-(\sqrt{3})^2} + \frac{\sqrt{5}-\sqrt{4}}{(\sqrt{5})^2-(\sqrt{4})^2} + \frac{\sqrt{6}-\sqrt{5}}{(\sqrt{6})^2-(\sqrt{5})^2} + \frac{\sqrt{7}-\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2} + \frac{\sqrt{8}-\sqrt{7}}{(\sqrt{8})^2-(\sqrt{7})^2} + \frac{\sqrt{9}-\sqrt{8}}{(\sqrt{9})^2-(\sqrt{8})^2}$

$= \frac{\sqrt{2}-1}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \frac{\sqrt{4}-\sqrt{3}}{4-3} + \frac{\sqrt{5}-\sqrt{4}}{5-4} + \frac{\sqrt{6}-\sqrt{5}}{6-5} + \frac{\sqrt{7}-\sqrt{6}}{7-6} + \frac{\sqrt{8}-\sqrt{7}}{8-7} + \frac{\sqrt{9}-\sqrt{8}}{9-8}$

$= \cancel{\sqrt{2}}-1+\cancel{\sqrt{3}}-\cancel{\sqrt{2}}+\cancel{\sqrt{4}}-\cancel{\sqrt{3}}+\cancel{\sqrt{5}}-\cancel{\sqrt{4}}+\cancel{\sqrt{6}}-\cancel{\sqrt{5}}+\cancel{\sqrt{7}}-\cancel{\sqrt{6}}+\cancel{\sqrt{8}}-\cancel{\sqrt{7}}+\sqrt{9}-\cancel{\sqrt{8}}$

$= \sqrt{9}-1 = 3-1 = \underline{\underline{2}}$

14) When $x=1, y=1$, $5+3=a$

$\therefore a=\underline{\underline{8}}$

15) $(998)^3=(1000-2)^3$ $[(a-b)^3=a^3-b^3-3ab(a-b)]$

$=1000^3-2^3-3\times1000\times2\,(1000-2)$

$=1000000000-8-6000\times998$

$=1000000000-8-5988000$

$=1000000000-5988008=994011992$ //

16) Put $x+y=a$

Then, $2a^2-9a-5=2a^2-10a+a-5$ S P

$=2a(a-5)+1(a-5)$ -9 -10 $<$ -10, 1

$=(2a+1)(a-5)$

$=[2(x+y)+1][x+y-5]$

$=\underline{\underline{(2x+2y+1)(x+y-5)}}$

17) Class-size $=52-47=5$ [difference between two Consecutive class-marks]

Let the class mark be 47

lower limit $=$ Class mark $-\dfrac{\text{classsize}}{2}=47-\dfrac{5}{2}=44.5$

upper limit $=$ classmark $+\dfrac{\text{classsize}}{2}=47+\dfrac{5}{2}=49.5$

$\therefore$ Class limits are 44.5 – 49.5
49.5 – 54.5
54.5 – 59.5
59.5 – 64.5
64.5 – 69.5
69.5 – 74.5
74.5 – 79.5
79.5 – 84.5

True class limits are 45 – 49
50 – 54
55 – 59
60 – 64
65 – 69
70 – 74
75 – 79
80 – 84

18) $\dfrac{3+\sqrt{2}}{3-\sqrt{2}} = \dfrac{(3+\sqrt{2})^2}{3^2-(\sqrt{2})^2} = \dfrac{9+2+6\sqrt{2}}{9-2}$

$= \dfrac{11+6\sqrt{2}}{7}$

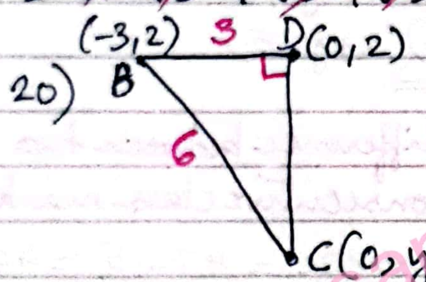
19) $\dfrac{1}{2+\sqrt{3}} + \dfrac{2}{\sqrt{5}-\sqrt{3}} + \dfrac{1}{2-\sqrt{5}}$

$= \dfrac{2-\sqrt{3}}{2^2-(\sqrt{3})^2} + \dfrac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} + \dfrac{2+\sqrt{5}}{(2)^2-(\sqrt{5})^2}$

$= \dfrac{2-\sqrt{3}}{4-3} + \dfrac{2(\sqrt{5}+\sqrt{3})}{5-3} + \dfrac{2+\sqrt{5}}{4-5}$

$= \dfrac{2-\sqrt{3}}{1} + \dfrac{\cancel{2}^{1}(\sqrt{5}+\sqrt{3})}{\cancel{2}_{1}} + \dfrac{2+\sqrt{5}}{-1}$

$= \cancel{2}-\cancel{\sqrt{3}}+\cancel{\sqrt{5}}+\cancel{\sqrt{3}}-\cancel{2}-\cancel{\sqrt{5}} = 0$

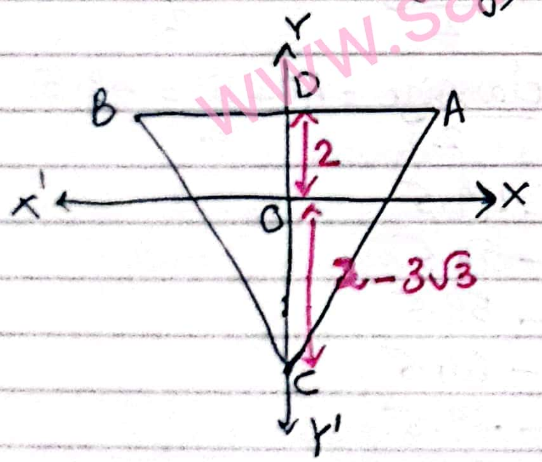
20) 

Using Pythagoras Theorem in rt. $\triangle BDC$,

$DC^2 = BC^2 - BD^2 = 6^2 - 3^2$

$= 36 - 9 = 27$

$DC = \sqrt{27} = 3\sqrt{3}$ units.



$\therefore$ Coordinates of vertex C $= (0, 2-3\sqrt{3})$

21) $3x+4y = 7$

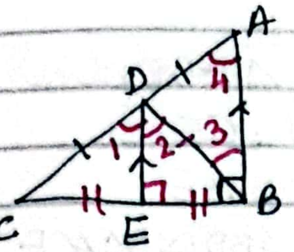
$4y = 7-3x$

$y = \dfrac{7-3x}{4}$

When $x=0$, $y = \dfrac{7}{4}$ $\quad\therefore$ The solution is $(0, \frac{7}{4})$

When $x=1$, $y = \dfrac{7-3}{4} = \dfrac{4}{4} = 1$ $\quad\therefore$ The solution is $(1,1)$

22)



Given: in $\triangle ABC$, $\angle B = 90°$
D is the mid-point of AC.
$DE \parallel AB$

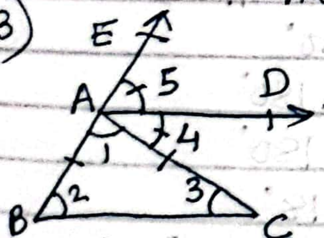
To prove: (i) E is the mid-point of BC
(ii) $DE \perp BC$ (iii) $BD = AD$

Proof: (i) Since D is the mid-point of AC and $DE \parallel AB$,
using mid-point theorem, E is also the mid-point of BC.
Thus, $CE = EB \longrightarrow (1)$

(ii) Since $DE \parallel AB$ and EB is the transversal,
$\angle ABE = \angle DEC = 90°$ (corresponding angles)
Thus, $DE \perp BC$.

(iii) In $\triangle DEC$ and $\triangle DEB$, $\angle DEC = \angle DEB$ (each $90°$)
$CE = EB$ (from eq: (1))
$DE = DE$ (common side)
$\therefore \triangle DEC \cong \triangle DEB$ (SAS congruency)
Thus, $\angle 1 = \angle 2$ (by CPCT) $\longrightarrow (2)$
Also, $\angle 2 = \angle 3$ (alternate interior angles, $DE \parallel BA$) $\to (2)$
and $\angle 1 = \angle 4$ (corresponding angles, $DE \parallel AB$) $\to (3)$
$\therefore \angle 3 = \angle 4$ (from eq: (1), (2) and (3))
$\Rightarrow BD = AD$ (sides opposite to equal angles)
Hence Proved.

23)

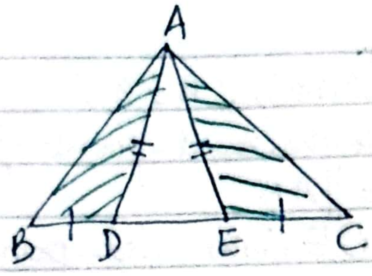


Given: in isosceles $\triangle ABC$, $AB = AC$
AD bisects $\angle EAC$. i.e, $\angle 4 = \angle 5$

To prove: $AD \parallel BC$.

Proof :- Since $AB = AC$, $\angle 2 = \angle 3$ (angles opposite to equal sides) $\longrightarrow (1)$
Also $\angle 4 = \angle 5$ (given) $\rightarrow (2)$
Using exterior angle property in $\triangle ABC$,
$\angle 2 + \angle 3 = \angle 4 + \angle 5$
$\Rightarrow 2\angle 3 = 2\angle 4$ [from eq: (1) and (2)]
$\Rightarrow \angle 3 = \angle 4$
These angles form a pair of alternate interior angles only when $AD \parallel BC$.
Hence Proved.

24)



Given: in $\triangle ABC$,
$BD = CE$
$AD = AE$

To prove: $\triangle ABD \cong \triangle ACE$

Proof:- Since $AD = AE$, $\angle ADE = \angle AED$ (angles opposite to equal sides)

$\Rightarrow 180^\circ - \angle ADE = 180^\circ - \angle AED$ (linear pair)

$\Rightarrow \angle ADB = \angle AEC \longrightarrow (1)$

In $\triangle ABD$ and $\triangle ACE$,

$AD = AE$ (given)

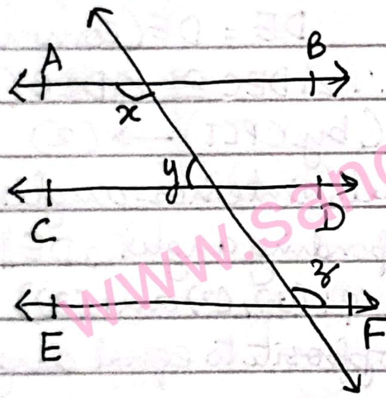
$\angle ADB = \angle AEC$ (proved above)

$BD = CE$ (given)

$\therefore \triangle ABD \cong \triangle ACE$ (SAS congruency)

Hence Proved.

25)



Since $AB \parallel CD$ and $CD \parallel EF$,
then $AB \parallel CD \parallel EF$.

Let $y = 3a$ and $z = 7a$

$x = z$ (Since $AB \parallel EF$, alternate interior angles)

Also, $x + y = 180^\circ$ (cointerior angles, $AB \parallel CD$)

$\Rightarrow z + y = 180^\circ$

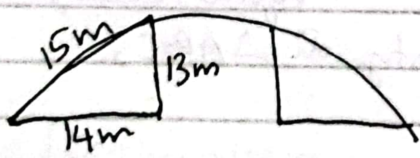
$\Rightarrow 7a + 3a = 180^\circ$

$\Rightarrow 10a = 180^\circ$

$a = 18^\circ$

$\therefore x = z = 7a = 7 \times 18^\circ = \underline{\underline{126^\circ}}$

26)



Let $a = 15m$, $b = 13m$, $C = 14m$

$S = \frac{a+b+C}{2} = \frac{15+13+14}{2} = \frac{42}{2}$

$= 21m$

area of 1 triangular side wall $= \sqrt{S(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-15)(21-13)(21-14)} = \sqrt{\underset{7\times3}{21} \times \underset{2\times3}{6} \times \underset{2\times4}{8} \times 7} = 7\times2\times3\times2$

$= 84\,m^2$

$\therefore$ Rent to be paid $= 84 \times \cancel{2000}^{1000} \times \frac{1}{\cancel{2}_1}$

$= \underline{\underline{Rs\ 84000}}$