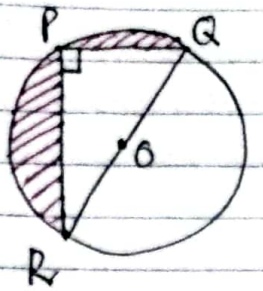
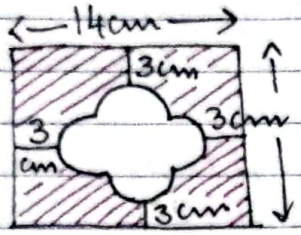
X Elite work – 14 (Areas Related to Circles)

1) 

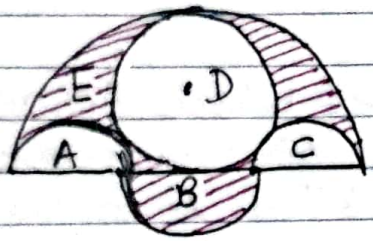
The area of the shaded region if PR = 12cm, PQ = 5cm and O is the centre of the circle is ($\pi = \frac{22}{7}$)

(a) 36.39cm² (b) 48.24cm² (c) 28.76cm² (d) 62.62cm²

2) 

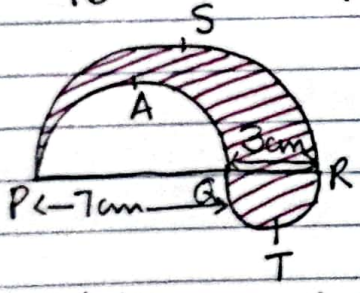
The area of the shaded region

(a) $(180 - 2\pi)$ cm² (b) $(90 - 8\pi)$ cm²
(c) $(180 - 8\pi)$ cm² (d) $(90 - 2\pi)$ cm²

3) 

There are three semi-circles A, B and C having diameter 3cm each and another semicircle E having a circle D with diameter 4.5cm as shown. The area of shaded region is

(a) $\frac{15}{16}\pi$ cm² (b) $\frac{18}{19}\pi$ cm² (c) $\frac{37}{54}\pi$ cm² (d) $\frac{63}{16}\pi$ cm²

4) 

PSR, RTQ and PAQ are three semi-circles of diameters 10cm, 3cm and 7cm respectively. The perimeter of the shaded region is

(a) 22.5cm (b) 31.4cm (c) 36.6cm (d) none of these

5) The difference of the areas of two segments of a circle formed by a chord of radius 5cm subtending an angle of 90° at the centre is

(a) $\left(\frac{25\pi}{4} - \frac{25}{2}\right)$cm² (b) $\left(\frac{15\pi}{4} - \frac{7}{2}\right)$cm² (c) $\left(\frac{7\pi}{4} - \frac{3}{2}\right)$cm² (d) none of these

6) All the vertices of a rhombus lie on a circle. The area of the rhombus, if the area of the circle is 1256cm² is

(a) 300 cm² (b) 600 cm² (c) 800 cm² (d) 900 cm²

7) The area of the largest circle that can be drawn inside the given rectangle of length 'a' cm and breadth 'b' cm (a > b) is (a) $\frac{1}{2}\pi b^2$ cm² (b) $\frac{1}{3}\pi b^2$ cm² (c) $\frac{1}{4}\pi b^2$ cm² (d) πb^2 cm²

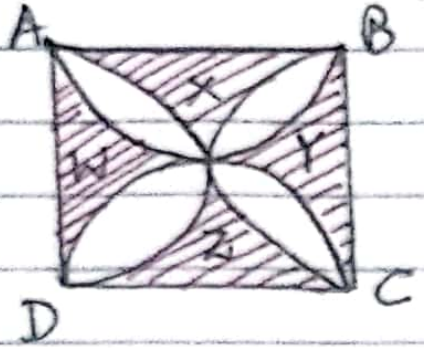
8) The area of the sector of a circle of radius 5cm, if

the corresponding arc length is 3.5cm is

(a) 3.25 cm^2 (b) 8.75 cm^2 (c) 4.60 cm^2 (d) 5.50 cm^2

9) The short and long hands of a clock are 4cm and 6cm long respectively. The sum of distances travelled by their tips in 2 days is

(a) 1148cm (b) 1426.35cm (c) 1910.85cm (d) none of these

10) 

ABCD is a square of side 14cm. Semicircles are drawn with each side of square as diameter. The area of shaded region is ($\pi = 22/7$)

(a) 84 cm^2 (b) 96 cm^2 (c) 110 cm^2 (d) 220 cm^2

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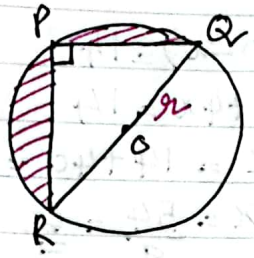
1)



$PR = 12cm$.

$PQ = 5cm$

Since angle in a semi-circle is a right angle, in rt. ΔPQR using Pythagoras Theorem,

$QR^2 = PR^2 + PQ^2 = 12^2 + 5^2$

$= 144 + 25 = 169$.

$QR = \sqrt{169} = 13cm$.

$r = \frac{QR}{2} = \frac{13}{2}cm$ //

Area of shaded region = area of semicircle – area (ΔPQR)

$= \frac{\pi r^2}{2} - \frac{1}{2} \times PQ \times PR$

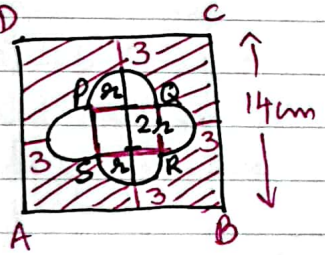
$= \frac{\cancel{22}^{11}}{7} \times \frac{1}{\cancel{2}_1} \times \frac{\cancel{13}^{6.5}}{\cancel{2}_1} \times \frac{\cancel{13}^{6.5}}{\cancel{2}_1} - \frac{1}{\cancel{2}_1} \times 5 \times \cancel{12}^{6}$

$= \frac{464.75}{7} - 30$

$= \frac{464.75 - 210}{7} = \frac{254.75}{7}$

$= 36.39 cm^2$ (a)

2)



$3 + r + 2r + r + 3 = 14$

$4r + 6 = 14$

$4r = 8$

$r = 2cm$

$PQ = 2r = 4cm$

Area of shaded region = area of square$_{ABCD}$ – (area of 2 circles + area of square$_{PQRS}$)

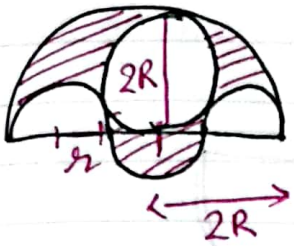
$= AB \times BC - (\pi r^2 \times 2 + PQ \times QR)$

$= 14 \times 14 - (\pi \times 4 \times 2 + 4 \times 4)$

$= 196 - (8\pi + 16) = 196 - 16 - 8\pi = (180 - 8\pi) cm^2$

(c)

3)



$r = \frac{3}{2}$ cm

$2R = 4.5\text{cm} = \frac{9}{2}$ cm

$R = \frac{9}{4}$ cm

Area of shaded region = Area of big semicircle − area of circle − area of 2 small semicircles + area of 1 small semicircle

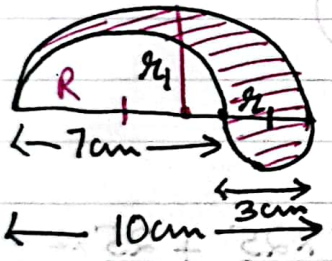
$= \frac{\pi(2R)^2}{2} - \pi R^2 - 2\times\frac{\pi r^2}{2} + \frac{\pi r^2}{2}$

$= 2\pi R^2 - \pi R^2 - \frac{\pi r^2}{2} = \pi R^2 - \frac{\pi r^2}{2}$

$= \pi\left(R^2 - \frac{r^2}{2}\right) = \pi\left(\left(\frac{9}{4}\right)^2 - \left(\frac{3}{2}\right)^2 \times \frac{1}{2}\right)$

$= \pi\left(\frac{81}{16} - \frac{9\times 2}{8\times 2}\right) = \pi \times \left(\frac{81-18}{16}\right) = \frac{63\pi}{16}\text{ cm}^2$ (d)

4)



$r_1 = \frac{10}{2} = 5$cm

$R = \frac{7}{2}$ cm

$r = \frac{3}{2}$ cm

Perimeter of the shaded region

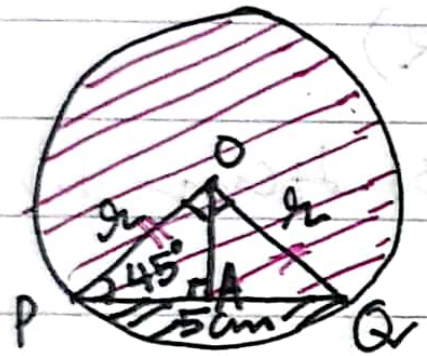
$= \overset{\frown}{PSR} + \overset{\frown}{RTQ} + \overset{\frown}{QAP}$

$= \pi r_1 + \pi r + \pi R = \pi(r_1 + r + R)$

$= \frac{22}{7}\left(5 + \frac{3}{2} + \frac{7}{2}\right) = \frac{22}{7}\times$ $= 31.4\text{cm}$ (b)

Q

5)



In rt. $\triangle OPQ$, $OP = OQ = r$ cm

$\Rightarrow \angle OPQ = \angle OQP = \frac{90^\circ}{2} = 45^\circ$

Draw $OA \perp PQ$.

In rt. $\triangle OPA$, $\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{AP}{OP}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{5}{OP}$

$\Rightarrow OP = r = \frac{5}{2} \times \sqrt{2} = \frac{5}{\sqrt{2}}$ cm

area of minor segment = area of Sector − area($\triangle OPQ$)

$$= \frac{\pi r^2}{4} - \frac{1}{2} r^2$$

$$= \frac{\pi}{4} \times \frac{25}{2} - \frac{1}{2} \times \frac{25}{2} = \frac{25\pi}{8} - \frac{25}{4} \text{ cm}^2$$

Area of major segment = area of circle − area of minor segment

$$= \pi r^2 - \frac{25\pi}{8} + \frac{25}{4}$$

$$= \frac{25\pi^{\times 4}}{2^{\times 4}} - \frac{25\pi}{8} + \frac{25}{4}$$

$$= \frac{100\pi - 25\pi}{8} + \frac{25}{4} = \frac{75\pi}{8} + \frac{25}{4} \text{ cm}^2$$

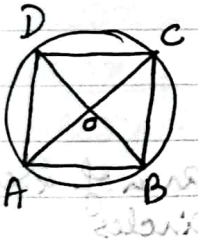
∴ Required difference = area of major segment − area of minor segment

$$= \frac{75\pi}{8} + \frac{25}{4} - \frac{25\pi}{8} + \frac{25}{4}$$

$$= \frac{50\pi}{8_4} + \frac{50^{25}}{4_2}$$

$$= \left(\frac{25\pi}{4} + \frac{25}{2}\right) \text{cm}^2 \quad \text{(a)}$$

6)



Since all the vertices of rhombus lie on circle, then it is a square.

Then AC = BD.

AC and BD pass through the centre of the circle

area of circle = 1256

$$\pi r^2 = 1256$$

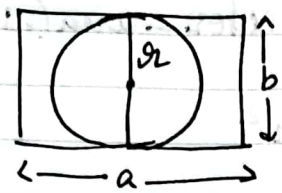
$$r^2 = \frac{1256}{3.14} = 400$$

$$r = 20 \text{ cm}$$

∴ area of rhombus ABCD $= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 2r \times 2r$

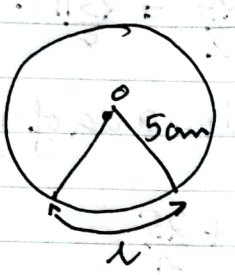
$$= \frac{4}{2} r^2 = 2r^2 = 2 \times 400$$

$$= 800 \text{ cm}^2 \quad \text{(c)}$$

7) 

$r = \frac{b}{2}$ cm

area of circle $= \pi r^2 = \frac{\pi b^2}{4}$ cm² (c)

8) 

area of sector $= \frac{lr}{2} = \frac{3.5 \times 5}{2}$

$= \frac{17.5}{2} = 8.75 \text{cm}^2$ (b)

9) $R = 6$cm

$r = 4$cm

Distance travelled by the tip of long hand in 2 days

$= 2\pi R \times 48 = 2\pi \times 6 \times 48 = 576\pi$ cm

Distance travelled by the tip of short hand in 2 days

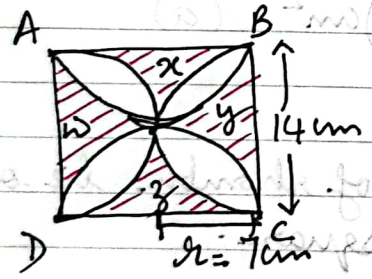
$= 2\pi r \times 4 = 8\pi \times 4 = 32\pi$ cm

∴ Total distance travelled $= 576\pi + 32\pi$

$= 608\pi$

$= \frac{608 \times 22}{7}$

$= \frac{13376}{7} = 1910.85$cm (c)

10) 

area of region $x + z$

= area of Square − area of 2 semi-circles with AD and BC as diameters

$= 14 \times 14 - 2 \times \frac{\pi r^2}{2}$

$= 14 \times 14 - \pi r^2 = 196 - \pi r^2$

Similarly, area of region $w + y$ = area of Square − area of 2 semi-circles on AB and DC

$= 14 \times 14 - \pi r^2 = 196 - \pi r^2$

∴ Total area of shaded region $= 2(196 - \pi r^2)$

$= 392 - 2 \times \frac{22}{7} \times 49 = 392 - 308 = 84 \text{cm}^2$ (a)