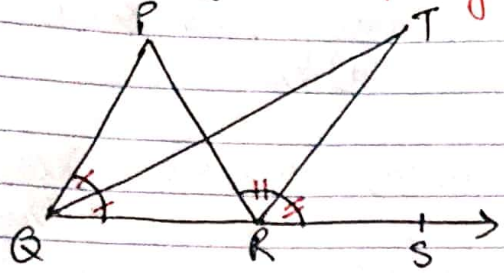
IX

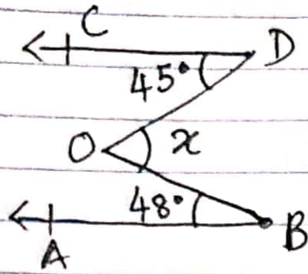
Test-12 (Lines & Angles / Triangles) Time : 1 hr Max. Marks : 20

1) (3)



The side QR of ΔPQR is produced to S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at T, then prove that $\angle QTR = \frac{1}{2}\angle QPR$

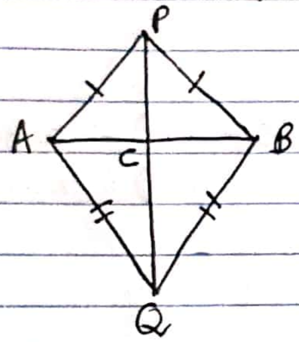
2) (2)



$AB \parallel CD$. Find the value of x.

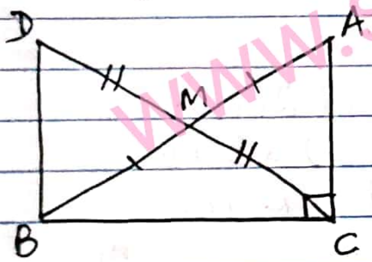
(3) 3) Prove that the sum of angles of a triangle is 180°

4) (3)



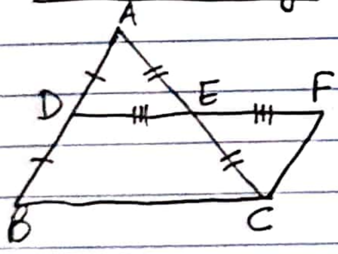
AB is a line segment. P and Q are points on opposite sides of AB such that each of them is equidistant from A and B. Show that PQ is $\perp$ bisector of AB

5) (4)



In right ΔABC, $\angle C = 90°$.
M is the mid-point of AB.
DM = CM.
Show that (i) $\Delta AMC \cong \Delta BMD$
(ii) $\Delta DBC \cong \Delta ACB$.

6) Case-Study (5)



Reena and Manisha Study in class 9, One day they were discussing geometrical shapes together.
For this they did the following:
* drew a ΔABC
* D and E are found as the mid-points of AB and AC respectively.
* FC was joined.

Answer the following questions :

(i) ΔADE and ΔCFE are congruent by which criteria?
(a) SSS (b) RHS (c) SAS (d) ASA

(ii) $\angle EFC$ is equal to:

(a) $\angle DAE$ (b) $\angle ADE$ (c) $\angle AED$ (d) $\angle B$

(iii) $\angle ECF$ is equal to :

(a) $\angle DAE$ (b) $\angle ADE$ (c) $\angle AED$ (d) $\angle B$

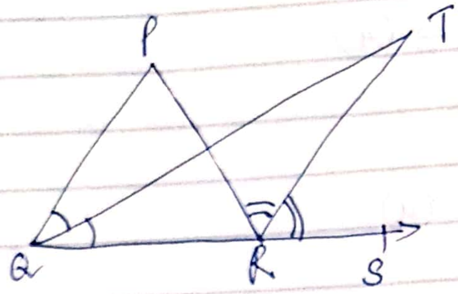
(iv) CF is equal to:

(a) BD (b) CE (c) AD (d) EF

(v) CF is parallel to which of the following ?

(a) AB (b) CE (c) BD (d) EF

IX Test-12 (Answers)

1) 

Given: in ΔPQR,
QT bisects $\angle PQR$
RT bisects $\angle PRS$
QT and RT meet at T.
To prove : $\angle QTR = \frac{1}{2}\angle QPR$

Proof:- Using exterior angle property in ΔPQR,

$\angle PQR + \angle QPR = \angle PRS$

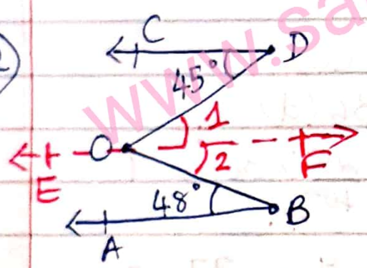
$\Rightarrow \frac{1}{2}\angle PQR + \frac{1}{2}\angle QPR = \frac{1}{2}\angle PRS$ [$\because$ QT and RT are angle bisectors]

$\Rightarrow \angle TQR + \frac{1}{2}\angle QPR = \angle TRS \rightarrow (1)$

Similarly, in ΔTQR, $\angle TQR + \angle QTR = \angle TRS \rightarrow (2)$

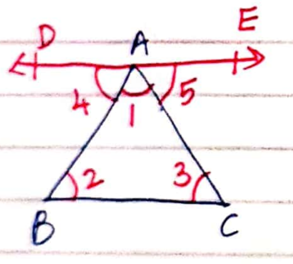
From (1) and (2), $\cancel{\angle TQR} + \frac{1}{2}\angle QPR = \cancel{\angle TQR} + \angle QTR$

$\therefore \angle QTR = \frac{1}{2}\angle QPR$, Hence Proved

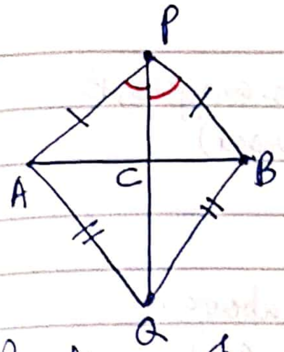
2) 

Construction: draw EF || AB || CD.
Since CD || EF and OD is the transversal,
$\angle 1 = 45°$ (alternate interior angles)
Since EF || AB and OB is the transversal
$\angle 2 = 48°$ (alternate interior angles)

$\therefore x = \angle 1 + \angle 2 = 45° + 48° = \underline{93°}$

3) 

Given : a ΔABC
To prove : $\angle A + \angle B + \angle C = 180°$
Construction : draw DE || BC through A.
Proof :- Since DE || BC and AB is the transversal, $\angle 2 = \angle 4$ (alternate interior angles) $\rightarrow$ (1)
Similarly, since DE || BC and AC is the transversal,
$\angle 3 = \angle 5$ (alternate interior angles) $\rightarrow$ (2)
Also, $\angle 4 + \angle 1 + \angle 5 = 180°$ (angles on a straight line)
$\Rightarrow \angle 2 + \angle 1 + \angle 3 = 180°$ (from eq: (1) and (2))
$\Rightarrow \angle 1 + \angle 2 + \angle 3 = 180°$
Hence Proved.

4) 

Given: $AP = PB$

$AQ = QB$

To prove: PQ is the perpendicular bisector of AB.

i.e., $AC = BC$ and $\angle ACP = 90^\circ$.

Proof:- In $\triangle PAQ$ and $\triangle PBQ$, $AP = BP$ (given)

$AQ = QB$ (given)

$PQ = PQ$ (common side)

$\therefore \triangle PAQ \cong \triangle PBQ$ (SSS congruency)

Thus $\angle APQ = \angle BPQ$ (by CPCT)

$\Rightarrow \angle APC = \angle BPC \longrightarrow (1)$

In $\triangle APC$ and $\triangle BPC$, $AP = BP$ (given)

$\angle APC = \angle BPC$ (from eq:(1))

$PC = PC$ (common side)

$\therefore \triangle APC \cong \triangle BPC$ (SAS congruency)

Thus $AC = BC$ } by CPCT

$\angle ACP = \angle BCP$ }

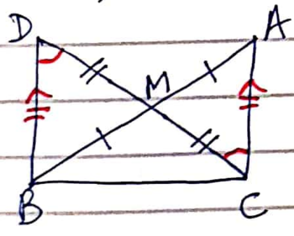
But these angles form linear pair, thus

$\angle ACP + \angle BCP = 180^\circ$

$2\angle ACP = 180^\circ$ [$\because \angle ACP = \angle BCP$]

$\angle ACP = 90^\circ$

Hence, PQ is the perpendicular bisector of AB.

5) 

Given: in right $\triangle ABC$, $\angle C = 90^\circ$

$AM = BM$

$DM = CM$

To prove: (i) $\triangle AMC \cong \triangle BMD$

(ii) $\triangle DBC \cong \triangle ACB$.

Proof:- In $\triangle AMC$ and $\triangle BMD$, $AM = BM$ (given)

$\angle AMC = \angle BMD$ (V.O.A)

$MC = DM$ (given)

$\therefore \triangle AMC \cong \triangle BMD$ (by SAS congruency)

Thus, $\angle MCA = \angle MDB$ } by CPCT

$AC = BD$ }

These angles form a pair of alternate interior angles only when $AC \parallel BD$.

Thus $\angle ACB + \angle DBC = 180°$ (co-interior angles, BC is the transversal)

$$90° + \angle DBC = 180°$$
$$\therefore \angle DBC = 90°$$

In $\triangle DBC$ and $\triangle ACB$, $DB = AC$ (proved above)

$\angle DBC = \angle ACB$ (each $90°$)

$BC = BC$ (common side)

$\therefore \triangle DBC \cong \triangle ACB$ (SAS congruency)

Hence Proved.

6) 

In $\triangle ADE$ and $\triangle CFE$, $AE = EC$ (given)

$\angle AED = \angle CEF$ (VOA)

$DE = FE$

$\therefore \triangle ADE \cong \triangle CFE$

(i) SAS (c)
(ii) $\angle ADE$ (b)
(iii) $\angle DAE$ (a)
(iv) AD (c)
(v) AB (a)