

X Elite work - 10 (TRIGONOMETRY)

- 1) $(\sec\theta + \tan\theta)(1 - \sin\theta) =$ (a) $\sec\theta$ (b) $\sin\theta$ (c) $\operatorname{cosec}\theta$ (d) $\cos\theta$
- 2) $2\cos^2\theta + \frac{2}{1+\cot^2\theta} =$ (a) 1 (b) 2 (c) 0 (d) $\frac{1}{2}$
- 3) Simplified form of $\frac{3 - \tan\theta}{3\operatorname{cosec}\theta - \sec\theta}$ is (a) $\cos\theta$ (b) $\sin\theta$ (c) $\operatorname{cosec}\theta$ (d) $\tan\theta$
- 4) $\frac{2\sin^2 63^\circ + 1 + 2\sin^2 27^\circ}{3\cos^2 17^\circ - 2 + 3\cos^2 73^\circ} =$ (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 0 (d) 3
- 5) $\cos(40^\circ + \theta) - \sin(50^\circ - \theta) + \frac{\cos^2 40^\circ + \cos^2 50^\circ}{\sin^2 40^\circ + \sin^2 50^\circ} =$
(a) $\frac{1}{2}$ (b) 1 (c) 0 (d) none of these
- 6) If $\sqrt{3}\tan\theta = 3\sin\theta$, then $\sin^2\theta - \cos^2\theta =$
(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{3}$ (d) $\frac{1}{\sqrt{3}}$
- 7) $\frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta} =$ (a) $2\sin\theta$ (b) $2\cos\theta$ (c) $2\tan\theta$ (d) $2\operatorname{cosec}\theta$
- 8) If $\sin\theta = \frac{a}{\sqrt{a^2 + b^2}}$, then $\tan\theta =$
(a) $\frac{b}{a}$ (b) $\frac{a}{b}$ (c) $-\frac{a}{b}$ (d) $-\frac{b}{a}$
- 9) $\frac{\cos\theta}{1 - \sin\theta} + \frac{\cos\theta}{1 + \sin\theta} =$ (a) $2\sin\theta$ (b) $2\cos\theta$ (c) $2\operatorname{cosec}\theta$ (d) $2\sec\theta$
- 10) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} =$ (a) $\cot\theta$ (b) $\tan\theta$ (c) $\frac{1}{\sin\theta}$ (d) $\sec\theta$

Elite work - 10 (TRIGONOMETRY - Answers)

$$1) (\sec \theta + \tan \theta)(1 - \sin \theta) \quad [\sec \theta = \frac{1}{\cos \theta}; \tan \theta = \frac{\sin \theta}{\cos \theta}]$$

$$= \left(\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \right) (1 - \sin \theta) = \frac{(1 + \sin \theta)(1 - \sin \theta)}{\cos \theta} = \frac{1 - \sin^2 \theta}{\cos \theta}$$

$$= \frac{\cos^2 \theta}{\cos \theta} \quad [\cos^2 \theta = 1 - \sin^2 \theta]$$

$$= \cos \theta \quad (d)$$

$$2) \frac{2 \cos^2 \theta + 2}{1 + \cot^2 \theta} = \frac{2 \cos^2 \theta + 2}{\operatorname{cosec}^2 \theta} = 2 \cos^2 \theta + 2 \sin^2 \theta$$

$$= 2(\sin^2 \theta + \cos^2 \theta) \quad [1 + \cot^2 \theta = \operatorname{cosec}^2 \theta; \sin \theta = \frac{1}{\operatorname{cosec} \theta}]$$

$$= 2(b) \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$3) \frac{3 - \tan \theta}{3 \operatorname{cosec} \theta - \sec \theta} = \frac{3 - \frac{\sin \theta}{\cos \theta}}{\frac{3}{\sin \theta} - \frac{1}{\cos \theta}} = \frac{3 \cos \theta - \sin \theta}{\frac{3 \cos \theta - \sin \theta}{\sin \theta \cos \theta}}$$

$$= \frac{3 \cos \theta - \sin \theta}{\cos \theta} \times \frac{\sin \theta \cos \theta}{3 \cos \theta - \sin \theta} = \sin \theta \quad (b)$$

$$4) \frac{2 \sin^2 63^\circ + 1 + 2 \sin^2 27^\circ}{3 \cos^2 17^\circ - 2 + 3 \cos^2 73^\circ} = \frac{2 \sin^2 63^\circ + 1 + 2 \cos^2 (90^\circ - 27^\circ)}{3 \sin^2 (90^\circ - 17^\circ) - 2 + 3 \cos^2 73^\circ}$$

$$= \frac{2 \sin^2 63^\circ + 2 \cos^2 63^\circ + 1}{2 \sin^2 73^\circ + 3 \cos^2 73^\circ - 2}$$

$$[\sin(90^\circ - \theta) = \cos \theta; \cos(90^\circ - \theta) = \sin \theta]$$

$$= \frac{2(\sin^2 63^\circ + \cos^2 63^\circ) + 1}{3(\sin^2 73^\circ + \cos^2 73^\circ) - 2}$$

$$= \frac{2 + 1}{3 - 2} \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= 3 \quad (d)$$

$$5) \frac{\cos(40^\circ + \theta) - \sin(50^\circ - \theta) + \cos^2 40^\circ + \cos^2 50^\circ}{\sin^2 40^\circ + \sin^2 50^\circ}$$

$$= \frac{\sin(90^\circ - (40^\circ + \theta)) - \sin(50^\circ - \theta) + \sin^2(90^\circ - 40^\circ) + \cos^2 50^\circ}{\sin^2 40^\circ + \cos^2(90^\circ - 50^\circ)}$$

$$= \frac{\sin(90^\circ - 40^\circ - \theta) - \sin(50^\circ - \theta) + \sin^2 50^\circ + \cos^2 50^\circ}{\sin^2 40^\circ + \cos^2 40^\circ}$$

$$= \frac{\sin(50^\circ - \theta) - \sin(50^\circ - \theta) + 1}{1} \quad [\sin(90^\circ - \theta) = \cos \theta; \sin^2 \theta + \cos^2 \theta = 1]$$

$$= 1 \quad (b)$$

$$6) \sqrt{3} \tan \theta = 3 \sin \theta$$

$$\sqrt{3} \times \frac{\sin \theta}{\cos \theta} = 3 \sin \theta \quad \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$\cos \theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\therefore \sin^2 \theta - \cos^2 \theta = 1 - \cos^2 \theta - \cos^2 \theta \quad \left[\because \sin^2 \theta = 1 - \cos^2 \theta \right]$$

$$= 1 - 2 \cos^2 \theta = 1 - 2 \times \frac{1}{3} = \frac{3-2}{3} = \frac{1}{3} \quad (c)$$

$$7) \frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{\sin^2 \theta + (1 + \cos \theta)^2}{\sin \theta (1 + \cos \theta)}$$

$$= \frac{\sin^2 \theta + 1 + \cos^2 \theta + 2 \cos \theta}{\sin \theta (1 + \cos \theta)}$$

$$= \frac{1 + 1 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)}$$

$$= \frac{2 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)} = \frac{2(1 + \cos \theta)}{\sin \theta (1 + \cos \theta)}$$

$$= \frac{2}{\sin \theta} = 2 \operatorname{cosec} \theta \quad (d)$$

$$8) \sin \theta = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\begin{aligned}\cos^2 \theta &= 1 - \sin^2 \theta \\ &= 1 - \frac{a^2}{a^2 + b^2} \\ &= \frac{a^2 + b^2 - a^2}{a^2 + b^2} = \frac{b^2}{a^2 + b^2}\end{aligned}$$

$$\therefore \cos \theta = \frac{b}{\sqrt{a^2 + b^2}}$$

$$\begin{aligned}\text{Then, } \tan \theta &= \frac{\sin \theta}{\cos \theta} = \frac{a}{\sqrt{a^2 + b^2}} \times \frac{\sqrt{a^2 + b^2}}{b} \\ &= \frac{a}{b} \quad (b)\end{aligned}$$

$$\begin{aligned}9) \quad \frac{\cos \theta}{1 - \sin \theta} + \frac{\cos \theta}{1 + \sin \theta} &= \cos \theta \left[\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} \right] \\ &= \cos \theta \left[\frac{1 + \sin \theta + 1 - \sin \theta}{1 - \sin^2 \theta} \right] \\ &= \cos \theta \times \frac{2}{\cos^2 \theta} \quad [\because \cos^2 \theta = 1 - \sin^2 \theta] \\ &= \frac{2}{\cos \theta}\end{aligned}$$

$$\begin{aligned}10) \quad \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} &= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)} \\ &= \frac{\tan \theta (1 - 2 \sin^2 \theta)}{(2(1 - \sin^2 \theta) - 1)} \quad [\because \tan \theta = \frac{\sin \theta}{\cos \theta}] \\ &= \frac{\tan \theta (1 - 2 \sin^2 \theta)}{2 - 2 \sin^2 \theta - 1} \quad [\because \cos^2 \theta = 1 - \sin^2 \theta] \\ &= \frac{\tan \theta (1 - 2 \sin^2 \theta)}{1 - 2 \sin^2 \theta} = \tan \theta \quad (b)\end{aligned}$$