X

ELITE WORK – 9

1) If $\tan\theta + \cot\theta = 2$, then $\tan^{100}\theta + \cot^{100}\theta =$ ___

(a) 100 (b) $\frac{1}{100}$ (c) -2 (d) 2

2) If $\sin\theta + \cos\theta = \sqrt{2}\sin(90^\circ - \theta)$, then $\cot\theta =$ ___

(a) $\sqrt{2}$ (b) $\sqrt{2}-1$ (c) $\sqrt{2}+1$ (d) $\frac{1}{\sqrt{2}}$

3) If $a\cos\theta - b\sin\theta = c$, then $a\sin\theta + b\cos\theta =$

(a) $\pm\sqrt{a^2+b^2+c^2}$ (b) $\pm\sqrt{a^2-b^2+c^2}$ (c) $\pm\sqrt{a^2-b^2-c^2}$

(d) $\pm\sqrt{a^2+b^2-c^2}$

4) $\frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\operatorname{cosec} 40^\circ}{\sec 50^\circ} - 4\cos 50^\circ \cdot \operatorname{cosec} 40^\circ =$

(a) 1 (b) 2 (c) -2 (d) 0

5) If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then $\cos\theta - \sin\theta =$

(a) $\sqrt{2}\tan\theta$ (b) $\sqrt{2}\cot\theta$ (c) $\frac{1}{\sqrt{2}}\sin\theta$ (d) $\sqrt{2}\sin\theta$

6) If $\sec\theta + \tan\theta = p$, then $\frac{p^2-1}{p^2+1}$

(a) $\tan\theta$ (b) $\cos\theta$ (c) $\sin\theta$ (d) $\operatorname{cosec}\theta$

7) $\sin^6\theta + \cos^6\theta + 3\sin^2\theta \cdot \cos^2\theta =$

(a) 0 (b) 1 (c) 2 (d) -2

8) If $\sin\theta + \cos\theta = \sqrt{3}$, then $\tan\theta + \cot\theta =$

(a) 1 (b) -1 (c) 2 (d) -2

9) If $\tan A = \frac{3}{4}$, then $\sin A \cdot \cos A =$

(a) $\frac{11}{25}$ (b) $\frac{13}{25}$ (c) $\frac{12}{25}$ (d) $\frac{17}{25}$

10) If $\sqrt{3}\tan\theta = 1$, then $\sin^2\theta - \cos^2\theta =$

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) $\frac{5}{2}$

Elite work - 9 (Answers) TRIGONOMETRY

1) $\tan\theta + \cot\theta = 2$

$\Rightarrow \tan\theta + \frac{1}{\tan\theta} = 2$ $\left[\cot\theta = \frac{1}{\tan\theta}\right]$

$\Rightarrow \tan^2\theta + 1 = 2\tan\theta$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow (\tan\theta - 1)^2 = 0$

$\Rightarrow \tan\theta - 1 = 0$

$\therefore \tan\theta = 1$

$\theta = 45^\circ$ $[\because \tan 45^\circ = 1]$

Thus, $\tan^{100}\theta + \cot^{100}\theta$

$= (\tan\theta)^{100} + (\cot\theta)^{100}$

$= (\tan 45^\circ)^{100} + (\cot 45^\circ)^{100}$

$= 1^{100} + 1^{100}$ $[\because \tan 45^\circ = \cot 45^\circ = 1]$

$= 1 + 1$

$= 2$ (d)

2) $\sin\theta + \cos\theta = \sqrt{2}\sin(90^\circ - \theta)$

$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$\Rightarrow \sin\theta = \cos\theta(\sqrt{2} - 1)$

$\Rightarrow \frac{1}{\sqrt{2}-1} = \frac{\cos\theta}{\sin\theta}$

$\Rightarrow \cot\theta = \frac{1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{\sqrt{2}+1}{2-1} = \sqrt{2}+1$ (c)

3) $a\cos\theta - b\sin\theta = c$

$\Rightarrow (a\cos\theta - b\sin\theta)^2 = c^2$

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta = c^2$

$\Rightarrow \underline{2ab\sin\theta\cos\theta} = a^2\cos^2\theta + b^2\sin^2\theta - c^2$

$\therefore (a\sin\theta + b\cos\theta)^2 = a^2\sin^2\theta + b^2\cos^2\theta + \underline{2ab\sin\theta\cos\theta}$

$= a^2\sin^2\theta + b^2\cos^2\theta + a^2\cos^2\theta + b^2\sin^2\theta - c^2$

$= a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta) - c^2$

$= a^2 + b^2 - c^2$ $[\because \sin^2\theta + \cos^2\theta = 1]$

Thus, $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$ (d)

4) $\frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\operatorname{cosec} 40^\circ}{\sec 50^\circ} - 4\cos 50^\circ . \operatorname{cosec} 40^\circ$

$= \frac{\cos(90^\circ - 50^\circ)}{\cos 40^\circ} + \frac{\sec(90^\circ - 40^\circ)}{\sec 50^\circ} - 4\cos 50^\circ . \sec(90^\circ - 40^\circ)$

$= \frac{\cancel{\cos 40^\circ}}{\cancel{\cos 40^\circ}} + \frac{\cancel{\sec 50^\circ}}{\cancel{\sec 50^\circ}} - 4\cos 50^\circ . \sec 50^\circ$ $[\because \cos(90^\circ - \theta) = \sin\theta$, $\sec(90^\circ - \theta) = \operatorname{cosec}\theta]$

$= 1 + 1 - 4 \times 1$ $[\cos\theta . \sec\theta = 1]$

$= 2 - 4$

$= -2$ (c)

5) $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\Rightarrow (\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 2\cos^2\theta$

$2\sin\theta\cos\theta = 2\cos^2\theta - 1 \rightarrow (1)$

Thus, $(\cos\theta - \sin\theta)^2 = \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta$

$= 1 - (2\cos^2\theta - 1)$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= 1 - 2\cos^2\theta + 1$

$= 2 - 2\cos^2\theta$

$= 2(1 - \cos^2\theta)$

$= 2\sin^2\theta$ $[\because \sin^2\theta = 1 - \cos^2\theta]$

$\therefore \cos\theta - \sin\theta = \sqrt{2}\sin\theta$ (d)

6) $\sec\theta + \tan\theta = p \rightarrow (1)$

$\frac{1}{\sec\theta + \tan\theta} = \frac{1}{p}$

$\Rightarrow \frac{\sec^2\theta - \tan^2\theta}{\sec\theta + \tan\theta} = \frac{1}{p}$

$\Rightarrow \frac{\cancel{(\sec\theta + \tan\theta)}(\sec\theta - \tan\theta)}{\cancel{\sec\theta + \tan\theta}} = \frac{1}{p}$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p} \rightarrow (2)$

(1) − (2), $2\tan\theta = p - \frac{1}{p} = \frac{p^2 - 1}{p} \rightarrow (3)$

(1) + (2), $2\sec\theta = p + \frac{1}{p} = \frac{p^2 + 1}{p} \rightarrow (4)$

(3)
(4) , $\frac{2\tan\theta}{2\sec\theta} = \frac{\frac{p^2-1}{p}}{\frac{p^2+1}{p}}$

$\Rightarrow \frac{\sin\theta}{\cancel{\cos\theta}} \times \cancel{\cos\theta} = \frac{p^2-1}{p^2+1}$

$\therefore \frac{p^2-1}{p^2+1} = \sin\theta$ (c)

7) $\sin^6\theta + \cos^6\theta + 3\sin^2\theta.\cos^2\theta$ $[a^3+b^3 = (a+b)^3 - 3ab(a+b)]$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta.\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta.\cos^2\theta(\sin^2\theta + \cos^2\theta) + 3\sin^2\theta\cos^2\theta$

$= 1 - \cancel{3\sin^2\theta\cos^2\theta} + \cancel{3\sin^2\theta\cos^2\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= 1$ (b)

8) $\sin\theta + \cos\theta = \sqrt{3}$

$\Rightarrow (\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 3$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$2\sin\theta\cos\theta = 2$

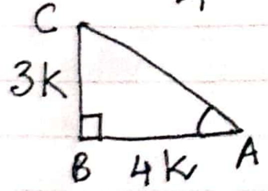
$\sin\theta\cos\theta = 1 \rightarrow (1)$

Thus, $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta} = \frac{1}{1} = 1$ (a)

9) $\tan A = \frac{3}{4} = \frac{3k}{4k}$



Using Pythagoras Theorem, $AC = 5k$

$\sin A = \frac{3k}{5k} = \frac{3}{5}$

$\cos A = \frac{4k}{5k} = \frac{4}{5}$

$\therefore \sin A.\cos A = \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$ (c)

10) $\sqrt{3}\tan\theta = 1$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\therefore \theta = 30^\circ$

Thus, $\sin^2\theta - \cos^2\theta = \sin^2 30^\circ - \cos^2 30^\circ$

$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{4} - \frac{3}{4}$

$= -\frac{2}{4} = -\frac{1}{2}$ (a)