Goyal

X Homework - 15 (TRIGONOMETRY)

1) If $\tan A = n \tan B$ and $\sin A = m \sin B$, Prove that $\cos^2 A = \frac{m^2-1}{n^2-1}$

2) If $\operatorname{cosec}\theta = x + \frac{1}{4x}$, Prove that $\operatorname{cosec}\theta + \cot\theta = 2x$ or $\frac{1}{2x}$

3) If $\operatorname{cosec}\theta - \sin\theta = l$ and $\sec\theta - \cos\theta = m$, show that $l^2m^2(l^2+m^2+3) = 1$

4) Prove that $\frac{\sin^2 A - \sin^2 B}{\cos^2 A \cdot \cos^2 B} = \tan^2 A - \tan^2 B$

5) Prove that $\sin^8\theta - \cos^8\theta = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$

6) If $\cot\theta + \tan\theta = x$ and $\sec\theta - \cos\theta = y$, Prove that $(x^2y)^{\frac{2}{3}} - (xy^2)^{\frac{2}{3}} = 1$

7) If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$

8) If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, then Show that $(m^2-n^2)^2 = 16mn$

9) If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

10) If $x = r\sin\alpha\cos\beta$, $y = r\sin\alpha\sin\beta$ and $z = r\cos\alpha$; prove that $r^2 = x^2+y^2+z^2$

MCQs

11) If $x = a\cos^3\theta$, $y = b\sin^3\theta$, then $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} =$ ____

(a) $1/3$ (b) $2/3$ (c) $3/2$ (d) 1

12) $\sec^4\theta(1 - \sin^2\theta) - \tan^2\theta =$ ____

(a) 0 (b) 1 (c) -1 (d) none of these

13) If $\operatorname{cosec}\theta - \sin\theta = a$, $\sec\theta - \cos\theta = b$, then $a^2b^2(a^2+b^2+3)$

(a) 3 (b) -3 (c) 1 (d) -1

14) $5\tan^2\theta - 5\sec^2\theta =$

(a) -5 (b) 5 (c) 0 (d) 1

15) If $\cot\theta = \frac{15}{8}$, then $\frac{(3+3\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(3-3\cos\theta)} =$

(a) $\frac{25}{64}$ (b) $\frac{64}{225}$ (c) $\frac{225}{64}$ (d) $-\frac{225}{64}$

16) If $\operatorname{cosec}\theta + \cot\theta = p$, then $\frac{p^2-1}{p^2+1} =$

(a) $\sin\theta$ (b) $\tan\theta$ (c) $\operatorname{cosec}\theta$ (d) $\cos\theta$

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17) The value of $(\sin 30^\circ + \cos 30^\circ) - (\sin 60^\circ + \cos 60^\circ)$ is
(a) -1 (b) 0 (c) 1 (d) 2

18) The value of $\frac{\tan 30^\circ}{\cot 60^\circ}$ is
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) 1

19) The value of $\sin 45^\circ + \cos 45^\circ$ is
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

20) $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} =$
(a) $2\sin\theta$ (b) $2\operatorname{cosec}\theta$ (c) $2\tan\theta$ (d) $2\sec\theta$

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X Homework - 15 (TRIGONOMETRY - answers)

1) $\tan A = n \tan B$

$\Rightarrow \tan B = \dfrac{\tan A}{n}$

$\Rightarrow \cot B = \dfrac{n}{\tan A} \longrightarrow (1)$

$\sin A = m \sin B$

$\Rightarrow \sin B = \dfrac{\sin A}{m}$

$\Rightarrow \operatorname{cosec} B = \dfrac{m}{\sin A} \rightarrow (2)$

We know that $\operatorname{cosec}^2 B - \cot^2 B = 1$

$\Rightarrow \dfrac{m^2}{\sin^2 A} - \dfrac{n^2}{\tan^2 A} = 1$

$\Rightarrow \dfrac{m^2}{\sin^2 A} - \dfrac{n^2}{\sin^2 A} \times \cos^2 A = 1$

$\Rightarrow m^2 - n^2\cos^2 A = \sin^2 A$

$\Rightarrow m^2 - n^2\cos^2 A = 1 - \cos^2 A$

$\Rightarrow \cos^2 A - n^2\cos^2 A = 1 - m^2$

$\Rightarrow \cos^2 A (1 - n^2) = 1 - m^2$

$\therefore \cos^2 A = \dfrac{1-m^2}{1-n^2} = \dfrac{m^2-1}{n^2-1}$

2) $\operatorname{cosec}\theta = x + \dfrac{1}{4x}$

$\cot^2\theta = \operatorname{cosec}^2\theta - 1$

$= \left(x + \dfrac{1}{4x}\right)^2 - 1$

$= x^2 + \dfrac{1}{16x^2} + \dfrac{1}{2} - 1$

$= x^2 + \dfrac{1}{16x^2} - \dfrac{1}{2} = \left(x - \dfrac{1}{4x}\right)^2$

$\therefore \cot\theta = \pm\left(x - \dfrac{1}{4x}\right)$

$\operatorname{cosec}\theta + \cot\theta = x + \dfrac{1}{4x} + x - \dfrac{1}{4x} = \underline{\underline{2x}}$

(Or) $\operatorname{cosec}\theta + \cot\theta = x + \dfrac{1}{4x} - x + \dfrac{1}{4x} = \underline{\underline{\dfrac{1}{2x}}}$

3) $\operatorname{cosec}\theta - \sin\theta = l$

$\Rightarrow \dfrac{1}{\sin\theta} - \sin\theta = l$

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$$\Rightarrow \frac{1-\sin^2\theta}{\sin\theta} = l$$

$$\Rightarrow \frac{\cos^2\theta}{\sin\theta} = l \longrightarrow (1)$$

Also, $\sec\theta - \cos\theta = m$

$$\Rightarrow \frac{1}{\cos\theta} - \cos\theta = m$$

$$\Rightarrow \frac{1-\cos^2\theta}{\cos\theta} = m$$

$$\Rightarrow \frac{\sin^2\theta}{\cos\theta} = m \longrightarrow (2)$$

$$\therefore l^2m^2(l^2+m^2+3) = \frac{\cos^{\cancel{4}\,2}\theta}{\cancel{\sin^2\theta}} \times \frac{\sin^{\cancel{4}\,2}\theta}{\cancel{\cos^2\theta}}\left(\frac{\cos^4\theta}{\sin^2\theta} + \frac{\sin^4\theta}{\cos^2\theta} + 3\right)$$

$$= \cancel{\cos^2\theta} \times \cancel{\sin^2\theta}\left(\frac{\cos^6\theta + \sin^6\theta + 3\sin^2\theta\cos^2\theta}{\cancel{\sin^2\theta}\,\cancel{\cos^2\theta}}\right)$$

$$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta) + 3\sin^2\theta\cos^2\theta$$

$[a^3+b^3 = (a+b)^3 - 3ab(a+b)]$

$$= 1 - 3\sin^2\theta\cos^2\theta \times 1 + 3\sin^2\theta\cos^2\theta \quad [\because \sin^2\theta+\cos^2\theta=1]$$

$= \underline{\underline{1}}$, RHS

4) LHS, $\frac{\sin^2A - \sin^2B}{\cos^2A \cdot \cos^2B} = \frac{\cancel{1} - \cos^2A - \cancel{1} + \cos^2B}{\cos^2A \cdot \cos^2B}$ $[\because \sin^2\theta = 1-\cos^2\theta]$

$$= \frac{\cos^2B - \cos^2A}{\cos^2A \cdot \cos^2B} = \frac{\cancel{\cos^2B}}{\cos^2A \cdot \cancel{\cos^2B}} - \frac{\cancel{\cos^2A}}{\cancel{\cos^2A} \cdot \cos^2B}$$

$$= \frac{1}{\cos^2A} - \frac{1}{\cos^2B} = \sec^2A - \sec^2B \quad [\because \sec\theta = 1/\cos\theta]$$

$$= \cancel{1} + \tan^2A - \cancel{1} - \tan^2B \quad [\because \sec^2\theta = 1+\tan^2\theta]$$

$= \underline{\underline{\tan^2A - \tan^2B}}$; RHS

5) LHS, $\sin^8\theta - \cos^8\theta = (\sin^4\theta)^2 - (\cos^4\theta)^2$

$a^2 - b^2 = (a+b)(a-b)$

$$= (\sin^4\theta + \cos^4\theta)(\sin^4\theta - \cos^4\theta)$$

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$a^2+b^2 = (a+b)^2 - 2ab$

$= \left[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta\right](\sin^2\theta+\cos^2\theta)(\sin^2\theta-\cos^2\theta)$

$= (1-2\sin^2\theta\cos^2\theta)(\sin^2\theta-\cos^2\theta)$, RHS $[\because \sin^2\theta+\cos^2\theta=1]$

6) $\cot\theta + \tan\theta = x$

$\Rightarrow \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = x$ $\left[\because \cot\theta = \cos\theta/\sin\theta ; \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$\Rightarrow \frac{\cos^2\theta + \sin^2\theta}{\sin\theta . \cos\theta} = x$

$\Rightarrow x = \frac{1}{\sin\theta\cos\theta}$ $[\because \sin^2\theta+\cos^2\theta=1]$

$\sec\theta - \cos\theta = y$

$\Rightarrow \frac{1}{\cos\theta} - \cos\theta = y$ $[\because \sec\theta = 1/\cos\theta]$

$\Rightarrow \frac{1-\cos^2\theta}{\cos\theta} = y$

$\Rightarrow y = \frac{\sin^2\theta}{\cos\theta}$ $[\because \sin^2\theta = 1-\cos^2\theta]$

$\therefore x^2y = \frac{1}{\cancel{\sin^2\theta}\cos^2\theta} \times \frac{\cancel{\sin^2\theta}}{\cos\theta} = \frac{1}{\cos^3\theta} = \sec^3\theta$

$xy^2 = \frac{1}{\cancel{\sin\theta}\cos\theta} \times \frac{\sin^{\cancel{4}\,3}\theta}{\cos^2\theta} = \frac{\sin^3\theta}{\cos^3\theta} = \tan^3\theta$

$\therefore (x^2y)^{\frac{2}{3}} - (xy^2)^{\frac{2}{3}}$

$= (\sec^3\theta)^{\frac{2}{3}} - (\tan^3\theta)^{\frac{2}{3}}$

$= \sec^2\theta - \tan^2\theta = 1$, RHS

7) $m^2 - n^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$= \cancel{\tan^2\theta} + \cancel{\sin^2\theta} + 2\tan\theta\sin\theta$

$- \cancel{\tan^2\theta} - \cancel{\sin^2\theta} + 2\tan\theta\sin\theta$

$= 4\tan\theta\sin\theta$

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$$= 4\sqrt{\tan^2\theta \sin^2\theta}$$

$$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta}(1-\cos^2\theta)} \quad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}, \sin^2\theta = 1-\cos^2\theta\right]$$

$$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$$

$$= 4\sqrt{\tan^2\theta - \sin^2\theta}$$

$$= 4\sqrt{(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)}$$

$$= 4\sqrt{m\times n} = 4\sqrt{mn}, \text{ RHS}$$

8) LHS, $m^2-n^2 = (\tan A+\sin A)^2-(\tan A-\sin A)^2$

$$= \cancel{\tan^2 A}+\cancel{\sin^2 A}+2\tan A\sin A - \cancel{\tan^2 A} - \cancel{\sin^2 A} + 2\tan A \sin A$$

$$= 4\tan A\sin A$$

$$\therefore (m^2-n^2)^2 = 16\tan^2 A\sin^2 A$$

$$= 16\times\frac{\sin^2 A}{\cos^2 A}(1-\cos^2 A) \quad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}; \sin^2 A = 1-\cos^2 A\right]$$

$$= 16\left(\frac{\sin^2 A}{\cos^2 A} - \sin^2 A\right)$$

$$= 16(\tan^2 A - \sin^2 A)$$

$$= 16(\tan A+\sin A)(\tan A-\sin A)$$

$$= \underline{16mn}$$

9) Given, $\cos\theta+\sin\theta = \sqrt{2}\cos\theta$

Squaring on both sides, $(\cos\theta+\sin\theta)^2 = 2\cos^2\theta$

$$\Rightarrow \cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta = 2\cos^2\theta$$

$$\Rightarrow 1+2\sin\theta\cos\theta = 2\cos^2\theta \quad [\because \sin^2\theta+\cos^2\theta = 1]$$

$$\therefore 2\sin\theta\cos\theta = 2\cos^2\theta-1 \rightarrow (1)$$

Thus, $(\cos\theta-\sin\theta)^2 = \cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta$

$$= 1-(2\cos^2\theta-1) \quad [\text{from eq:(1)}]$$

$$= 1-2\cos^2\theta+1$$

$$= 2-2\cos^2\theta = 2(1-\cos^2\theta)$$

$$= 2\sin^2\theta$$

$$\therefore \cos\theta-\sin\theta = \sqrt{2}\sin\theta //$$

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10) $x = r\sin\alpha\cos\beta \Rightarrow x^2 = r^2\sin^2\alpha\cos^2\beta$

$y = r\sin\alpha\sin\beta \Rightarrow y^2 = r^2\sin^2\alpha\sin^2\beta$

$z = r\cos\alpha \Rightarrow z^2 = r^2\cos^2\alpha$

$\therefore x^2+y^2+z^2 = r^2\sin^2\alpha(\cos^2\beta+\sin^2\beta)+r^2\cos^2\alpha$

$= r^2(\sin^2\alpha+\cos^2\alpha)$ $[\because \sin^2\theta+\cos^2\theta=1]$

$= \underline{\underline{r^2}}$

MCQs

11) $x = a\cos^3\theta$

$\frac{x}{a} = \cos^3\theta$

$\left(\frac{x}{a}\right)^{\frac{2}{3}} = (\cos^3\theta)^{\frac{2}{3}} = \cos^2\theta$ //

$y = b\sin^3\theta$

$\frac{y}{b} = \sin^3\theta$

$\left(\frac{y}{b}\right)^{\frac{2}{3}} = (\sin^3\theta)^{\frac{2}{3}} = \sin^2\theta$ //

$\therefore \left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = \cos^2\theta+\sin^2\theta = 1$ (d)

12) $\sec^4\theta(1-\sin^2\theta) - \tan^2\theta$

$= \sec^4\theta.\cos^2\theta - \tan^2\theta$

$= \sec^{\cancel{4}\,2}\theta \times \frac{1}{\cancel{\sec^2\theta}} - \tan^2\theta$

$= \sec^2\theta - \tan^2\theta = 1$ (b)

13) $\operatorname{cosec}\theta - \sin\theta = a$

$\Rightarrow \frac{1}{\sin\theta} - \sin\theta = a$

$\Rightarrow \frac{1-\sin^2\theta}{\sin\theta} = a$

$\Rightarrow a = \frac{\cos^2\theta}{\sin\theta}$

$\sec\theta - \cos\theta = b$

$\Rightarrow \frac{1}{\cos\theta} - \cos\theta = b$

$\Rightarrow \frac{1-\cos^2\theta}{\cos\theta} = b$

$\Rightarrow b = \frac{\sin^2\theta}{\cos\theta}$

$a^2b^2(a^2+b^2+3) = \frac{\cos^{\cancel{4}\,2}\theta}{\cancel{\sin^2\theta}} \times \frac{\sin^{\cancel{4}\,2}\theta}{\cancel{\cos^2\theta}}\left(\frac{\cos^4\theta}{\sin^2\theta}+\frac{\sin^4\theta}{\cos^2\theta}+3\right)$

$= \cancel{\cos^2\theta}.\cancel{\sin^2\theta}\left(\frac{\cos^6\theta+\sin^6\theta+3\sin^2\theta\cos^2\theta}{\cancel{\sin^2\theta}\cancel{\cos^2\theta}}\right)$

$[a^3+b^3 = (a+b)^3 - 3ab(a+b)]$

$= (\cos^2\theta+\sin^2\theta)^3 - 3\cos^2\theta\sin^2\theta(\cos^2\theta+\sin^2\theta) + 3\sin^2\theta\cos^2\theta$

$= 1 - \cancel{3\cos^2\theta\sin^2\theta} + \cancel{3\sin^2\theta\cos^2\theta}$

$= 1$ (c)

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14) $5\tan^2\theta - 5\sec^2\theta = 5(\tan^2\theta - \sec^2\theta)$

$= -5(\sec^2\theta - \tan^2\theta)$

$= -5$ (a)

15) $\cot\theta = \frac{15}{8}$

$$\frac{(3+3\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(3-3\cos\theta)} = \frac{\cancel{3}(1+\sin\theta)(1-\sin\theta)}{\cancel{3}(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$$

$$= \left(\frac{15}{8}\right)^2 = \frac{225}{64} \text{ (c)}$$

16) $\csc\theta + \cot\theta = p \rightarrow (1)$

$$\Rightarrow \frac{1}{\csc\theta + \cot\theta} = \frac{1}{p}$$

$$\Rightarrow \frac{\csc^2\theta - \cot^2\theta}{\csc\theta + \cot\theta} = \frac{1}{p}$$

$$\Rightarrow \frac{\cancel{(\csc\theta+\cot\theta)}(\csc\theta - \cot\theta)}{\cancel{\csc\theta + \cot\theta}} = \frac{1}{p}$$

$$\Rightarrow \csc\theta - \cot\theta = \frac{1}{p} \rightarrow (2)$$

$(1)+(2)$, $2\csc\theta = p + \frac{1}{p} = \frac{p^2+1}{p}$

$$\Rightarrow \csc\theta = \frac{p^2+1}{2p} \rightarrow (3)$$

$(1)-(2)$, $2\cot\theta = p - \frac{1}{p} = \frac{p^2-1}{p}$

$$\Rightarrow \cot\theta = \frac{p^2-1}{2p} \rightarrow (4)$$

$\frac{(4)}{(3)}$, $$\frac{\cot\theta}{\csc\theta} = \frac{\frac{p^2-1}{\cancel{2p}}}{\frac{p^2+1}{\cancel{2p}}} = \frac{p^2-1}{p^2+1}$$

$$\therefore \frac{p^2-1}{p^2+1} = \frac{\cos\theta}{\cancel{\sin\theta}} \times \cancel{\sin\theta} = \cos\theta \text{ (d)}$$

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17) $(\sin 30^\circ + \cos 30^\circ) - (\sin 60^\circ + \cos 60^\circ)$

$= \frac{\cancel{1}}{\cancel{2}} + \frac{\cancel{\sqrt{3}}}{\cancel{2}} - \frac{\cancel{\sqrt{3}}}{2} - \frac{\cancel{1}}{2} = \underline{\underline{0}}$ (b)

18) $\frac{\tan 30^\circ}{\cot 60^\circ} = \frac{1}{\sqrt{3}} \times \sqrt{3} = 1$ (d)

19) $\sin 45^\circ + \cos 45^\circ = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \overset{\sqrt{2}}{\cancel{2}} \times \frac{1}{\cancel{\sqrt{2}}_{\,1}}$

$= \sqrt{2}$ (b)

20) $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}}$

$= \sqrt{\frac{(\sec\theta - 1)^2}{\sec^2\theta - 1}} + \sqrt{\frac{(\sec\theta + 1)^2}{\sec^2\theta - 1}}$

$= \sqrt{\frac{(\sec\theta - 1)^2}{\tan^2\theta}} + \sqrt{\frac{(\sec\theta + 1)^2}{\tan^2\theta}}$

$= \frac{\sec\theta - 1}{\tan\theta} + \frac{\sec\theta + 1}{\tan\theta}$

$= \frac{\sec\theta - \cancel{1} + \sec\theta + \cancel{1}}{\tan\theta} = \frac{2\sec\theta}{\tan\theta}$

$= 2 \times \frac{1}{\cancel{\cos\theta}} \times \frac{\cancel{\cos\theta}}{\sin\theta} = 2\operatorname{cosec}\theta$ (b)

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