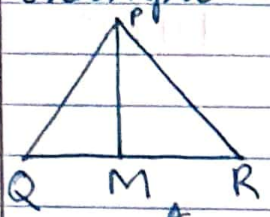
xam

IX Homework-12 (FIRST TERM - TRIANGLES)

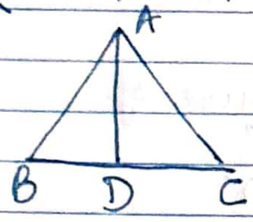
1) Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR. Show that (i) $\Delta ABM \cong \Delta PQN$
(ii) $\Delta ABC \cong \Delta PQR$

2) In ΔPQR, $PQ = PR$ and $\angle Q = 65°$. Then find $\angle R$

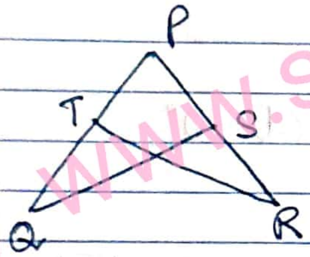
3) If the corresponding angles of two triangles are equal, then they are not always congruent. Justify by giving an example.

4) 

PM is the bisector of $\angle P$ and $PQ = PR$. Show that $\Delta PQM \cong \Delta PRM$.

5) 

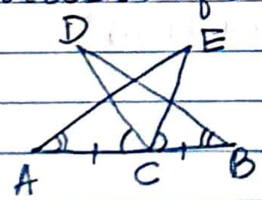
If $AB = AC$ and $BD = DC$, then show that $\angle ADB = 90°$

6) 

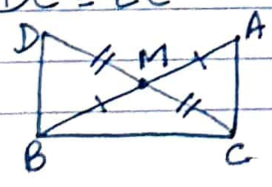
$PQ = PR$ and $\angle Q = \angle R$,
Prove that $\Delta PQS \cong \Delta PRT$

7) ΔABC is an isosceles triangle in which $AB = AC$. Side BA is produced to D such that $AD = AB$. Show that $\angle BCD$ is a right angle.

8) ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, $AB = AC$ and $DB = DC$. Show that AD is the perpendicular bisector of BC.

9) 

$AC = BC$, $\angle DCA = \angle ECB$ and $\angle DBC = \angle EAC$. Prove that triangles DBC and EAC are congruent and hence show that $DC = EC$.

10) 

In right ΔABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to

M and produced to a point D such that DM = CM. Point D is joined to point B. Show that

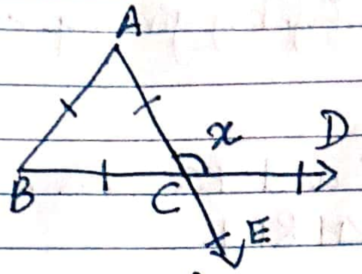
(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle

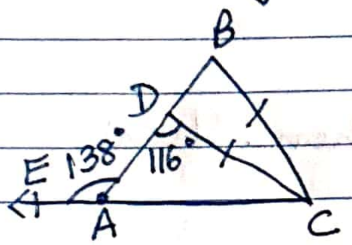
(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2} AB$

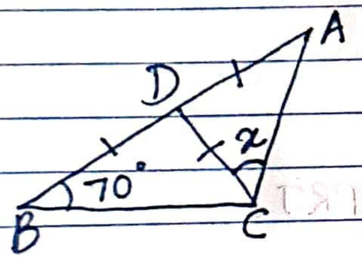
MCQs

1) 

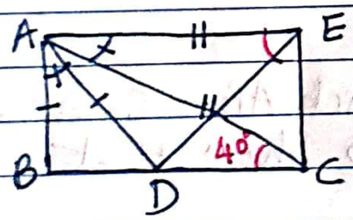
The measure of x is
(a) 80° (b) 100° (c) 120° (d) 140°

2) 

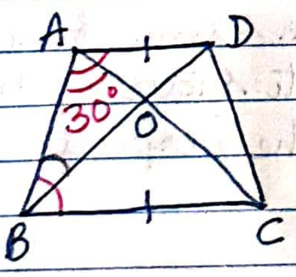
If BC = CD, then the measure of $\angle ACB$ = (a) 74° (b) 78° (c) 58° (d) 50°

3) 

The measure of x =
(a) 36° (b) 68° (c) 20° (d) 80°

4) 

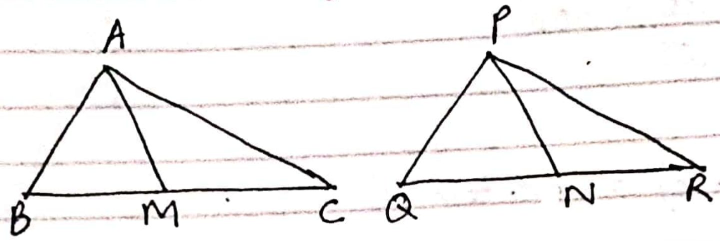
AC = AE, AB = AD and $\angle BAD = \angle EAC$. If $\angle ACB = 40^\circ$, then $\angle AED$ = (a) 35° (b) 40° (c) 45° (d) 50°

5) 

ABCD is a quadrilateral in which AD = BC and $\angle DAB = \angle CBA$. If $\angle CAB = 30^\circ$, then $\angle AOB$ =
(a) 80° (b) 100° (c) 120° (d) 135°

IX Homework-12 (Answers)

1)



Given: in ΔABC and ΔPQR,
AM and PN are medians respectively and $AB = PQ$, $BC = QR$ and $AM = PN$

To prove: (i) $\Delta ABM \cong \Delta PQN$
(ii) $\Delta ABC \cong \Delta PQR$

Proof :- $BC = QR$ (given)

$\Rightarrow \frac{1}{2} BC = \frac{1}{2} QR$

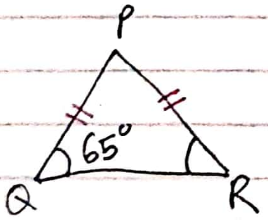
$\Rightarrow BM = QN$ [$\because$ AM and PN are medians] $\rightarrow$ (1)

(i) In ΔABM and ΔPQN, $AB = PQ$ (given)
$BM = QN$ (proved above)
$AM = PN$ (given)
$\therefore \Delta ABM \cong \Delta PQN$ (SSS congruence)

Thus $\angle ABM = \angle PQN$ (by CPCT)
$\Rightarrow \angle B = \angle Q \longrightarrow (2)$

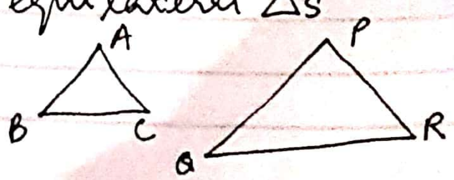
(ii) In ΔABC and ΔPQR, $AB = PQ$ (given)
$\angle B = \angle Q$ (from eq: (2))
$BC = QR$ (given)
$\therefore \Delta ABC \cong \Delta PQR$ (SAS Congruency)
Hence Proved.

2)



Since $PQ = PR$, $\angle Q = \angle R = 65°$ [angles opposite to equal sides]

3) If the corresponding angles of two triangles are equal, the two triangles are not always congruent.
For example, consider two equilateral Δs ΔABC and ΔPQR.



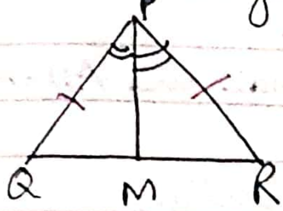
Since each angle measures 60°, $\angle A = \angle P$

$\angle B = \angle Q$

and $\angle C = \angle R$

But the sides need not be same and hence they are not congruent.

4)



Given: in $\triangle PQR$, $PQ = PR$.

Since PM bisects $\angle P$, $\angle QPM = \angle RPM \rightarrow (1)$

To prove: $\triangle PQM \cong \triangle PRM$

Proof:- In $\triangle PQM$ and $\triangle PRM$,

$PQ = PR$ (given)

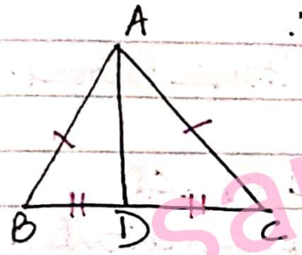
$\angle QPM = \angle RPM$ (from eq:(1))

$PM = PM$ (common side)

$\therefore \triangle PQM \cong \triangle PRM$ (SAS congruency)

Hence Proved.

5)



Given: in $\triangle ABC$, $AB = AC$

$BD = DC$

To prove: $\angle ADB = 90°$

Proof:- In $\triangle ABD$ and $\triangle ACD$, $AB = AC$ (given)

$BD = DC$ (given)

$AD = AD$ (common side)

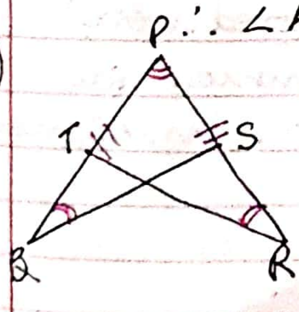
$\therefore \triangle ABD \cong \triangle ACD$ (SSS congruency)

Thus $\angle ADB = \angle ADC$ (by CPCT)

These angles form a linear pair since BC is a straight line.

$\therefore \angle ADB = \angle ADC = \frac{180°}{2} = 90°$. Hence Proved.

6)



Given: $PQ = PR$ and $\angle Q = \angle R$

To prove: $\triangle PQS \cong \triangle PRT$

Proof:- In $\triangle PQS$ and $\triangle PRT$,

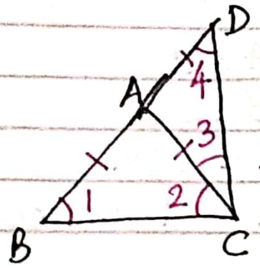
$\angle PQS = \angle PRT$ (given)

$PQ = PR$ (given)

$\angle QPS = \angle RPT$ (common angle

$\therefore \triangle PQ \cong \triangle PRT$ (ASA Congruency)

Hence Proved.

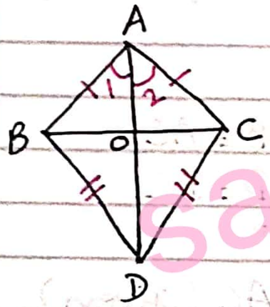
7) 

Given: In $\triangle ABC$, $AB = AC$
and BA is produced to D,
then, $AD = AB$.
To prove: $\angle BCD = 90^\circ$

Proof :- Since $AB = AC$, $\angle 1 = \angle 2 \longrightarrow (1)$
and also since $AC = AD$ [$\because AB = AC$ and $AD = AB$],
then, $\angle 3 = \angle 4 \rightarrow (2)$
In $\triangle BCD$, using angle sum property,

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$$
$$2\angle 2 + 2\angle 3 = 180^\circ \quad [\text{from } (1) \text{ and } (2)]$$
$$2(\angle 2 + \angle 3) = 180^\circ$$
$$\therefore \angle 2 + \angle 3 = \frac{180^\circ}{2} = \underline{\underline{90^\circ}}$$

Thus, $\angle BCD = 90^\circ$. Hence Proved.

8) 

Given: $AB = AC$ and $BD = CD$
To prove: AD is the perpendicular bisector of BC.

Proof :- In $\triangle ABD$ and $\triangle ACD$,

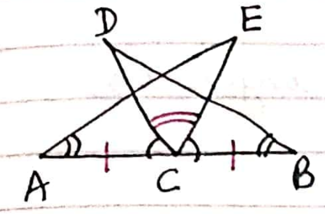
$$\left.\begin{aligned} AB &= AC \\ BD &= DC \end{aligned}\right\} \text{(given)}$$
$AD = AD$ (common side)
$\therefore \triangle ABD \cong \triangle ACD$ (SSS congruency)

Thus $\angle 1 = \angle 2$ (by CPCT) $\longrightarrow (1)$
In $\triangle AOB$ and $\triangle AOC$, $AB = AC$ (given)
$\angle 1 = \angle 2$ (proved above)
$OA = OA$ (common side)
$\therefore AOB \cong \triangle AOC$ (by SAS congruency)

$$\left.\begin{aligned} \text{Thus } \angle AOB &= \angle AOC \\ OB &= OC \end{aligned}\right\} \text{by CPCT.}$$

$\angle AOB$ and $\angle AOC$ form linear pair.
$$\therefore \angle AOB = \angle AOC = \frac{180^\circ}{2} = 90^\circ$$

Hence AD is the perpendicular bisector of BC.

9) 

Given : $AC = BC$

$\angle DCA = \angle ECB$

$\angle EAC = \angle DBC$

To prove : $\triangle DBC \cong \triangle EAC$

$DC = EC$

Proof :- Given, $\angle DCA = \angle ECB$

$\Rightarrow \angle DCA + \angle DCE = \angle ECB + \angle DCE$ (on ~~con~~ adding $\angle DCE$ on both sides)

$\Rightarrow \angle ECA = \angle DCB \longrightarrow$ (i)

In $\triangle DBC$ and $\triangle EAC$, $BC = AC$ (given)

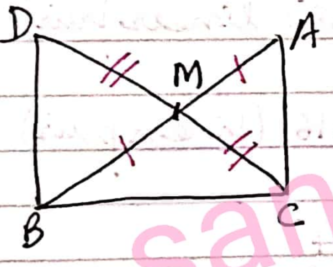
$\angle DBC = \angle EAC$ (given)

$\angle DCB = \angle ECA$ (proved above)

$\therefore \triangle DBC \cong \triangle EAC$ (by ASA congruency)

Thus, $DC = EC$ (by CPCT)

Hence Proved.

10) 

Given : in $\triangle ABC$, $\angle C = 90°$

$AM = BM$

$DM = CM$

To prove : (i) $\triangle AMC \cong \triangle BMD$

(ii) $\angle DBC$ is a right angle

(iii) $\triangle DBC \cong \triangle ACB$

(iv) $CM = \frac{1}{2} AB$

Proof :- (i) In $\triangle AMC$ and $\triangle BMD$, $AM = BM$ (given)

$CM = DM$ (given)

$\angle AMC = \angle BMD$ (V.O.A)

$\therefore \triangle AMC \cong \triangle BMD$ (SAS Congruency)

Thus, $\left.\begin{array}{l} \angle ACM = \angle BDM \\ AC = BD \end{array}\right\}$ (by CPCT)

But $\angle ACM$ and $\angle BDM$ form a pair of alternate interior angles only when $AC \parallel BD$.

(ii) Then $\angle ACB + \angle DBC = 180°$ (co-interior angles)

$\therefore \angle DBC = 180° - 90° = 90°$

$\Rightarrow \angle DBC$ is a right angle

(iii) In $\triangle DBC$ and $\triangle ACB$, $DB = AC$ (proved above)

$\angle DBC = \angle ACB$ (each $90°$)

$BC = BC$ (Common side)

$\therefore \triangle DBC \cong \triangle ACB$ (by SAS congruency)

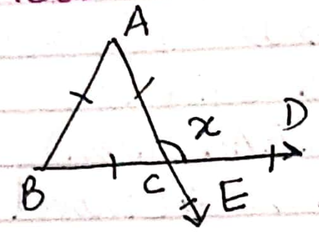
(iv) Thus $DC = AB$ (by CPCT)

$2CM = AB$ [$\because DM = CM$]

$\therefore CM = \frac{1}{2} AB$. Hence Proved

MCQs

(1)

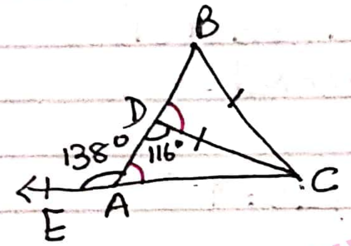


$\angle ACB = 60°$ [$\because \triangle ABC$ is an equilateral $\triangle$]

$x = 180° - 60°$ (linear pair)

$= \underline{\underline{120°}}$ (c)

(2)



$\angle BAC = 180° - 138°$ (linear pair)

$= \underline{42°}$

$\angle BDC = 180° - 116°$ (linear pair)

$= \underline{64°}$

Since $DC = BC$, $\angle BDC = \angle DBC = 64°$

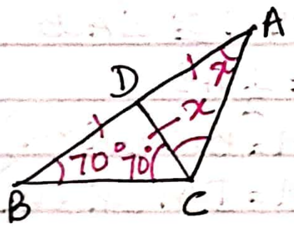
In $\triangle ABC$, using angle sum property,

$\angle ACB = 180° - (42° + 64°)$

$= 180° - 106°$

$= 74°$ (a)

3)



Since $BD = DC$, $\angle DBC = \angle DCB = 70°$

Since $DA = DC$, $\angle DAC = \angle DCA = x$

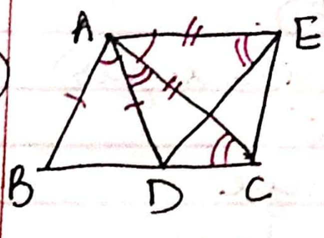
Using angle sum property in $\triangle BCA$,

$70° + 70° + x + x = 180°$

$2x = 180° - 140° = 40°$

$\therefore x = \frac{40°}{2} = \underline{\underline{20°}}$ (c)

4)

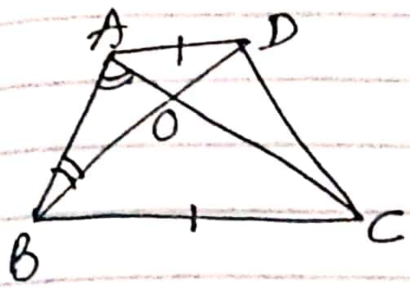


$\triangle BAC \cong \triangle DAE$ (SAS congruency)

Thus $\angle ACB = \angle AED = 40°$ (by CPCT)

(b)

5)



$\triangle DAB \cong \triangle CBA$ (SAS Congruency)

Thus, $\angle CAB = \angle DBA = 30^\circ$ (by CPCT)

In $\triangle AOB$, by using angle sum property,

$\angle AOB = 180^\circ - (30^\circ + 30^\circ)$

$= 180^\circ - 60^\circ$

$= 120^\circ$