IX

Elite work – 6

1) The factors of $x^3 - x^2y - xy^2 + y^3$ are

(a) $(x+y)(x^2-xy+y^2)$ (b) $(x+y)(x^2+xy+y^2)$

(c) $(x+y)^2(x-y)$ (d) $(x-y)^2(x+y)$

Solution :-

$$x^3 - x^2y - xy^2 + y^3 = x^2(x-y) - y^2(x-y)$$
$$= (x^2-y^2)(x-y)$$
$$= (x-y)(x+y)(x-y)$$
$$= (x-y)^2(x+y) \quad (d)$$

2) The factors of $x^3 - 1 + y^3 + 3xy$ are

(a) $(x-1+y)(x^2+1+y^2+x+y-xy)$ (b) $(x+y+1)(x^2+y^2+1-xy-x-y)$

(c) $(x-1+y)(x^2-1-y^2+x+y+xy)$ (d) $3(x+y-1)(x^2+y^2-1)$

Solution :-

$$x^3 - 1 + y^3 + 3xy = (x)^3 + (-1)^3 + (y)^3 - 3 \times x \times (-1) \times y$$

$[a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)]$

$$= (x-1+y)(x^2+1+y^2+x+y-xy) \quad (a)$$

3) The factors of $8a^3 + b^3 - 6ab + 1$ are

(a) $(2a+b-1)(4a^2+b^2+1-3ab-2a)$ (b) $(2a-b+1)(4a^2+b^2-4ab+1-2a+b)$

(c) $(2a+b+1)(4a^2+b^2+1-2ab-b-2a)$ (d) $(2a-1+b)(4a^2+1-4a-b-2ab)$

Solution :-

$$8a^3 + b^3 - 6ab + 1 = (2a)^3 + (b)^3 + (1)^3 - 3 \times 2a \times b \times 1$$

$[x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)]$

$$= (2a+b+1)(4a^2+b^2+1-2ab-b-2a) \quad (c)$$

4) $(x+y)^3 - (x-y)^3$ can be factorized as

(a) $2y(3x^2+y^2)$ (b) $2x(3x^2+y^2)$ (c) $2y(3y^2+x^2)$ (d) $2x(x^2+3y^2)$

Solution :-

$[a^3 - b^3 = (a-b)(a^2+ab+b^2)]$; $a = (x+y)$, $b = (x-y)$

$$(x+y)^3 - (x-y)^3 = (x+y-x+y)\left((x+y)^2 + (x+y)(x-y) + (x-y)^2\right)$$
$$= 2y(x^2+y^2+2xy+x^2-y^2+x^2+y^2-2xy)$$
$$= 2y(3x^2+y^2)$$

5) The expression $(a-b)^3 + (b-c)^3 + (c-a)^3$ can be factorized as

(a) $(a-b)(b-c)(c-a)$ (b) $3(a-b)(b-c)(c-a)$

(c) $-3(a-b)(b-c)(c-a)$ (d) $(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Solution :-

Checking :- $a - b + b - c + c - a = 0$

[if $x+y+z=0$, then $x^3+y^3+z^3=3xyz$]

$(a-b)^3+(b-c)^3+(c-a)^3=3(a-b)(b-c)(c-a)$ (b)

6) The value of $\frac{(2.3)^3-0.027}{(2.3)^2+0.69+0.09}$ =

(a) 2 (b) 3 (c) 2.327 (d) 2.273

Solution:-

$$\frac{(2.3)^3-(0.3)^3}{(2.3)^2+2.3\times0.3+(0.3)^2} \quad [\because a^3-b^3=(a-b)(a^2+ab+b^2)]$$

$$=\frac{(2.3-0.3)((2.3)^2+2.3\times0.3+(0.3)^2)}{(2.3)^2+2.3\times0.3+(0.3)^2}=2.3-0.3=2 \text{ (a)}$$

7) The value of $\frac{(0.013)^3+(0.007)^3}{(0.013)^2-0.013\times0.007+(0.007)^2}$ is

(a) 0.006 (b) 0.02 (c) 0.0091 (d) 0.00185

Solution:-

$$\frac{[0.013+0.007][(0.13)^2-0.013\times0.007+(0.007)^2]}{(0.013)^2-0.013\times0.007+(0.007)^2} \quad [a^3+b^3=(a+b)(a^2-ab+b^2)]$$

$=0.02$ (b)

8) The factors of $a^2-1-2x-x^2$ are

(a) $(a-x+1)(a-x-1)$ (b) $(a+x-1)(a-x+1)$

(c) $(a+x+1)(a-x-1)$ (d) none of these

Solution:-

$a^2-(1+2x+x^2)=a^2-(1+x)^2 \quad [\because a^2-b^2=(a+b)(a-b)]$

$=(a+1+x)(a-1-x)$ (c)

9) The factors of x^4+x^2+25 are

(a) $(x^2+3x+5)(x^2-3x+5)$ (b) $(x^2+3x+5)(x^2+3x-5)$

(c) $(x^2+x+5)(x^2-x+5)$ (d) none of these

Solution:-

$x^4+x^2+25=x^4+25+10x^2-9x^2$

$=((x^2)^2+(5)^2+2\times x^2\times5)-9x^2 \quad [a^2+b^2+2ab=(a+b)^2]$

$=(x^2+5)^2-(3x)^2 \quad [\because a^2-b^2=(a+b)(a-b)]$

$=(x^2+5+3x)(x^2+5-3x)$

$=(x^2+3x+5)(x^2-3x+5)$ (a)

10) The factors of $x^2+4y^2+4y-4xy-2x-8$ are
(a) $(x-2y-4)(x-2y+2)$ (b) $(x-y+2)(x-4y-4)$ (c) $(x+2y-4)(x+2y+2)$ (d) none

Solution:-

$x^2+4y^2-4xy+4y-2x-8$

$=(x-2y)^2+2(2y-x)-8$ $[\because a^2+b^2-2ab=(a-b)^2]$

$=(x-2y)^2-2(x-2y)-8$

Put $x-2y=a$

Then, a^2-2a-8

$\Rightarrow (a-4)(a+2)$

S P
-2 -8 $<$ -4, 2

$=(x-2y-4)(x-2y+2)$ (a)

11) The factors of x^3-7x+6 are
(a) $x(x-6)(x-1)$ (b) $(x^2-6)(x-1)$
(c) $(x+1)(x+2)(x-3)$ (d) $(x-1)(x+3)(x-2)$

Solution:-

$x^3-7x+6 = x^3-1+7-7x = (x^3-1)+7(1-x)$

$=(x^3-1)-7(x-1)=(x-1)(x^2+x+1)-7(x-1)$

$=(x-1)[x^2+x+1-7]$

$=(x-1)(x^2+x-6)$

S P
1 -6 $<$ 3, -2

$=(x-1)(x+3)(x-2)$ (d)

12) The expression x^4+4 can be factorised as
(a) $(x^2+2x+2)(x^2-2x+2)$ (b) $(x^2+2x+2)(x^2+2x-2)$
(c) $(x^2-2x-2)(x^2-2x+2)$ (d) $(x^2+2)(x^2-2)$

Solution:-

$x^4+4=(x^4+4+4x^2)-4x^2$

$=((x^2)^2+(2)^2+2\times x^2\times 2)-(2x)^2$

$=(x^2+2)^2-(2x)^2$

$=(x^2+2+2x)(x^2+2-2x)$ (a)

13) If $3x=a+b+c$, then the value of $(x-a)^3+(x-b)^3+(x-c)^3-3(x-a)(x-b)(x-c)$ is
(a) $a+b+c$ (b) $(a-b)(b-c)(c-a)$ (c) 0 (d) none of these

Solution:-

$[a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)]$

$(x-a)^3+(x-b)^3+(x-c)^3-3(x-a)(x-b)(x-c)$

$=(x-a+x-b+x-c)((x-a)^2+(x-b)^2+(x-c)^2-(x-a)(x-b)-(x-b)(x-c)$

$=[3x-(a+b+c)][(x-a)^2+(x-b)(x-c)^2-(x-a)(x-b)-(x-b)(x-c)-(x-a)(x-b)]$

$=0$ $[\because a+b+c=3x]$ (c)

14) If $(x+y)^3-(x-y)^3-6y(x^2-y^2)=ky^3$, then $k=$

(a) 1 (b) 2 (c) 4 (d)

Solution:- [$(a+b)^3=a^3+3a^2b+3ab^2+b^3$; $(a-b)^3=a^3-3a^2b+3ab^2+b^3$]

$(x^3+3x^2y+3xy^2+y^3)-(x^3-3x^2y+3xy^2-y^3)-6x^2y+6y^3$

$=x^3+3x^2y+3xy^2+y^3-x^3+3x^2y-3xy^2+y^3-6x^2y+6y^3$

$=8y^3$

$\therefore 8y^3=ky^3 \Rightarrow k=8$ (d)

15) If $x^3-3x^2+3x-7=(x+1)(ax^2+bx+c)$, then $a+b+c=$

(a) 4 (b) 12 (c) −10 (d) 3

Solution:-

$(x+1)(ax^2+bx+c)=ax^3+bx^2+cx+ax^2+bx+c$

$=ax^3+x^2(a+b)+x(b+c)+c$

On comparing with x^3-3x^2+3x-7,

$\therefore a=1$

$a+b=-3$

$\therefore b=-3-1=-4$

$b+c=3$

$\therefore c=3+4=7$

Then, $a+b+c=1-4+7=4$ (a)
