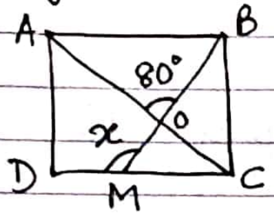
IX Homework – 11 (REVISION WORKSHEET)

1) Find the zeroes of the polynomial $6x^3 - 7x^2 - 11x + 12$, if $(x-1)$ is a factor of the polynomial.

2) If $a = \frac{\sqrt{10}+\sqrt{5}}{\sqrt{10}-\sqrt{5}}$ and $b = \frac{\sqrt{10}-\sqrt{5}}{\sqrt{10}+\sqrt{5}}$, then show that $\sqrt{a} - \sqrt{b} - 2\sqrt{ab} = 0$

3) If $x = \frac{\sqrt{5}-2}{\sqrt{5}+2}$ and $y = \frac{\sqrt{5}+2}{\sqrt{5}-2}$, find the value of $x^2 + y^2 - xy$

4)



ABCD is a square. A line BM intersects CD at M and diagonal AC at O such that $\angle AOB = 80°$. Find the value of x.

5) Plot the points A(0,4), B(−3,0), C(0,−4), D(3,0). Name the figure obtained by joining the points A, B, C and D. Also name the quadrants in which sides AB and AD lie.

6) The polynomials $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + a$ when divided by $(x-2)$ leave same remainder. Find the value of a.

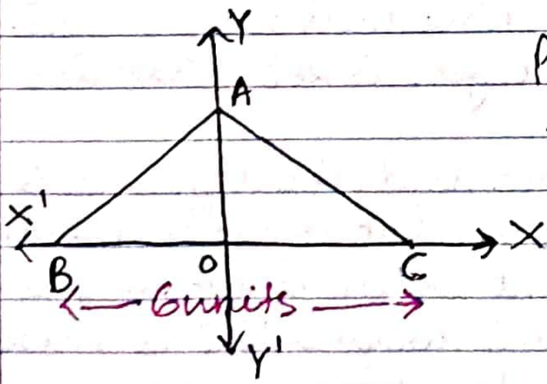
7) Draw the graph of the equation $x + 3y = 15$. Find the coordinates of the point where the graph intersects the x-axis.

8) Determine a and b, if $\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} - \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = a + \sqrt{35}\,b$

9) If $a+b+c=0$, then prove that

$$\frac{(b+c)^2}{3bc} + \frac{(c+a)^2}{3ac} + \frac{(a+b)^2}{3ab} = 1$$

10)



Point A is chosen on y-axis in such a way that $\triangle ABC$ is an equilateral $\triangle$. The base BC of the $\triangle ABC$ is 6 units. Find the coordinates of

(i) the mid-point of BC

(ii) the area of the $\triangle ABC$

(iii) the vertices of the $\triangle$.

IX Homework-11 (Answers)

1) Let $p(x) = 6x^3 - 7x^2 - 11x + 12$.

On dividing $p(x)$ by $(x-1)$,

quotient $= 6x^2 - x - 12$

remainder $= 0$

$$\begin{array}{r} 6x^2 - x - 12 \\ x-1 \enclose{longdiv}{6x^3 - 7x^2 - 11x + 12} \\ \underline{\overset{(-)}{}\cancel{6x^3} \overset{(+)}{-} 6x^2} \\ -\cancel{x^2} - 11x + 12 \\ \underline{\overset{(+)}{-}\cancel{x^2} \overset{(-)}{+} x} \\ -\cancel{12x} + \cancel{12} \\ \underline{\overset{(+)}{-}\cancel{12x} \overset{(-)}{+} \cancel{12}} \\ 0 \end{array}$$

Using division algorithm,

$p(x) = (x-1)(6x^2 - x - 12) + 0$

$= (x-1)(6x^2 - 9x + 8x - 12)$ S P

$= (x-1)[3x(2x-3) + 4(2x-3)]$ -1 -72

$= (x-1)(3x+4)(2x-3)$ $8, -9$

Hence the zeroes of $p(x)$ are $1, \frac{-4}{3}, \frac{3}{2}$.

2) $a = \frac{\sqrt{10}+\sqrt{5}}{\sqrt{10}-\sqrt{5}} = \frac{(\sqrt{10}+\sqrt{5})(\sqrt{10}+\sqrt{5})}{(\sqrt{10}-\sqrt{5})(\sqrt{10}+\sqrt{5})} = \frac{(\sqrt{10}+\sqrt{5})^2}{(\sqrt{10})^2-(\sqrt{5})^2} = \frac{(\sqrt{10}+\sqrt{5})^2}{5}$

$\therefore \sqrt{a} = \frac{\sqrt{10}+\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{2}\times\sqrt{5}+\sqrt{5}}{\sqrt{5}} = \frac{\cancel{\sqrt{5}}(\sqrt{2}+1)}{\cancel{\sqrt{5}}} = \sqrt{2}+1$ //

$b = \frac{\sqrt{10}-\sqrt{5}}{\sqrt{10}+\sqrt{5}} = \frac{(\sqrt{10}-\sqrt{5})(\sqrt{10}-\sqrt{5})}{(\sqrt{10}+\sqrt{5})(\sqrt{10}-\sqrt{5})} = \frac{(\sqrt{10}-\sqrt{5})^2}{(\sqrt{10})^2-(\sqrt{5})^2} = \frac{(\sqrt{10}-\sqrt{5})^2}{5}$

$\sqrt{b} = \frac{\sqrt{10}-\sqrt{5}}{\sqrt{5}} = \frac{\cancel{\sqrt{5}}(\sqrt{2}-1)}{\cancel{\sqrt{5}}} = \sqrt{2}-1$ //

$\sqrt{ab} = \sqrt{a}\times\sqrt{b} = (\sqrt{2}+1)(\sqrt{2}-1) = 2-1 = 1$

$\therefore \sqrt{a}-\sqrt{b}-2\sqrt{ab} = \sqrt{2}+1-(\sqrt{2}-1)-2\times 1$

$= \cancel{\sqrt{2}}+1-\cancel{\sqrt{2}}+1-2 = 2-2 = \underline{0}$

3) $x = \frac{\sqrt{5}-2}{\sqrt{5}+2} = \frac{(\sqrt{5}-2)(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)} = \frac{(\sqrt{5}-2)^2}{(\sqrt{5})^2-2^2} = \frac{5+4-4\sqrt{5}}{5-4} = 9-4\sqrt{5}$

$x^2 = (9-4\sqrt{5})^2 = 81+80-72\sqrt{5} = 161-72\sqrt{5}$

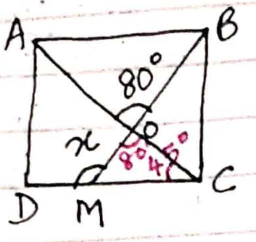
$y = \frac{\sqrt{5}+2}{\sqrt{5}-2} = \frac{(\sqrt{5}+2)(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)} = \frac{(\sqrt{5}+2)^2}{(\sqrt{5})^2-2^2} = \frac{5+4+4\sqrt{5}}{5-4} = 9+4\sqrt{5}$

$y^2 = (9+4\sqrt{5})^2 = 81+80+72\sqrt{5} = 161+72\sqrt{5}$

$xy = (9-4\sqrt{5})(9+4\sqrt{5}) = 81-80 = \underline{\underline{1}}$

$\therefore x^2+y^2-xy = 161-72\sqrt{5}+161+72\sqrt{5}-1$
$= 322-1 = \underline{\underline{321}}$

4)



$\angle AOB = \angle COM = 80°$ (V.O.A)

Since diagonal AC bisects $\angle C$ of square ABCD, $\angle OCM = 45°$.

Using exterior angle property in $\triangle OMC$,

$\angle MOC + \angle MCO = x$

$\therefore x = 80° + 45° = \underline{\underline{125°}}$

5) (graph) The figure obtained is a rhombus.
Side AB lies in second quadrant.
Side AD lies in first quadrant.

6) Let $p_1(x) = ax^3+3x^2-13$ and $p_2(x) = 2x^3-5x+a$

ATQ, $p_1(2) = p_2(2)$

$\Rightarrow a(2)^3+3(2)^2-13 = 2(2)^3-5\times2+a$

$\Rightarrow 8a+12-13 = 16-10+a$

$\Rightarrow 8a-a = 6+1$

$\Rightarrow 7a = 7$

$\therefore a = \underline{\underline{1}}$

7) (graph)

$x+3y = 15$

$3y = 15-x$

$y = \frac{15-x}{3}$

When $x=0$, $y = \frac{15}{3} = 5$

x	0	3	15
y	5	4	0

When $x=3$, $y = \frac{15-3}{3} = \frac{12}{3} = 4$

When $x=15$, $y = \frac{15-15}{3} = \frac{0}{3} = 0$

From the graph :- the line intersects the x-axis at (15,0)

8) $\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \dfrac{(\sqrt{7}+\sqrt{5})(\sqrt{7}+\sqrt{5})}{(\sqrt{7}-\sqrt{5})(\sqrt{7}+\sqrt{5})} = \dfrac{(\sqrt{7}+\sqrt{5})^2}{(\sqrt{7})^2-(\sqrt{5})^2} = \dfrac{7+5+2\sqrt{35}}{7-5}$

$= \dfrac{12+2\sqrt{35}}{2} = \dfrac{\cancel{2}(6+\sqrt{35})}{\cancel{2}} = \underline{\underline{6+\sqrt{35}}}$

$\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \dfrac{(\sqrt{7}-\sqrt{5})(\sqrt{7}-\sqrt{5})}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})} = \dfrac{(\sqrt{7}-\sqrt{5})^2}{(\sqrt{7})^2-(\sqrt{5})^2} = \dfrac{7+5-2\sqrt{35}}{7-5}$

$= \dfrac{12-2\sqrt{35}}{2} = \dfrac{\cancel{2}(6-\sqrt{35})}{\cancel{2}} = \underline{\underline{6-\sqrt{35}}}$

$\therefore \dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} - \dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = (6+\sqrt{35})-(6-\sqrt{35}) = \cancel{6}+\sqrt{35}-\cancel{6}+\sqrt{35}$

$= 2\sqrt{35}$

$= 0+2\sqrt{35}$

On comparing with $a+\sqrt{35}\,b$, $a=0$

$\underline{\underline{b=2}}$

9) $a+b+c=0$

$b+c=-a \rightarrow (1)$

$c+a=-b \rightarrow (2)$

$a+b=-c \rightarrow (3)$

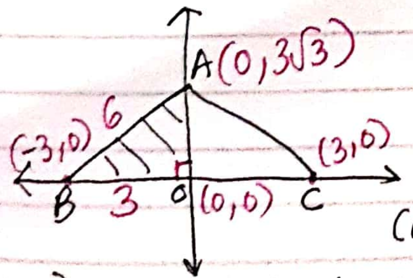
$\therefore \dfrac{(b+c)^2}{3bc} + \dfrac{(c+a)^2}{3ac} + \dfrac{(a+b)^2}{3ab}$

$= \dfrac{(-a)^2}{3bc} + \dfrac{(-b)^2}{3ac} + \dfrac{(-c)^2}{3ab}$

$= \dfrac{a^2 \times a}{3bc \times a} + \dfrac{b^2 \times b}{3ac \times b} + \dfrac{c^2 \times c}{3ab \times c}$

$= \dfrac{a^3+b^3+c^3}{3abc} = \dfrac{3abc}{3abc} = \underline{\underline{1}}$ [$\because$ If $a+b+c=0$, then $a^3+b^3+c^3=3abc$]

10)



Using Pythagoras Theorem in ΔAOB,

$OA^2 = AB^2 - OB^2 = 6^2 - 3^2 = 36-9 = 27$

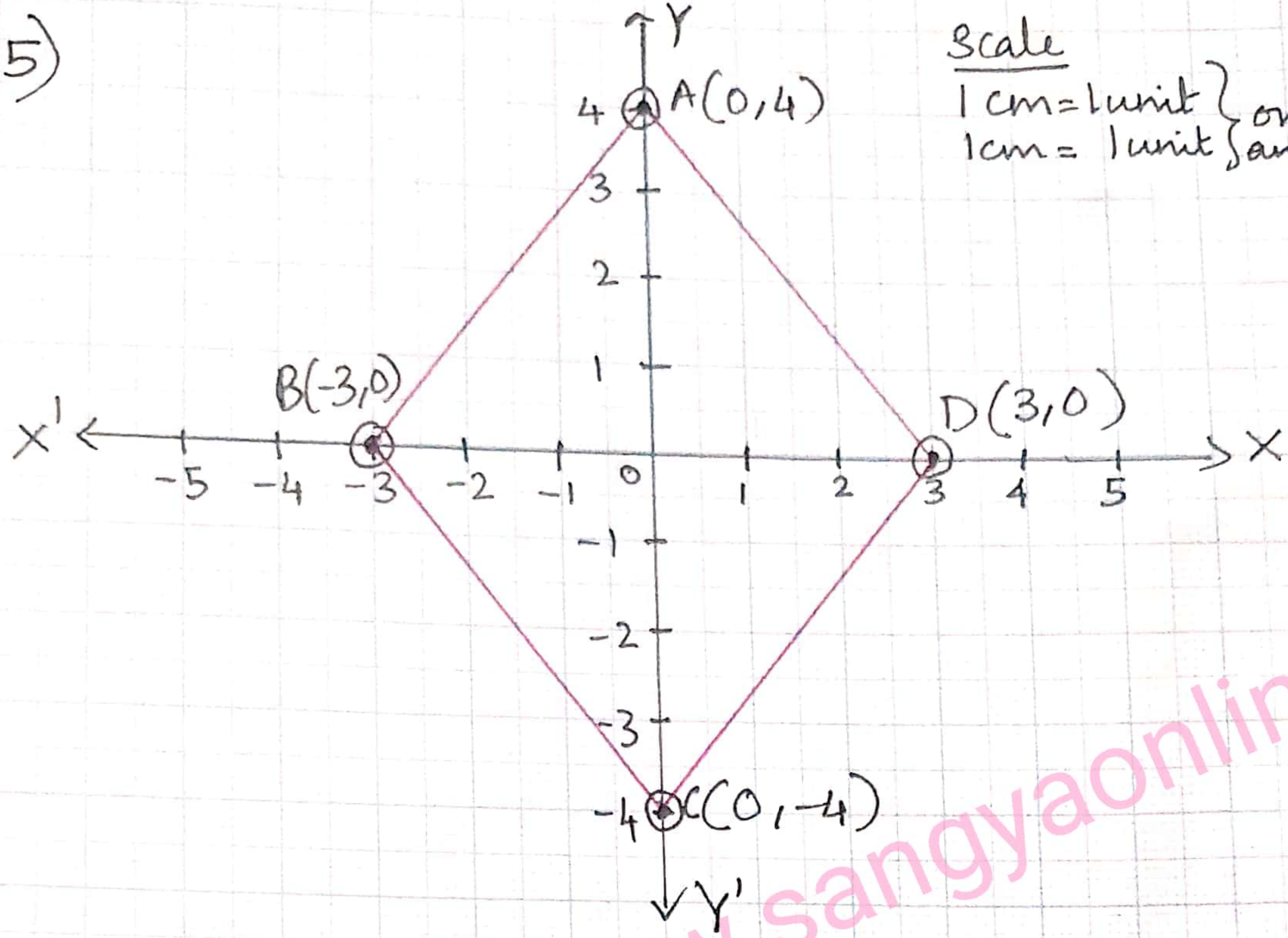
$OA = \sqrt{27} = 3\sqrt{3}$ units

(i) mid-point of BC is origin, O (0,0)

(ii) area (ΔABC) $= \frac{1}{2} \times BC \times OA = \frac{1}{2} \times 6 \times 3\sqrt{3} = 9\sqrt{3}$ sq. units //

(iii) Vertices of ΔABC are $A(0, 3\sqrt{3})$, $B(-3,0)$ and $C(3,0)$

5)

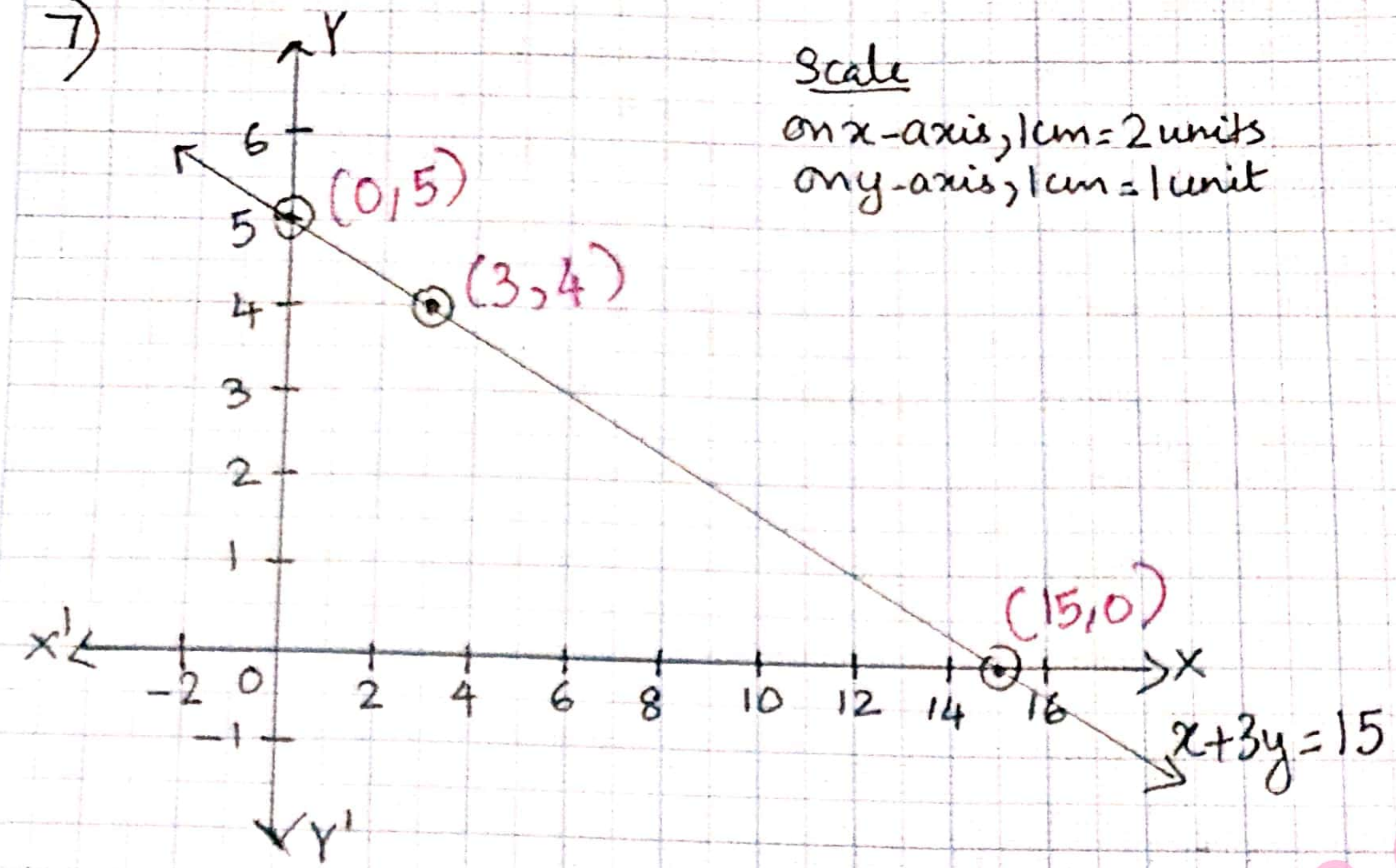


Scale

1 cm = 1 unit on x-axis

1 cm = 1 unit and y-axis

7)



Y
Scale
on x-axis, 1cm = 2 units
on y-axis, 1cm = 1 unit
6
5
4
3
2
1
-1
(0,5)
(3,4)
(15,0)
X'
X
-2
0
2
4
6
8
10
12
14
16
Y'
x+3y=15