X Homework-14 (BOARD EXAM QUESTIONS)

1) A person can row 8 km upstream and 24 km downstream in 4 hours. He can row 12 km downstream and 12 km upstream in 4 hours. Find the speed of the person in still water and also the speed of the current.

2) Using quadratic formula, solve the following equation for $x$: $abx^2 + (b^2 - ac)x - bc = 0$

3) The difference of two natural numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

4) P is the mid-point of side BC of $\triangle ABC$, Q is the mid-point of AP, BQ when produced meets AC at L. Prove that $AL = \frac{1}{3} AC$.

5) In a seminar, the number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required if in each room the same number of participants are to be seated and all of them being of the same subject.

6) Find k, if the sum of the zeroes of the polynomial $x^2 - (k+6)x + 2(2k-1)$ is half of their product.

7) If $7\sin^2 A + 3\cos^2 A = 4$, show that $\tan A = \frac{1}{\sqrt{3}}$

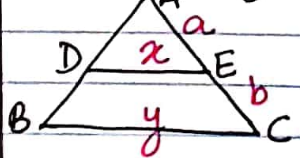
8) Prove that: $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

9) If two positive integers p and q are written as $p = a^2 b^3$ and $q = a^3 b$; a, b are prime numbers, then verify that $\text{LCM}(p,q) \times \text{HCF}(p,q) = pq$

10) Prove that $\frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \operatorname{cosec}\theta + \cot\theta$

1 mark Questions

1) Find the zeroes of the polynomial $p(x) = 4x^2 - 12x + 9$

2) 


DE || BC. Find $\frac{x}{y}$

3) If $2k+1, 6, 3k+1$ are in AP, then find the value of $k$.

4) If the sum of $n$ terms of an AP is $2n^2+5n$, then find the $2^{nd}$ term.

5) If $x=a, y=b$ is the solution of the pair of equations $x-y=2$ and $x+y=4$, then find the values of $a$ and $b$.

6) If $\cot\theta = \frac{7}{8}$, then find the value of $\frac{(1+\cos\theta)(1-\cos\theta)}{(1-\sin\theta)(1+\sin\theta)}$

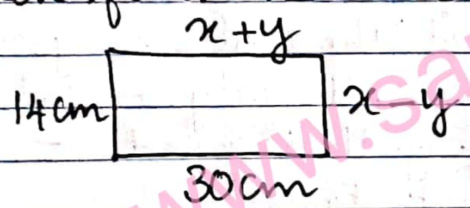
7) Write whether the rational number $\frac{7}{75}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

8) Write the HCF of the smallest compositi number and the smallest prime number.

9) Find the value of $k$ if the quadratic equation $3x^2 - k\sqrt{3}x + 4 = 0$ has equal roots.

10) If $-5$ is a root of the quadratic equation $2x^2+px-15=0$ and the quadratic equation $p(x^2+x)+k=0$ has equal roots, then find the value of $k$.

11)




ABCD is a rectangle. Find the values of $x$ and $y$.

12) Find the value of $\sin 30° + \cos 60°$

X Homework-14 (Answers)

1) Let the speed of the person in still water be $x$ km/hr and speed of current be $y$ km/hr.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Speed of boat downstream $= (x+y)$ km/hr

Speed of boat upstream $= (x-y)$ km/hr

ATQ, $\dfrac{8}{x-y} + \dfrac{24}{x+y} = 4 \longrightarrow (1)$

Also, $\dfrac{12}{x+y} + \dfrac{12}{x-y} = 4 \longrightarrow (2)$

Let $\dfrac{1}{x+y} = a$ and $\dfrac{1}{x-y} = b$

Thus, $24a + 8b = 4 \overset{\div 2}{\Rightarrow} 12a + 4b = 2$

and $12a + 12b = 4 \Rightarrow 12a + 12b = 4$

$(-)$, $-8b = -2$

$\therefore b = \dfrac{-2}{-8} = \dfrac{1}{4}$

Then, $12a + \cancel{4}^{1} \times \dfrac{1}{\cancel{4}_{1}} = 2$

$12a = 2 - 1 = 1$

$\therefore a = \dfrac{1}{12}$

$\therefore x + y = 12$

$x - y = 4$

$$x + y = 12$$
$$x - y = 4$$
$$(+), \quad 2x = 16$$
$$x = 8$$
$$y = 4$$

Hence speed of the person in still water = 8 km/hr
Speed of stream = 4 km/hr

2) Let the given equation be of the form $Ax^2 + Bx + C = 0$;
where $A = ab$ ; $B = b^2 - ac$ and $C = -bc$

$$B^2 - 4AC = (b^2 - ac)^2 - 4ab \times -bc$$
$$= b^4 + a^2c^2 - 2ab^2c + 4ab^2c$$
$$= b^4 + a^2c^2 + 2ab^2c$$
$$= (b^2 + ac)^2$$

$$\therefore x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-(b^2 - ac) \pm (b^2 + ac)}{2ab}$$

$$x = \frac{\cancel{-b^2} + ac + \cancel{b^2} + ac}{2ab} = \frac{\cancel{2}ac}{\cancel{2}ab} = \underline{\underline{\frac{c}{b}}}$$

$$x = \frac{-b^2 + \cancel{ac} - b^2 - \cancel{ac}}{2ab} = \frac{-\cancel{2}b^{\cancel{2}}}{\cancel{2}a\cancel{b}} = \underline{\underline{-\frac{b}{a}}}$$



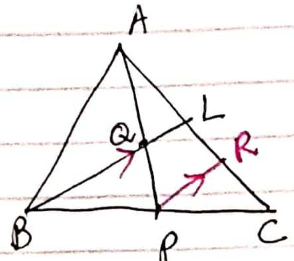
3) Let the numbers be $x$ and $y$.
ATQ, $x - y = 5 \Rightarrow x = 5 + y \rightarrow (1)$
Also, $\frac{1}{y} - \frac{1}{x} = \frac{1}{10} \rightarrow (2)$

On substituting (1) in (2), $\frac{1}{y} - \frac{1}{5+y} = \frac{1}{10}$

$$\Rightarrow \frac{5 + \cancel{y} - \cancel{y}}{y(5+y)} = \frac{1}{10}$$
$$\Rightarrow 50 = 5y + y^2$$
$$\Rightarrow y^2 + 5y - 50 = 0$$
$$\Rightarrow (y + 10)(y - 5) = 0$$
$$\therefore y = -10, 5$$

S P
5 -50
^
10, -5

Since -10 is not a natural number, the required value of $y = 5$.
Then, $x = 10$
Hence, the numbers are 10 and 5.

4) 

Given: in $\triangle ABC$, P is the mid-point of BC.
Q is the mid-point of AP.

To prove: $AL = \frac{1}{3}AC$

Construction :- draw $PR \parallel BL$ to meet AC at R.

Proof :- Since $QL \parallel PR$ in $\triangle APR$, using Thales theorem,

$$\frac{AQ}{QP} = \frac{AL}{LR} \longrightarrow (1)$$

But $AQ = QP$ ($\because$ Q is the mid-point of AP)

$$\Rightarrow \frac{AQ}{QP} = 1 \longrightarrow (2)$$

From (1) and (2), $\frac{AL}{LR} = 1$

$$\Rightarrow AL = LR \Rightarrow (3)$$

Similarly in $\triangle BCL$, since $PR \parallel BL$, using Thales theorem,

$$\frac{CP}{BP} = \frac{CR}{LR} \rightarrow (4)$$

But $CP = BP$ [$\because$ P is the mid-point of BC]

$$\Rightarrow \frac{CP}{BP} = 1 \rightarrow (5)$$

From (4) and (5), $\frac{CR}{LR} = 1$

$$\Rightarrow CR = LR \longrightarrow (6)$$

From (3) and (6), $AL = LR = CR \longrightarrow (7)$

Now, $AC = AL + LR + CR$

$\Rightarrow AC = 3AL$ [from eq: (7)]

$\therefore AC = \frac{1}{3}AC$. Hence Proved

5) $60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

$108 = 2^2 \times 3^3$

HCF $(60, 84, 108) = 2^2 \times 3 = 12$ participants in 1 room

$\therefore$ Minimum no. of rooms required $= \frac{60}{12} + \frac{84}{12} + \frac{108}{12}$

$= 5 + 7 + 9 = \underline{\underline{21 \text{ rooms}}}$

6) Let the given equation be of the form $ax^2+bx+c$; where $a=1$, $b=-(k+6)$, $c=2(2k-1)$ and $\alpha, \beta$ be the zeroes.

Sum of zeroes, $\alpha+\beta = -\frac{b}{a} = k+6$

Product of zeroes, $\alpha\beta = \frac{c}{a} = 2(2k-1)$

ATQ, $\alpha+\beta = \frac{1}{2} \times \alpha\beta$

$\Rightarrow k+6 = \frac{1}{\cancel{2}} \times \cancel{2}(2k-1)$

$\Rightarrow k+6 = 2k-1$

$\Rightarrow k-2k = -1-6$

$\Rightarrow -k = -7$

$\therefore \underline{\underline{k=7}}$

7) $7\sin^2 A + 3\cos^2 A = 4$

$\Rightarrow 4\sin^2 A + (3\sin^2 A + 3\cos^2 A) = 4$

$\Rightarrow 4\sin^2 A + 3(\sin^2 A + \cos^2 A = 4$

$\Rightarrow 4\sin^2 A + 3 = 4 \quad [\because \sin^2 A + \cos^2 A = 1]$

$\Rightarrow 4\sin^2 A = 4-3 = 1$

$\Rightarrow \sin^2 A = \frac{1}{4}$

$\therefore \sin A = \frac{1}{2} \Rightarrow \underline{\underline{A = 30^\circ}}$

Thus, $\tan A = \tan 30^\circ = \frac{1}{\sqrt{3}}$ //

8) LHS, $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$\Rightarrow \frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)} = \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{\cos A(1+\sin A)}$

$= \frac{1+1+2\sin A}{\cos A(1+\sin A)} \quad [\because \sin^2 A + \cos^2 A = 1]$

$= \frac{2+2\sin A}{\cos A(1+\sin A)} = \frac{2\cancel{(1+\sin A)}}{\cos A\cancel{(1+\sin A)}}$

$= \frac{2}{\cos A} = 2\sec A$, RHS

9) $p = a^2b^3$

$q = a^3b$

$LCM(p,q) = a^3b^3$

$HCF(p,q) = a^2b$

LHS, $LCM(p,q) \times HCF(p,q) = a^3b^3 \times a^2b = a^5b^4$

RHS, $pq = a^2b^3 \times a^3b = a^5b^4$

$\therefore$ LHS = RHS. Hence verified

10) LHS, $\dfrac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1}$

$\div \sin\theta$, $\dfrac{\dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\sin\theta} + \dfrac{1}{\sin\theta}}{\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\sin\theta} - \dfrac{1}{\sin\theta}}$

$= \dfrac{\cot\theta - 1 + \operatorname{cosec}\theta}{\cot\theta + 1 - \operatorname{cosec}\theta}$

$= \dfrac{(\operatorname{cosec}\theta + \cot\theta) - (\operatorname{cosec}^2\theta - \cot^2\theta)}{\cot\theta + 1 - \operatorname{cosec}\theta}$ $[\because \operatorname{cosec}^2\theta - \cot^2\theta = 1]$

$= \dfrac{(\operatorname{cosec}\theta + \cot\theta) - (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)}{\cot\theta + 1 - \operatorname{cosec}\theta}$

$= \dfrac{(\operatorname{cosec}\theta + \cot\theta)\,[\overset{1}{\cancel{1 - \operatorname{cosec}\theta + \cot\theta}}]}{\underset{1}{\cancel{\cot\theta + 1 - \operatorname{cosec}\theta}}}$

$= \operatorname{cosec}\theta + \cot\theta$, RHS

## 1 mark Questions

1) $p(x) = 4x^2 - 12x + 9$

$= (2x-3)^2$

$= (2x-3)(2x-3)$

$\therefore$ The zeroes are $\frac{3}{2}, \frac{3}{2}$

2) $\triangle ADE \sim \triangle ABC$ (AA Similarity)

Thus, $\dfrac{AE}{AC} = \dfrac{DE}{BC}$ [$\because$ corresponding sides are in proportion]

$\Rightarrow \dfrac{a}{a+b} = \dfrac{x}{y}$

$\therefore \frac{x}{y} = \frac{a}{a+b}$ //

3) Since the given terms are in A.P,

$6-(2k+1) = 3k+1-6$

$\Rightarrow 6-2k-1 = 3k-5$

$\Rightarrow -2k+5 = 3k-5$

$\Rightarrow -2k-3k = -5-5$

$\Rightarrow -5k = -10$

$\therefore k = \frac{10}{5} = 2$ //

4) $S_n = 2n^2+5n$

$a_1 = S_1 = 2\times1^2+5\times1 = 2+5 = 7$

$S_2 = a_1+a_2 = 2\times2^2+5\times2 = 8+10 = 18$

$\therefore 2^{nd}$ term $= a_2 = 18-7$ $[\because S_2 - S_1]$

$= \underline{\underline{11}}$

5) When $x=a, y=b$ ; $a-b=2$

$a+b=4$

$(+)$, $2a = 6$

$a = 3$ //

$b = 1$ //

6) $\cot\theta = \frac{7}{8} \longrightarrow (1)$

$$\frac{(1+\cos\theta)(1-\cos\theta)}{(1-\sin\theta)(1+\sin\theta)} = \frac{1-\cos^2\theta}{1-\sin^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} = \left(\frac{\sin\theta}{\cos\theta}\right)^2$$

$$= (\tan\theta)^2 = \frac{1}{\cot^2\theta} = \left(\frac{8}{7}\right)^2 = \underline{\underline{\frac{64}{49}}}$$

7) $\frac{7}{75} = \frac{7}{5^2\times3}$

$$\begin{array}{r|l} 5 & 75 \\ \hline 5 & 15 \\ \hline & 3 \end{array}$$

Since the denominator is not of the form $2^m \times 5^n$; where m and n are non-negative integers, the decimal expansion is non-terminating repeating

8) smallest composite number $= 4 = 2^2$

Smallest prime number $= 2$

$\therefore$ HCF $(4,2) = \underline{\underline{2}}$

9) Let the given eq: be of the form $ax^2+bx+c=0$; Where $a=3$, $b=-k\sqrt{3}$, $c=4$.

For equal roots, $b^2-4ac=0$

$$(-k\sqrt{3})^2-4\times3\times4=0$$

$$3k^2-48=0$$

$$k^2=\frac{48}{3}=16$$

$$\therefore k=\pm4$$

10) Since $-5$ is a root of $2x^2+px-15=0$,

$$2(-5)^2+p(-5)-15=0$$

$$\Rightarrow 50-5p-15=0$$

$$\Rightarrow 35-5p=0$$

$$-5p=-35$$

$$p=7$$

Thus, $7(x^2+x)+k=0$

$7x^2+7x+k=0$, be of the form $ax^2+bx+c=0$; where $a=7$, $b=7$, $c=k$

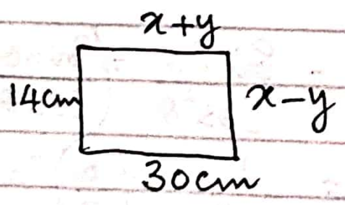
For equal roots, $b^2-4ac=0$

$$\Rightarrow 49-4\times7\times k=0$$

$$\Rightarrow -28k=-49$$

$$\therefore k=\frac{\cancel{49}\,7}{\cancel{28}\,4}=\frac{7}{4}$$



11) 


Since opposite sides are equal in a rectangle,

$$x+y=30$$

$$x-y=14$$

$$(+)\quad 2x=44$$

$$x=22\text{ cm}$$

$$y=8\text{ cm}$$

12) $\sin 30^\circ+\cos 60^\circ=\frac{1}{2}+\frac{1}{2}=1$ $\left[\because \sin 30^\circ=\frac{1}{2},\ \cos 60^\circ=\frac{1}{2}\right]$