Long Answer Questions

64. If $a\sin\theta + \cos\theta = c$, then prove that $a\cos\theta - b\sin\theta = \sqrt{a^2 + b^2 - c^2}$.

65. Given that $\sin\theta + 2\cos\theta = 1$, then prove that $2\sin\theta - \cos\theta = 2$.

66. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, then prove that $\tan\theta = 1$ or $\frac{1}{2}$.

67. If $\sec\theta + \tan\theta = p$, show that $\frac{p^2 - 1}{p^2 + 1} = \sin\theta$.

68. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, prove that $a^2 + b^2 = m^2 + n^2$.

Prove that the following identities (69–80)

69. $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$

70. $\sin A(1 + \tan A) + \cos A(1 + \cot A) = \sec A + \operatorname{cosec} A$

71. $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

72. $\dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta$

73. $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \dfrac{1 + \sin\theta}{\cos\theta}$

74. $(1 + \cot\theta + \tan\theta)(\sin\theta - \cos\theta) = \dfrac{\sec\theta}{\operatorname{cosec}^2\theta} - \dfrac{\operatorname{cosec}\theta}{\sec^2\theta}$

75. $\dfrac{\sin A + \cos A}{\sin A - \cos A} + \dfrac{\sin A - \cos A}{\sin A + \cos A} = \dfrac{2}{\sin^2 A - \cos^2 A} = \dfrac{2}{2\sin^2 A - 1}$

76. $(\sin\theta - \sec\theta)^2 + (\cos\theta - \operatorname{cosec}\theta)^2 = (1 - \sec\theta\operatorname{cosec}\theta)^2$

77. $\dfrac{1 + \cos\theta - \sin^2\theta}{\sin\theta(1 + \cos\theta)} = \cot\theta$

78. $\dfrac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \dfrac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2\sec^2 A$

79. $\dfrac{1}{\sec A - 1} + \dfrac{1}{\sec A + 1} = 2\operatorname{cosec} A\cot A$

80. $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta) = \dfrac{1}{\tan\theta + \cot\theta}$

81. If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$, prove that $x^2 - y^2 = a^2 - b^2$.

82. $\sqrt{\dfrac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\dfrac{\sec\theta + 1}{\sec\theta - 1}} = 2\operatorname{cosec}\theta$

83. $(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \tan A\sec A - \cot A\operatorname{cosec} A$.

84. $\sin A(1 + \tan\theta)^2 + \cos A(1 + \cot A) = \sec A + \operatorname{cosec} A$

85. $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

35. $\left(\frac{3\cos 43^\circ}{\sin 47^\circ}\right)^2 - \frac{\cos 37^\circ \operatorname{cosec} 53^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ}$

36. If $\sec 2A = \operatorname{cosec}(A - 42^\circ)$ where $2A$ is an acute angle, find the value of A.

37. If $\sin(A - B) = 0$, $\cos(A + B) = 0$, $0^\circ < A + B \le 90^\circ$, find A and B.

38. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = 0$, $0^\circ < A + B \le 90^\circ$, find $\sin(A + B)$ and $\cos(A - B)$.

39. Simplify: $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$.

40. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

41. Given that $\alpha + \beta = 90^\circ$, show that $\sqrt{\cos\alpha \operatorname{cosec}\beta - \cos\alpha \sin\beta} = \sin\alpha$.

42. If $\tan\theta = \frac{a}{b}$, prove that $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 + b^2}$.

43. If $\sec\theta = \frac{5}{4}$, find the value of $\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$.

44. If $\theta = 30^\circ$, verify that $\tan^2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

45. If $\theta = 30^\circ$, verify that $\cos^2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$

46. If $A = 30^\circ$ and $B = 60^\circ$, verify that $\cos(A + B) = \cos A \cos B - \sin A \sin B$.

47. If $A = 30^\circ$ and $B = 60^\circ$, verify that $\sin(A + B) = \sin A \cos B + \cos A \sin B$.

48. If $\cot\theta = \frac{15}{8}$, then evaluate $\frac{(2 + 2\sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(2 - 2\cos\theta)}$.

49. If $\sec\theta = x + \frac{1}{x}$, prove that $\sec\theta \tan\theta = 2x$ or $\frac{1}{2x}$.

50. If $\sqrt{3}\tan\theta = 3\sin\theta$, find the value of $\sin^2\theta - \cos^2\theta$.

51. If $\operatorname{cosec}\theta = \frac{13}{12}$, find the value of $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$.

52. If $\sin\theta = \frac{a^2 - b^2}{a^2 + b^2}$, find $1 + \tan\theta\cos\theta$.

53. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

Find the value of x : (54–55)

54. $\sqrt{3}\sin x = \cos x$

55. $\tan x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

Prove the following trigonometric identities: (56–63)

56. $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$

57. $\tan\theta + \tan(90^\circ - \theta) = \sec\theta \sec(90^\circ - \theta)$

58. $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

59. $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = 1 + \tan A + \cot A$

60. $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

61. $\cot\theta - \tan\theta = \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$

62. $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$

Trigonometry

35) $\left(\dfrac{3\sin(90^\circ-43^\circ)}{\sin 47^\circ}\right)^2 - \dfrac{\sin(90^\circ-37^\circ)\cdot \operatorname{cosec} 53^\circ}{\tan 5^\circ \cot(90^\circ-85^\circ)\cdot \tan 45^\circ \cdot \tan 25^\circ \cdot \cot(90^\circ-65^\circ)}$

$= \left(\dfrac{3\sin 47^\circ}{\sin 47^\circ}\right)^2 - \dfrac{\sin 53^\circ \cdot \operatorname{cosec} 53^\circ}{\tan 5^\circ \cdot \cot 5^\circ \cdot \tan 45^\circ \cdot \tan 25^\circ \cdot \cot 25^\circ}$

$= 9 - \dfrac{1}{1\times 1\times 1} = 9-1 = \underline{\underline{8}}$ $\left[\because \sin(90^\circ-\theta)=\cos\theta;\ \cot(90^\circ-\theta)=\tan\theta;\ \tan 45^\circ = 1 \ \middle|\ \sin\theta\cdot\operatorname{cosec}\theta = 1;\ \tan\theta\cdot\cot\theta = 1\right]$

36) $\sec 2A = \operatorname{cosec}(A-42^\circ)$

$\operatorname{cosec}(90^\circ-2A) = \operatorname{cosec}(A-42^\circ)$ $[\because \operatorname{cosec}(90^\circ-\theta) = \sec\theta]$

$\therefore 90^\circ - 2A = A - 42^\circ$

$3A = 132 \Rightarrow A = 44^\circ //$

37) $\sin(A-B)=0 \Rightarrow A-B=0^\circ$ [Since $\sin 0^\circ = 0$, $\cos 90^\circ = 0$]

$\cos(A+B)=0 \Rightarrow A+B = 90^\circ$

$2A = 90^\circ \Rightarrow A = 45^\circ, B = 45^\circ //$

38) $\tan(A+B) = \sqrt{3} \Rightarrow A+B = 60^\circ$ $[\because \tan 60^\circ = \sqrt{3}$, $\tan 0^\circ = 0]$

$\tan(A-B) = 0 \Rightarrow A-B = 0^\circ$

$2A = 60^\circ \Rightarrow A = 30^\circ, B = 30^\circ //$

$\sin(A+B) = \sin 60^\circ = \sqrt{3}/2$

$\cos(A-B) = \cos 0^\circ = 1$

39) $(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta)$

$= (1+\tan^2\theta)(1-\sin^2\theta)$ $[\because 1+\tan^2\theta = \sec^2\theta$, $1-\sin^2\theta = \cos^2\theta]$

$= \sec^2\theta \times \cos^2\theta = \underline{\underline{1}}$

40) $\sin\theta + \cos\theta = \sqrt{3}$

$\Rightarrow (\sin\theta+\cos\theta)^2 = 3$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 2\sin\theta\cos\theta = 3-1 = 2$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow \sin\theta\cos\theta = 1$

$\therefore \tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{1} = \underline{\underline{1}}$

41) $\alpha + \beta = 90^\circ \Rightarrow \alpha = 90^\circ - \beta$

$\sqrt{\cos\alpha \operatorname{cosec}\beta - \cos\alpha\sin\beta} = \sqrt{\cos(90^\circ-\beta)\cdot\operatorname{cosec}\beta - \cos(90^\circ-\beta)\cdot\sin\beta}$

$= \sqrt{\sin\beta\cdot\operatorname{cosec}\beta - \sin\beta\cdot\sin\beta} = \sqrt{1-\sin^2\beta} = \sqrt{\cos^2\beta} = \cos\beta = \cos(90^\circ-\alpha)$

$= \underline{\underline{\sin\alpha}}$

$[\because \cos(90^\circ-\theta) = \sin\theta;\ \sin\theta\cdot\operatorname{cosec}\theta = 1]$

LHS,

42) $\dfrac{a\dfrac{\sin\theta}{\cos\theta} - b\dfrac{\cos\theta}{\cos\theta}}{a\dfrac{\sin\theta}{\cos\theta} + b\dfrac{\cos\theta}{\cos\theta}} = \dfrac{a\tan\theta - b}{a\tan\theta + b} = \dfrac{a\times\frac{a}{b} - b}{a\times\frac{a}{b} + b} = \dfrac{\frac{a^2-b^2}{b}}{\frac{a^2+b^2}{b}}$

$= \dfrac{a^2-b^2}{a^2+b^2}$, RHS

43) $\sec\theta = \frac{5}{4} \Rightarrow \cos\theta = \frac{4}{5}$

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$ //

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3}{4}$ // $\Rightarrow \cot\theta = \frac{4}{3}$ //

$\therefore \dfrac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \dfrac{\frac{3}{5} - \frac{8}{5}}{\frac{3}{4} - \frac{4}{3}} = \dfrac{\frac{-5}{5}}{\frac{9-16}{12}} = -1 \times \frac{12}{-7} = \frac{12}{7}$

45) $\theta = 30^\circ \Rightarrow \tan 30^\circ = \frac{1}{\sqrt{3}}$

R.H.S, $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-\frac{1}{3}}{1+\frac{1}{3}} = \dfrac{\frac{2}{3}}{\frac{4}{3}} = \frac{1}{2}$ //

LHS, $\cos 2\times 30^\circ = \cos 60^\circ = \frac{1}{2}$ //

$\therefore$ LHS = RHS. Hence verified

44) $\theta = 30^\circ$, $\tan 2\theta = \tan 60^\circ = \sqrt{3}$ //

RHS, $\dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2\times\frac{1}{\sqrt{3}}}{1-\frac{1}{3}} = \dfrac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{\cancel{2}}{\cancel{\sqrt{3}}} \times \frac{\cancel{3}^{\sqrt{3}}}{\cancel{2}} = \sqrt{3}$ //

$\therefore$ LHS = RHS. Hence Proved

46) $A = 30^\circ, B = 60^\circ$

LHS, $\cos(A+B) = \cos 90^\circ = 0$

RHS, $\cos A\cos B - \sin A\sin B = \cos 30^\circ \cdot \cos 60^\circ - \sin 30^\circ \cdot \sin 60^\circ$

$= \frac{\sqrt{3}}{2}\times\frac{1}{2} - \frac{1}{2}\times\frac{\sqrt{3}}{2} = 0$

$\therefore$ LHS = RHS.

47. $A = 30°, B = 60°$

LHS, $\sin(A+B) = \sin(30°+60°) = \sin 90° = 1$

RHS, $\sin A \cos B + \cos A \sin B = \sin 30° . \cos 60° + \cos 30° . \sin 60°$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$$

$\therefore$ LHS = RHS, Hence Verified

48. $\cot\theta = \frac{15}{8} \Rightarrow \left[\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + \frac{225}{64} = \frac{289}{64} \right.$

$$\operatorname{cosec}\theta = \frac{17}{8} \Rightarrow \sin\theta = \frac{8}{17}$$

$$\left.\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - \frac{64}{289}} = \frac{15}{17}\right]$$ Not actually needed

$$\therefore \frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)} = \frac{2(1+\sin\theta)(1-\sin\theta)}{2(1-\cos\theta)(1+\cos\theta)} = \frac{\cancel{2}(1-\sin^2\theta)}{\cancel{2}(1-\cos^2\theta)}$$

$$= \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta = \frac{225}{64}$$

49. $\sec\theta = x + \frac{1}{4x}$

$$\tan^2\theta = \sec^2\theta - 1 = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1 = x^2 + \frac{1}{16x^2} - \frac{1}{2} = \left(x - \frac{1}{4x}\right)^2$$

$$\therefore \tan\theta = \pm\left(x - \frac{1}{4x}\right)$$

Thus $\sec\theta + \tan\theta = x + \cancel{\frac{1}{4x}} + x - \cancel{\frac{1}{4x}}$ or $\cancel{x} + \frac{1}{4x} - \cancel{x} + \frac{1}{4x}$

$$= 2x \text{ or } \frac{1}{2x}$$

50. $\sqrt{3}\tan\theta = 3\sin\theta \Rightarrow \sqrt{3} \times \frac{\cancel{\sin\theta}}{\cos\theta} = 3\cancel{\sin\theta} \Rightarrow \frac{1}{\cos\theta} = \frac{3}{\sqrt{3}}$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$$

$$\therefore \sin^2\theta - \cos^2\theta = 1 - \cos^2\theta - \cos^2\theta = 1 - 2\cos^2\theta$$

$$\cancel{= 1 - 2\times 3 = 1 - 6 = -5}$$

$$= 1 - 2 \times \frac{1}{3} = \frac{3-2}{3} = \frac{1}{3}$$

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51. $\operatorname{cosec}\theta = \frac{13}{12} \Rightarrow \cot^2\theta = \operatorname{cosec}^2\theta - 1$

$$= \frac{169}{144} - 1 = \frac{25}{144}$$

$$\Rightarrow \cot\theta = \frac{5}{12}$$

$$\frac{2\sin\theta\ 3\cos\theta}{4\sin\theta - 9\cos\theta} \Rightarrow \div \sin\theta,\ \frac{2 - 3\cot\theta}{4 - 9\cot\theta} = \frac{2 - \cancel{3} \times \frac{5}{\cancel{12}_4}}{4 - \cancel{9}^3 \times \frac{5}{\cancel{12}_4}}$$

$$= \frac{\frac{8-5}{4}}{\frac{16-15}{1}} = \frac{3}{\cancel{4}} \times \frac{\cancel{4}}{1}$$

$$= \underline{3}$$

52. $\sin\theta = \frac{a^2 - b^2}{a^2 + b^2}$

$$1 + \tan\theta \cdot \cos\theta = 1 + \frac{\sin\theta}{\cancel{\cos\theta}} \times \cancel{\cos\theta} = 1 + \frac{a^2 - b^2}{a^2 + b^2}$$

$$= \frac{a^2 + \cancel{b^2} + a^2 - \cancel{b^2}}{a^2 + b^2} = \underline{\underline{\frac{2a^2}{a^2 + b^2}}}$$

53. $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$$

$$\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta = 2\cos^2\theta$$

$$2\cos\theta\sin\theta = \cos^2\theta - \sin^2\theta.$$

Thus, $(\cos\theta - \sin\theta)^2 = \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta$

$$= \cancel{\cos^2\theta} + \sin^2\theta - \cancel{\cos^2\theta} + \sin^2\theta$$

$$= 2\sin^2\theta$$

$\therefore \cos\theta - \sin\theta = \sqrt{2}\sin\theta$ //

54. $\sqrt{3}\sin x = \cos x$

$\Rightarrow \tan x = \frac{1}{\sqrt{3}} \Rightarrow x = 30^\circ$ //

55. $\tan x = \sin 45^\circ \cdot \cos 45^\circ + \sin 30^\circ$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1$$

$\therefore x = 45^\circ$ //

56. LHS $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta \quad [a^3+b^3 = (a+b)(a^2+b^2-ab)]$

$= (\sin^2\theta+\cos^2\theta)(\sin^4\theta+\cos^4\theta-\sin^2\theta\cos^2\theta) + 3\sin^2\theta\cos^2\theta$

$= 1\,((\sin^2\theta)^2+(\cos^2\theta)^2 - \sin^2\theta\cos^2\theta) + 3\sin^2\theta\cos^2\theta$

$= (\sin^2\theta+\cos^2\theta)^2 - \cancel{2\sin^2\theta\cos^2\theta} - \cancel{\sin^2\theta\cos^2\theta} + \cancel{3\sin^2\theta\cos^2\theta}$

$= 1^2 = \underline{1}$, RHS

57. LHS $\tan\theta + \tan(90^\circ-\theta) = \tan\theta + \cot\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{\sin\theta\cos\theta}$

$= \operatorname{cosec}\theta \cdot \sec\theta$

$= \sec(90^\circ-\theta)\cdot\sec\theta$, RHS

58. LHS, $\tan^4\theta + \tan^2\theta$

$= \tan^2\theta(\tan^2\theta+1)$

$= \tan^2\theta \cdot \sec^2\theta$

$= (\sec^2\theta-1)\cdot\sec^2\theta$

$= \sec^4\theta - \sec^2\theta$, RHS

59. LHS, $\dfrac{\tan A}{1-\cot A} + \dfrac{\cot A}{1-\tan A} = \dfrac{\tan A}{1-\frac{1}{\tan A}} + \dfrac{\cot A}{1-\tan A}$

$= \dfrac{\tan^2 A}{\tan A-1} + \dfrac{\cot A}{1-\tan A} = \dfrac{\tan^2 A-\cot A}{\tan A-1}$

$= \dfrac{\tan^3 A-1}{\tan A(\tan A-1)} = \dfrac{\cancel{(\tan A-1)}(\tan^2 A+1+\tan A)}{\tan A\,\cancel{(\tan A-1)}} \quad [a^3-b^3=(a-b)(a^2+b^2+ab)]$

$= \dfrac{\tan^2 A}{\tan A} + \dfrac{1}{\tan A} + \dfrac{\tan A}{\tan A} = \underline{\tan A + \cot A + 1}$, RHS

60. LHS, $(1+\cot A-\operatorname{cosec} A)(1+\tan A+\sec A)$

$\left(1+\dfrac{\cos A}{\sin A}-\dfrac{1}{\sin A}\right)\left(1+\dfrac{\sin A}{\cos A}+\dfrac{1}{\cos A}\right) = \left(\dfrac{\sin A+\cos A-1}{\sin A}\right)\left(\dfrac{\cos A+\sin A+1}{\cos A}\right)$

$= \dfrac{(\sin A+\cos A)^2-1^2}{\sin A\cos A} = \dfrac{\cancel{\sin^2 A+\cos^2 A}+2\sin A\cos A-\cancel{1}}{\sin A\cos A} = \dfrac{2\sin A\cos A}{\sin A\cos A}$

$= \underline{2}$, RHS

61. LHS, $\cot\theta - \tan\theta = \dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{\cos^2\theta - (1-\cos^2\theta)}{\sin\theta\cos\theta} = \dfrac{\cos^2\theta - 1 + \cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$, RHS

62. LHS, $(\operatorname{cosec}\theta - \cot\theta)^2$

$= \left(\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}\right)^2 = \left(\dfrac{1-\cos\theta}{\sin\theta}\right)^2 = \dfrac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \dfrac{(1-\cos\theta)^2}{1-\cos^2\theta} = \dfrac{\cancel{(1-\cos\theta)}(1-\cos\theta)}{\cancel{(1-\cos\theta)}(1+\cos\theta)} = \dfrac{1-\cos\theta}{1+\cos\theta}$, RHS

63. LHS, $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \dfrac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$= \tan\theta \dfrac{(1-2(1-\cos^2\theta))}{2\cos^2\theta - 1}$

$= \dfrac{\tan\theta(1-2+2\cos^2\theta)}{2\cos^2\theta - 1} = \tan\theta \dfrac{(\cancel{2\cos^2\theta - 1})}{\cancel{2\cos^2\theta - 1}}$

$= \tan\theta$, RHS

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61. LHS, $\cot\theta - \tan\theta = \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cos^2\theta - (1-\cos^2\theta)}{\sin\theta\cos\theta} = \frac{\cos^2\theta - 1 + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}, \text{RHS}$$

62. LHS, $(\operatorname{cosec}\theta - \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{\cancel{(1-\cos\theta)}(1-\cos\theta)}{\cancel{(1-\cos\theta)}(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}, \text{RHS}$$

63. LHS, $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$$= \tan\theta\,\frac{(1-2(1-\cos^2\theta))}{2\cos^2\theta - 1}$$

$$= \frac{\tan\theta\,(1-2+2\cos^2\theta)}{2\cos^2\theta-1} = \tan\theta\,\frac{\cancel{(2\cos^2\theta-1)}}{\cancel{2\cos^2\theta-1}}$$

$$= \tan\theta, \text{RHS}$$

64. $a\sin\theta + b\cos\theta = c$

Squaring on both sides, $a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta = c^2$

$$2ab\sin\theta\cos\theta = c^2 - a^2\sin^2\theta - b^2\cos^2\theta$$

Then, $(a\cos\theta - b\sin\theta)^2 = a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta$

$$= a^2\cos^2\theta + b^2\sin^2\theta - c^2 + a^2\sin^2\theta + b^2\cos^2\theta$$

$$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) - c^2$$

$$= a^2 + b^2 - c^2$$

$$\therefore a\cos\theta - b\sin\theta = \sqrt{a^2+b^2-c^2}$$

65) $\sin\theta + 2\cos\theta = 1$

$\Rightarrow (\sin\theta + 2\cos\theta)^2 = 1$

$\Rightarrow \sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta = 1$

$\Rightarrow 4\sin\theta\cos\theta = 1 - \sin^2\theta \mp 4\cos^2\theta$

Thus $(2\sin\theta - \cos\theta)^2 = 4\sin^2\theta + \cos^2\theta - 4\sin\theta\cos\theta$

$= 4\sin^2\theta + \cos^2\theta - 1 + \sin^2\theta + 4\cos^2\theta$

$= 5\sin^2\theta + 5\cos^2\theta - 1$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= 5 - 1 = 4$

$\therefore 2\sin\theta - \cos\theta = 2$ //

66) $1 + \sin^2\theta = 3\sin\theta\cos\theta$

$\sin^2\theta + \cos^2\theta + \sin^2\theta = 3\sin\theta\cos\theta$

$2\sin^2\theta + \cos^2\theta = 3\sin\theta\cos\theta$

$\div \cos^2\theta$, $2\tan^2\theta + 1 = 3\tan\theta$

$2\tan^2\theta - 3\tan\theta + 1 = 0$

$2\tan^2\theta - \tan\theta - 2\tan\theta + 1 = 0$

$\tan\theta(2\tan\theta - 1) - (2\tan\theta - 1) = 0$

$(\tan\theta - 1)(2\tan\theta - 1) = 0$

$\tan\theta = 1$ or $\frac{1}{2}$

P	S
2	−3
	−1 −2

67) $\sec\theta + \tan\theta = p \rightarrow (1)$

$\frac{1}{\sec\theta + \tan\theta} = \frac{1}{p} \Rightarrow \frac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta} = \frac{1}{p}$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{p} \rightarrow (2)$

$(1) - (2)$, $2\tan\theta = p - \frac{1}{p} = \frac{p^2-1}{p} \rightarrow (3)$

$(1) + (2)$, $2\sec\theta = p + \frac{1}{p} = \frac{p^2+1}{p} \rightarrow (4)$

$\frac{(3)}{(4)}$, $\frac{2\tan\theta}{2\sec\theta} = \frac{p^2-1}{p^2+1}$

$\therefore \frac{p^2-1}{p^2+1} = \frac{\sin\theta}{\cos\theta} \times \cos\theta = \sin\theta$ //

68) $a\cos\theta + b\sin\theta = m$

$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta = m^2 \longrightarrow (1)$

$a\sin\theta - b\cos\theta = n$

$\Rightarrow a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = n^2 \rightarrow (2)$

$(1)+(2)$, $a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta) + \cos^2\theta) = m^2+n^2$

$\Rightarrow \underline{a^2+b^2 = m^2+n^2}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

69) LHS, $\sqrt{\sec^2\theta + \csc^2\theta} = \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$

$= \sqrt{1+\tan^2\theta + 1 + \cot^2\theta} = \sqrt{\tan^2\theta + \cot^2\theta + 2}$

$= \sqrt{\tan^2\theta + \cot^2\theta + 2\tan\theta\cdot\cot\theta}$

$= \sqrt{(\tan\theta + \cot\theta)^2}$ $[\because \tan\theta\cdot\cot\theta = 1]$

$= \underline{\tan\theta + \cot\theta}$, RHS

70) LHS, $\sin A(1+\tan A) + \cos A(1+\cot A)$

$= \sin A\left(\frac{\cos A + \sin A}{\cos A}\right) + \cos A\left(\frac{\sin A + \cos A}{\sin A}\right)$

$= (\sin A + \cos A)\left[\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right] = (\sin A + \cos A)\left[\frac{\sin^2 A + \cos^2 A}{\sin A\cos A}\right]$

$= \frac{\sin A + \cos A}{\sin A\cos A} = \frac{\sin A}{\sin A\cos A} + \frac{\cos A}{\sin A\cos A} = \underline{\sec A + \csc A}$, RHS

71) $\div \sin A$, $\frac{\cot A - 1 + \csc A}{\cot A + 1 - \csc A} = \frac{\cot A + \csc A - (\csc^2 A - \cot^2 A)}{\cot A + 1 - \csc A}$

$= \frac{(\cot A + \csc A) - (\csc A + \cot A)(\csc A - \cot A)}{\cot A + 1 - \csc A}$

$= \frac{(\cot A + \csc A)\,\cancel{[1 - \csc A + \cot A]}}{\cancel{\cot A + 1 - \csc A}} = \underline{\csc A + \cot A}$, RHS

72) LHS, $\frac{\tan^2\theta}{\tan\theta - 1} + \frac{\cot\theta}{1-\tan\theta} = \frac{\tan^2\theta - \cot\theta}{\tan\theta - 1} = \frac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)}$

$$[\because a^3-b^3=(a-b)(a^2+b^2+ab)]$$

$$=\frac{\cancel{(\tan\theta-1)}(\tan^2\theta+1+\tan\theta)}{\tan\theta\,\cancel{(\tan\theta-1)}} = \frac{\tan^2\theta}{\tan\theta}+\frac{1}{\tan\theta}+\frac{\tan\theta}{\tan\theta}$$

$$=\tan\theta+\cot\theta+1 = \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}+1$$

$$=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}+1 = \frac{1}{\sin\theta\cos\theta}+1 \quad [\because \sin^2\theta+\cos^2\theta=1]$$

$$=\operatorname{cosec}\theta\sec\theta+1, \text{RHS}$$

73. LHS, $\dfrac{\tan\theta+\sec\theta-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$

$$=\frac{(\tan\theta+\sec\theta)-(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)}{\tan\theta-\sec\theta+1}$$

$$=\frac{(\tan\theta+\sec\theta)\,[\cancel{1-\sec\theta+\tan\theta}]}{\cancel{\tan\theta-\sec\theta+1}} = \frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}$$

$$=\frac{\sin\theta+1}{\cos\theta}, \text{RHS}$$

74. LHS, $\left(1+\dfrac{\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{\cos\theta}\right)(\sin\theta-\cos\theta)$

$$=\frac{(\sin\theta\cos\theta+\cos^2\theta+\sin^2\theta)(\sin\theta-\cos\theta)}{\sin\theta\cos\theta}$$

$$=\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta} \quad [\because a^3-b^3=(a-b)(a^2+b^2+ab)]$$

$$=\frac{\sin^{\cancel{3}\,2}\theta}{\cancel{\sin\theta}\cos\theta}-\frac{\cos^{\cancel{3}\,2}\theta}{\sin\theta\cos\theta} = \sin^2\theta\cdot\sec\theta-\cos^2\theta\cdot\operatorname{cosec}\theta$$

$$=\frac{\sec\theta}{\operatorname{cosec}^2\theta}-\frac{\operatorname{cosec}\theta}{\sec^2\theta}, \text{RHS}$$

75. LHS, $\dfrac{(\sin A+\cos A)^2+(\sin A-\cos A)^2}{\sin^2A-\cos^2A} = \dfrac{\sin^2A+\cos^2A+\cancel{2\sin A\cos A}+\sin^2A+\cos^2A-\cancel{2\sin A\cos A}}{\sin^2A-\cos^2A}$

$$=\frac{1+1}{\sin^2A-\cos^2A} \quad [\because \sin^2A+\cos^2A=1] \quad = \frac{2}{\sin^2A-\cos^2A}$$

$$\frac{2}{\sin^2A-\cos^2A} = \frac{2}{\sin^2A-(1-\sin^2A)} = \frac{2}{\sin^2A-1+\sin^2A} = \frac{2}{2\sin^2A-1}, \text{RHS}$$

76. LHS, $\left(\dfrac{\sin\theta\cos\theta - 1}{\cos\theta}\right)^2 + \left(\dfrac{\sin\theta\cos\theta - 1}{\sin\theta}\right)^2$

$= (\sin\theta\cos\theta - 1)^2\left[\dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}\right]$

$= (\sin\theta\cos\theta - 1)^2\left[\dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}\right]$

$= \dfrac{(\sin\theta\cos\theta - 1)^2}{\sin^2\theta\cos^2\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \dfrac{\sin^2\theta\cos^2\theta + 1 - 2\sin\theta\cos\theta}{\sin^2\theta\cos^2\theta}$

$= \dfrac{\cancel{\sin^2\theta\cos^2\theta}}{\cancel{\sin^2\theta\cos^2\theta}} + \dfrac{1}{\sin^2\theta\cos^2\theta} - \dfrac{2\cancel{\sin\theta\cos\theta}}{\sin^{\cancel{2}1}\theta\cos^{\cancel{2}1}\theta}$

$= 1 + \operatorname{cosec}^2\theta\sec^2\theta - 2\operatorname{cosec}\theta\sec\theta$

$= (1 - \operatorname{cosec}\theta\sec\theta)^2$, RHS $[\because a^2 + b^2 - 2ab = (a-b)^2]$

77. LHS, $\dfrac{1 + \cos\theta - (1 - \cos^2\theta)}{\sin\theta(1 + \cos\theta)}$

$= \dfrac{\cancel{1} + \cos\theta \cancel{- 1} + \cos^2\theta}{\sin\theta(1 + \cos\theta)} = \dfrac{\cos\theta(\cancel{1 + \cos\theta})}{\sin\theta(\cancel{1 + \cos\theta})} = \cot\theta$, RHS

78. LHS, $\dfrac{\operatorname{cosec}A(\operatorname{cosec}A + 1) + \operatorname{cosec}A(\operatorname{cosec}A - 1)}{\operatorname{cosec}^2A - 1}$

$= \dfrac{\operatorname{cosec}^2A + \cancel{\operatorname{cosec}A} + \operatorname{cosec}^2A - \cancel{\operatorname{cosec}A}}{\operatorname{cosec}^2A - 1}$

$= \dfrac{2\operatorname{cosec}^2A}{\cot^2A} = 2 \times \dfrac{1}{\cancel{\sin^2A} \times \cos^2A} \times \cancel{\sin^2A} = 2\sec^2A$, RHS

79. LHS, $\dfrac{1}{\dfrac{1 - \cos A}{\cos A}} + \dfrac{1}{\dfrac{1 + \cos A}{\cos A}} = \dfrac{\cos A}{1 - \cos A} + \dfrac{\cos A}{1 + \cos A}$

$= \cos A\left[\dfrac{1}{1 - \cos A} + \dfrac{1}{1 + \cos A}\right] = \cos A\left[\dfrac{1 + \cancel{\cos A} + 1 - \cancel{\cos A}}{1 - \cos^2A}\right] = \dfrac{\cos A \times 2}{\sin^2A}$

$= 2 \times \dfrac{1}{\sin A} \times \dfrac{\cos A}{\sin A} = 2\operatorname{cosec}A\cot A$, RHS

80) LHS, $\left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right) = \frac{(1-\sin^2\theta)(1-\cos^2\theta)}{\sin\theta\cos\theta}$

$= \frac{\cos^2\theta \times \sin^2\theta}{\sin\theta\cos\theta} = \frac{\sin\theta\cos\theta}{1} = \frac{\sin\theta\cos\theta}{\sin^2\theta+\cos^2\theta}$

$= \frac{1}{\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}} = \frac{1}{\frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta}} = \frac{1}{\tan\theta+\cot\theta}$, RHS

81) $x = a\sec\theta + b\tan\theta$

$y = a\tan\theta + b\sec\theta$

$x^2 - y^2 = a^2\sec^2\theta + b^2\tan^2\theta + \cancel{2ab\sec\theta\tan\theta}$
$- a^2\tan^2\theta - b^2\sec^2\theta - \cancel{2ab\sec\theta\tan\theta}$

$= a^2(\sec^2\theta - \tan^2\theta) + b^2(\tan^2\theta - \sec^2\theta)$

$= a^2(\sec^2\theta - \tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta)$

$= a^2 - b^2$ $[\because \sec^2\theta - \tan^2\theta = 1]$

82) LHS, $\sqrt{\frac{(\sec\theta-1)^2}{\sec^2\theta-1}} + \sqrt{\frac{(\sec\theta+1)^2}{\sec^2\theta-1}} = \frac{\sec\theta-1}{\tan\theta} + \frac{\sec\theta+1}{\tan\theta}$

$= \frac{\sec\theta \cancel{-1} + \sec\theta \cancel{+1}}{\tan\theta} = 2 \times \frac{1}{\cos\theta \times \sin\theta} \times \cos\theta = 2\,\mathrm{cosec}\,\theta$, RHS

83) LHS, $\sec A + \sec A\tan A + \sec A\cot A - \mathrm{cosec}A - \mathrm{cosec}A\tan A$
$- \mathrm{cosec}A\cot A$.

$= \frac{1}{\cos A} + \sec A\tan A + \frac{1}{\cos A}\times\frac{\cos A}{\sin A} - \frac{1}{\sin A} - \frac{1}{\sin A}\times\frac{\sin A}{\cos A}$
$- \mathrm{cosec}A\cot A$

$= \cancel{\frac{1}{\cos A}} + \sec A\tan A + \cancel{\frac{1}{\sin A}} - \cancel{\frac{1}{\sin A}} - \cancel{\frac{1}{\cos A}} - \mathrm{cosec}A\cot A$

$= \tan A\sec A - \cot A\,\mathrm{cosec}A$, RHS

84) LHS, $\sin A\left(\frac{\cos A+\sin A}{\cos A}\right) + \cos A\left(\frac{\sin A+\cos A}{\sin A}\right)$

$= (\cos A+\sin A)\left[\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right] = (\cos A+\sin A)\left[\frac{\sin^2A+\cos^2A}{\sin A\cos A}\right]$

$= \frac{\cos A}{\sin A\cos A} + \frac{\sin A}{\sin A\cos A} = \mathrm{cosec}A + \sec A$, RHS

85. LHS, $\dfrac{\sin\theta\,(1-2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta-1)}$

$$= \tan\theta\,\frac{(1-2(1-\cos^2\theta))}{2\cos^2\theta-1}$$

$$= \tan\theta\,\frac{(1-2+2\cos^2\theta)}{2\cos^2\theta-1}$$

$$= \tan\theta\,\frac{(\cancel{2\cos^2\theta-1})}{\cancel{2\cos^2\theta-1}}$$

$= \underline{\underline{\tan\theta}}$, RHS

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