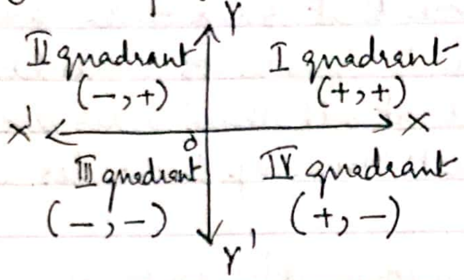
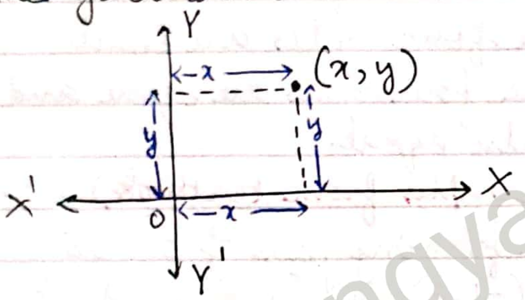
①

X COORDINATE GEOMETRY (notes and extra Questions)

* The angle between the horizontal and vertical axes is 90°
* Any line perpendicular to the X-axis is parallel to Y-axis
* Any line perpendicular to the Y-axis is parallel to X-axis
*

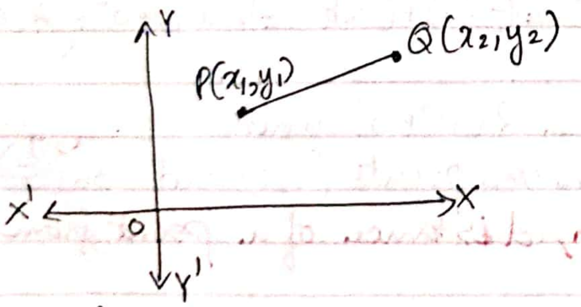


* The x-coordinate is also called the abscissa
* The y-coordinate is also called the ordinate
*



* Every point on the x-axis is of the form $(x, 0)$
* Every point on the y-axis is of the form $(0, y)$
* The coordinates of a point at origin is $(0, 0)$
* The equation of x-axis is $y = 0$
* The equation of y-axis is $x = 0$
* The equation of a line parallel to x-axis is $y = k$; where k is any real number
* The equation of a line parallel to y-axis is $x = a$; where a is any real number.

Distance Formula



The distance between any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

* Let $P(x, y)$ be a point and $O(0,0)$ be the origin, then

$$OP = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$$

* To show that the given points form a
- triangle : prove that sum of length of any two sides is greater than the length of third side.
- isosceles triangle : prove that any two sides are equal
- right angled triangle : prove that sum of squares of any two sides is equal to square of third largest side.
- equilateral triangle : prove that all three sides are equal.
- collinear points : prove that sum of two sides is equal to the third side.
- parallelogram : prove that the opposite sides are equal.
- rectangle : prove that the opposite sides are equal and the diagonals are also equal.
- rhombus : prove that the four sides are equal
- square : prove that the four sides are equal and the diagonals are also equal.

NCERT EX 7.1 (Learn examples from textbook)

1) Find the distance between the following pairs of points :
(i) (2, 3), (4, 1) (ii) (-5, 7), (-1, 3) (iii) (a, b), (-a, -b)

Using distance formula, $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

(i) $\underset{x_1, y_1}{(2, 3)}, \underset{x_2, y_2}{(4, 1)}$

$$d = \sqrt{(4-2)^2 + (1-3)^2} = \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units}$$

(ii) $\underset{x_1, y_1}{(-5, 7)}, \underset{x_2, y_2}{(-1, 3)}$

$$d = \sqrt{(-1+5)^2 + (3-7)^2} = \sqrt{4^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \text{ units}$$

(iii) $\underset{x_1, y_1}{(a, b)}, \underset{x_2, y_2}{(-a, -b)}$

$$d = \sqrt{(-a-a)^2 + (-b-b)^2} = \sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4a^2 + 4b^2} = \sqrt{4(a^2+b^2)}$$

$$= 2\sqrt{a^2+b^2} \text{ units}$$

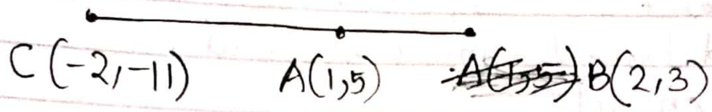
2) Find the distance between the points (0, 0) and $\overset{x, y}{(36, 15)}$

Using distance formula, distance of a point from the origin $= \sqrt{x^2+y^2}$

$$= \sqrt{36^2 + 15^2}$$

$$= \sqrt{1296 + 225} = \sqrt{1521} = 39 \text{ units}$$

3) Determine if the points (1,5), (2,3) and (−2,−11) are collinear.



Using distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB = \sqrt{(2-1)^2+(3-5)^2} = \sqrt{1^2+(-2)^2} = \sqrt{1+4} = \sqrt{5}$ units//

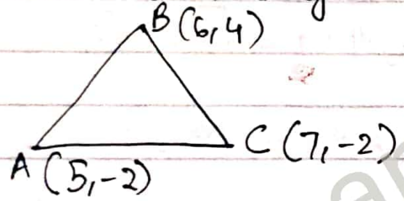
$AC = \sqrt{(-2-1)^2+(-11-5)^2} = \sqrt{(-3)^2+(-16)^2} = \sqrt{9+256} = \sqrt{265}$ units//

$BC = \sqrt{(-2-2)^2+(-11-3)^2} = \sqrt{(-4)^2+(-14)^2} = \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$ units//

Thus $AC + AB \neq BC$

Hence the given points are not collinear.

4) Check whether (5,−2), (6,4) and (7,−2) are the vertices of an isosceles triangle.



Using distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB = \sqrt{(6-5)^2+(4+2)^2} = \sqrt{1^2+6^2} = \sqrt{37}$ units//

$BC = \sqrt{(7-6)^2+(-2-4)^2} = \sqrt{1^2+(-6)^2} = \sqrt{37}$ units//

$AC = \sqrt{(7-5)^2+(-2+2)^2} = \sqrt{2^2+0^2} = \sqrt{4} = 2$ units//

Thus $AB = BC$

Hence the given vertices form an isosceles triangle.

5) In a class room, 4 friends are seated at the points A, B, C and D. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.

(See the figure in the text book)

Let the given points be A(3,4), B(6,7), C(9,4) and D(6,1)

Using distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB = \sqrt{(6-3)^2+(7-4)^2} = \sqrt{3^2+3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units//

$BC = \sqrt{(9-6)^2+(4-7)^2} = \sqrt{3^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units//

$CD = \sqrt{(6-9)^2+(1-4)^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units//

$AD = \sqrt{(6-3)^2+(1-4)^2} = \sqrt{3^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units//

diagonal AC $= \sqrt{(9-3)^2+(4-4)^2} = \sqrt{6^2} = 6$ units

diagonal BD $= \sqrt{(6-6)^2+(1-7)^2} = \sqrt{(-6)^2} = \sqrt{36} = 6$ units

Thus all sides and both diagonals are equal.

Hence Champa is correct.

6) Name the type of quadrilateral formed, if any, by the following points and give reasons for your answer.

(i) $(-1,-2), (1,0), (-1,2), (-3,0)$

(ii) $(-3,5), (3,1), (0,3), (-1,-4)$

(iii) $(4,5), (7,6), (4,3), (1,2)$

Using distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

(i) Let $A(-1,-2), B(1,0), C(-1,2), D(-3,0)$ be the given points.

$AB = \sqrt{(1+1)^2+(0+2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

$BC = \sqrt{(-1-1)^2+(2-0)^2} = \sqrt{(-2)^2+2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

$CD = \sqrt{(-3+1)^2+(0-2)^2} = \sqrt{(-2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

$AD = \sqrt{(-3+1)^2+(0+2)^2} = \sqrt{(-2)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

diagonal $AC = \sqrt{(-1+1)^2+(2+2)^2} = \sqrt{(4)^2} = \sqrt{16} = 4$ units

diagonal $BD = \sqrt{(-3-1)^2+(0-0)^2} = \sqrt{(-4)^2} = \sqrt{16} = 4$ units

Since all sides and diagonals are equal, the given points form a square.

✓ (ii) Let $A(-3,5), B(3,1), C(0,3)$ and $D(-1,-4)$ be the given points

$AB = \sqrt{(3+3)^2+(1-5)^2} = \sqrt{6^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ units

$BC = \sqrt{(0-3)^2+(3-1)^2} = \sqrt{(-3)^2+2^2} = \sqrt{9+4} = \sqrt{13}$ units

$CD = \sqrt{(-1-0)^2+(-4-3)^2} = \sqrt{(-1)^2+(-7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$ units

$AD = \sqrt{(-1+3)^2+(-4-5)^2} = \sqrt{2^2+(-9)^2} = \sqrt{4+81} = \sqrt{85}$ units

diagonal $AC = \sqrt{(0+3)^2+(3-5)^2} = \sqrt{3^2+(-2)^2} = \sqrt{9+4} = \sqrt{13}$ units

diagonal $BD = \sqrt{(-1-3)^2+(-4-1)^2} = \sqrt{(-4)^2+(-5)^2} = \sqrt{16+25} = \sqrt{41}$ units

Thus $AC+BC = \sqrt{13}+\sqrt{13} = 2\sqrt{13} = AB$; points A, B and C ~~form~~ are collinear. Hence the given points doesnot form any quadrilateral.

(iii) Let the given points be $A(4,5), B(7,6), C(4,3)$ and $D(1,2)$

$AB = \sqrt{(7-4)^2+(6-5)^2} = \sqrt{3^2+1^2} = \sqrt{9+1} = \sqrt{10}$ units

$BC = \sqrt{(4-7)^2+(3-6)^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$CD = \sqrt{(1-4)^2+(2-3)^2} = \sqrt{(-3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$ units

$AD = \sqrt{(1-4)^2+(2-5)^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

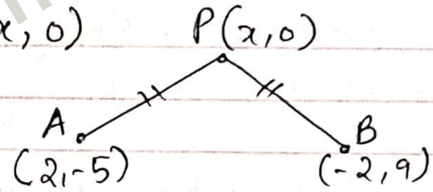
diagonal $AC = \sqrt{(4-4)^2+(3-5)^2} = \sqrt{0+(-2)^2} = \sqrt{4} = 2$ units

diagonal $BD = \sqrt{(1-7)^2+(2-6)^2} = \sqrt{(-6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ units

Since opposite sides are equal and the diagonals are unequal, the given quadrilateral is a parallelogram.

7) Find the point on the x-axis which is equidistant from $(2,-5)$ and $(-2,9)$

Let the point on the x-axis be $P(x,0)$



ATQ, $AP = PB$

$\Rightarrow AP^2 = PB^2$

$\Rightarrow (x-2)^2+(0+5)^2 = (-2-x)^2+(9-0)^2$

$\Rightarrow \cancel{x^2} - 4x + 4 + 25 = 4 + 4x + \cancel{x^2} + 81$

$\Rightarrow 29 - 85 = 8x$

$\Rightarrow 8x = -56$

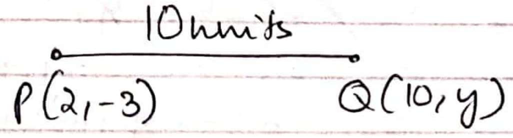
$\therefore x = -7$

Using distance formula

$d^2 = (x_2-x_1)^2+(y_2-y_1)^2$

Hence the required point is $(-7,0)$

8) Find the values of y for which the distance between the points $P(2,-3)$ and $Q(10,y)$ is 10 units.



Using distance formula,

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$PQ = \sqrt{(10-2)^2+(y+3)^2}$

$\Rightarrow 10 = \sqrt{8^2+y^2+6y+9}$

$\Rightarrow 10 = \sqrt{y^2+6y+73}$

$\Rightarrow 100 = y^2+6y+73$ (Squaring on both sides)

$\Rightarrow y^2+6y-27=0$

S	P	
6	-27	9, -3

$\Rightarrow (y+9)(y-3)=0$

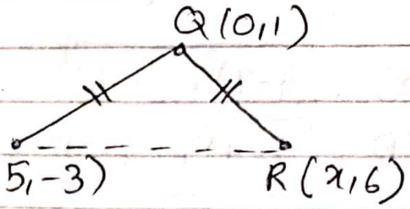
$y = -9, 3$

Hence the required values of y are -9 and 3.

9) If Q(0,1) is equidistant from P(5,−3) and R(x,6), find the value of x. Also find the distances QR and PR.

Using distance formula,

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$



ATQ, QP = QR

$\Rightarrow QP^2 = QR^2$

$\Rightarrow (5-0)^2+(-3-1)^2 = (x-0)^2+(6-1)^2$

$\Rightarrow \cancel{25}+16 = x^2+\cancel{25}$

$x = \pm 4$

$\underline{\underline{x = 4, -4}}$

when $x=4$, $QR = \sqrt{(4-0)^2+(6-1)^2} = \sqrt{16+25} = \sqrt{41}$ units

when $x=-4$, $QR = \sqrt{(-4-0)^2+(6-1)^2} = \sqrt{16+25} = \sqrt{41}$ units

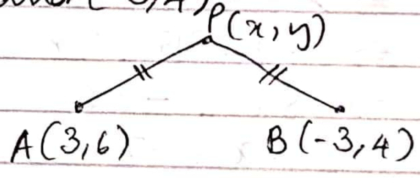
when $x=4$, $PR = \sqrt{(4-5)^2+(6+3)^2} = \sqrt{1+81} = \sqrt{82}$ units

when $x=-4$, $PR = \sqrt{(-4-5)^2+(6+3)^2} = \sqrt{81+81} = 9\sqrt{2}$ units

✓ 10) Find a relation between x and y such that the point (x,y) is equidistant from the points (3,6) and (−3,4).

Using distance formula,

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$



ATQ, AP = BP

$\Rightarrow AP^2 = BP^2$

$\Rightarrow (x-3)^2+(y-6)^2 = (x+3)^2+(y-4)^2$

$\Rightarrow \cancel{x^2}+\cancel{9}-6x+\cancel{y^2}+36-12y = \cancel{x^2}+\cancel{9}+6x+\cancel{y^2}+16-8y$

$\Rightarrow -6x-6x-12y+8y+36-16=0$

$\Rightarrow -12x-4y+20=0$

$\Rightarrow \underline{\underline{12x+4y-20=0}}$ is the required relation between x and y

ADDITIONAL QUESTIONS

MCQs

1) A triangle with vertices (4,0), (−1,−1) and (3,5) is a/an

(i) equilateral triangle (ii) right-angled triangle (iii) isosceles right-angled triangle (iv) none of these

Let the vertices be $A(4,0)$, $B(-1,-1)$, $C(3,5)$. Using distance formula, $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$AB=\sqrt{(-1-4)^2+(-1-0)^2}$$
$$=\sqrt{(-5)^2+(-1)^2}=\sqrt{25+1}=\sqrt{26}\text{ units}$$
$$BC=\sqrt{(3+1)^2+(5+1)^2}=\sqrt{(4)^2+(6)^2}=\sqrt{16+36}=\sqrt{52}\text{ units}$$
$$AC=\sqrt{(3-4)^2+(5-0)^2}=\sqrt{(-1)^2+(5)^2}=\sqrt{1^2+25}=\sqrt{26}\text{ units}$$

Thus $AB^2+AC^2=(\sqrt{26})^2+(\sqrt{26})^2=26+26=52=(\sqrt{52})^2=BC^2$

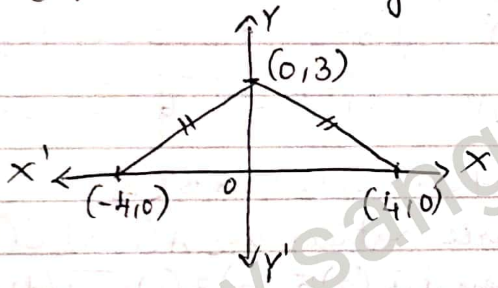
Hence the given points form a rt. angled Δ, by Converse of Pythagoras theorem, at $\angle A=90°$. Also $AB=AC$.

Right angled Isosceles triangle. (c)

2) The points $(-4,0)$, $(4,0)$ and $(0,3)$ are the vertices of a/an

(a) right triangle (b) isosceles triangle (c) equilateral Δ

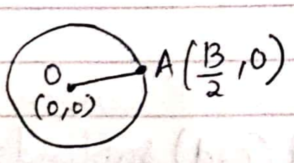
(d) Scalene triangle.



Isosceles triangle (b)

3) A circle drawn with origin as the Centre passes through $\left(\frac{13}{2},0\right)$. The point which does not lie in the interior of the circle is

(a) $\left(-\frac{3}{4},1\right)$ (b) $\left(2,\frac{7}{3}\right)$ (c) $\left(5,-\frac{1}{2}\right)$ (d) $\left(-6,\frac{5}{2}\right)$



Using distance formula,

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Radius of the circle, $OA=\sqrt{\left(\frac{13}{2}-0\right)^2+(0-0)^2}=\sqrt{\left(\frac{13}{2}\right)^2}=\frac{13}{2}$ units

The distance between the points $(0,0)$ and $\left(-6,\frac{5}{2}\right)$

$$=\sqrt{(-6-0)^2+\left(\frac{5}{2}-0\right)^2}=\sqrt{36+\frac{25}{4}}=\sqrt{\frac{144+25}{4}}=\sqrt{\frac{196}{4}}=\frac{13}{2}\text{ units}$$

= radius of the circle

Thus $\left(-6,\frac{5}{2}\right)$ (d) is the required value.

4) If the distance between the points (4, p) and (1, 0) is 5 units, then the value of p is (a) 4 only (b) ±4 (c) −4 only (d) 0

Using distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

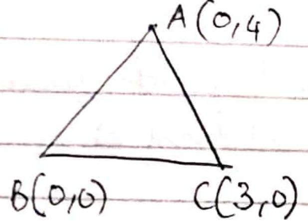
ATQ, $\sqrt{(1-4)^2+(0-p)^2} = 5$

$\Rightarrow (-3)^2+(-p)^2 = 25$

$\Rightarrow 9 + p^2 = 25$

$\Rightarrow p^2 = 16 \Rightarrow p = \pm 4$ (b)

5) The perimeter of a △ with vertices (0, 4), (0, 0) and (3, 0) is (a) 5 (b) 12 (c) 11 (d) $7+\sqrt{5}$



Using distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB = \sqrt{(0-0)^2+(0-4)^2} = \sqrt{(-4)^2} = \sqrt{16} = 4$ units

$BC = \sqrt{(3-0)^2+(0-0)^2} = \sqrt{3^2} = \sqrt{9} = 3$ units

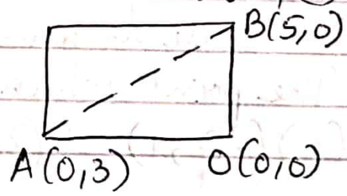
$AC = \sqrt{(3-0)^2+(0-4)^2} = \sqrt{3^2+(-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$ units

∴ Perimeter (△ABC) = AB + BC + AC = 4 + 3 + 5 = 12 units (b)

6) AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0) and B(5, 0). The length of its diagonal is (a) 5 (b) 3 (c) $\sqrt{34}$ (d) 4

Using distance formula,

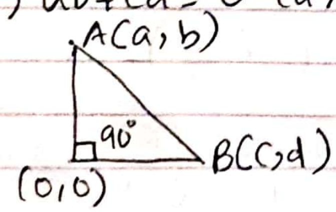
$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$



diagonal $AB = \sqrt{(5-0)^2+(0-3)^2}$

$= \sqrt{25+9} = \sqrt{34}$ units (c)

7) If the segment joining the points (a, b) and (c, d) subtends a right angle at the origin, then (a) ac − bd = 0 (b) ac + bd = 0 (c) ab + cd = 0 (d) ab − cd = 0



Using pythagoras theorem,

$OA^2 + OB^2 = AB^2$

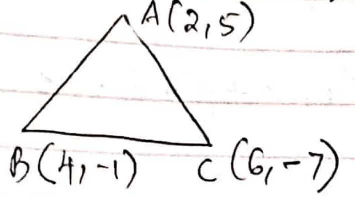
$\Rightarrow \left(\sqrt{(a-0)^2+(b-0)^2}\right)^2 + \left(\sqrt{(c-0)^2+(d-0)^2}\right)^2 = \left(\sqrt{(c-a)^2+(d-b)^2}\right)^2$

$\Rightarrow \cancel{a^2}+\cancel{b^2}+\cancel{c^2}+\cancel{d^2} = \cancel{c^2}+\cancel{a^2}-2ac+\cancel{d^2}+\cancel{b^2}-2bd$

$\Rightarrow 2ac + 2bd = 0$

$\Rightarrow ac + bd = 0$ (b)

8) The points (2,5), (4,−1) and (6,−7) are vertices of a/an
(a) isosceles △ (b) equilateral △ (c) right-angled △ (d) none of these



Using distance formula,
$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB = \sqrt{(4-2)^2+(-1-5)^2} = \sqrt{2^2+(-6)^2} = \sqrt{4+36} = \sqrt{40}$
$= 2\sqrt{10}$ units

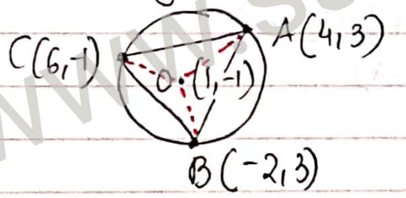
$BC = \sqrt{(6-4)^2+(-7+1)^2} = \sqrt{2^2+(-6)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$ units

$AC = \sqrt{(6-2)^2+(-7-5)^2} = \sqrt{4^2+(-12)^2} = \sqrt{16+144} = \sqrt{160} = 4\sqrt{10}$ units

Thus $AB + BC = AC$. The points A, B and C are collinear.
So it cannot form any triangle.
none of these (d)

1 mark, 2 marks, 3 marks and 4 marks Questions

1) Show that (1,−1) is the centre of the circle circumscribing the triangle whose angular points are (4,3), (−2,3) and (6,−1)

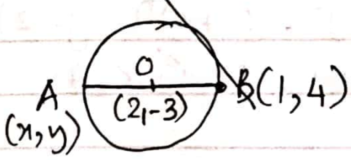


Using distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
To show:
ATQ, OA = OB = OC (radii of the same circle)

$OA = \sqrt{(4-1)^2+(3+1)^2} = \sqrt{9+16} = \sqrt{25} = 5$ units
$OB = \sqrt{(-2-1)^2+(3+1)^2} = \sqrt{9+16} = \sqrt{25} = 5$ units
$OC = \sqrt{(6-1)^2+(-1+1)^2} = \sqrt{25} = 5$ units

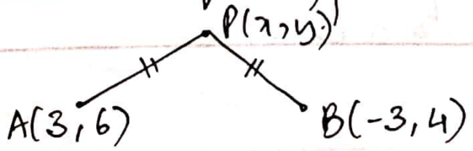
Hence the given point is the centre of the circle.

~~2) Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2,−3) and B is the point (1,4)~~



~~OA = OB (radii of the same circle)~~
$\Rightarrow OA^2 = OB^2$
$\Rightarrow (x-2)^2 +$

2) If the distances of P(x,y) from the points A(3,6) and B(−3,4) are equal, prove that $3x+y=5$.



Using distance formula,
$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

ATQ, $AP = BP$

$\Rightarrow AP^2 = BP^2$

$\Rightarrow (x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$

$\Rightarrow \cancel{x^2} + \cancel{9} - 6x + \cancel{y^2} + 36 - 12y = \cancel{x^2} + \cancel{9} + 6x + \cancel{y^2} + 16 - 8y$

$\Rightarrow -12x - 4y + 20 = 0$

$\Rightarrow 12x + 4y - 20 = 0$

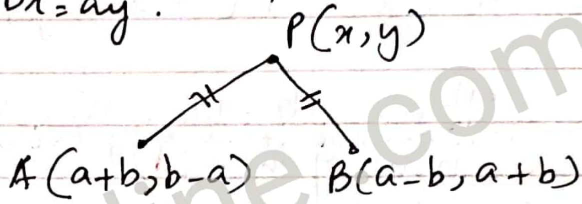
$\Rightarrow 3x + y - 5 = 0$

$\Rightarrow \underline{\underline{3x + y = 5}}$

3) If the point (x, y) is equidistant from the points $(a+b, b-a)$ and $(a-b, a+b)$, prove that $bx = ay$.

Using distance formula,

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$



ATQ, $AP = BP \Rightarrow AP^2 = BP^2$

$\Rightarrow (x-(a+b))^2 + (y-(b-a))^2 = (x-(a-b))^2 + (y-(a+b))^2$

$\Rightarrow \cancel{x^2} + \cancel{(a+b)^2} - 2x(a+b) + \cancel{y^2} + \cancel{(b-a)^2} - 2y(b-a)$

$= \cancel{x^2} + \cancel{(a-b)^2} - 2x(a-b) + \cancel{y^2} + \cancel{(a+b)^2} - 2y(a+b)$

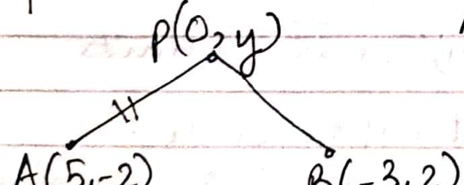
$\Rightarrow -2x(a+b) + 2x(a-b) = -2y(a+b) + 2y(b-a)$

$\Rightarrow \cancel{2}x(-a-\cancel{b}+\cancel{a}-b) = \cancel{2}y(-a-\cancel{b}+\cancel{b}-a)$

$\Rightarrow \cancel{-2}bx = \cancel{-2}ay$

$\therefore \underline{bx = ay}$

4) Find a point on y-axis which is equidistant from the points $(5,-2)$ and $(-3,2)$



Let the point on y-axis be $(0, y)$

Using distance formula,

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

ATQ, $AP = BP \Rightarrow AP^2 = BP^2$

$\Rightarrow (0-5)^2 + (y+2)^2 = (0+3)^2 + (y-2)^2$

$\Rightarrow 25 + \cancel{y^2} + \cancel{4} + 4y = 9 + \cancel{y^2} + \cancel{4} - 4y$

$8y = -16$

$y = -2$

Hence the required point is $(0, -2)$

5) Show that the points $A(7,5)$, $B(2,3)$ and $C(6,-7)$ are the vertices of a right $\triangle$. Also find its area.

Using distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB = \sqrt{(2-7)^2+(3-5)^2} = \sqrt{25+4} = \sqrt{29}$

$BC = \sqrt{(6-2)^2+(-7-3)^2} = \sqrt{16+100} = \sqrt{116}$

$AC = \sqrt{(6-7)^2+(-7-5)^2} = \sqrt{1+144} = \sqrt{145}$

$AB^2+BC^2 = 29+116 = 145 = AC^2$

Hence by the Converse of Pythagoras Theorem, $\triangle ABC$ is a right angled $\triangle$ at $\angle B = 90°$.

$2 \mid 116$, $2 \mid 58$, 29

Area $= \frac{1}{2} \times b \times h = \frac{1}{2} \times AB \times BC = \frac{1}{2} \times \sqrt{29} \times \sqrt{116}$

$= \frac{1}{\cancel{2}} \sqrt{29} \times \cancel{2}\sqrt{29}$

$= 29$ sq. units//

6) Two points $A(1,0)$ and $B(-1,0)$ with a variable point $P(x,y)$ satisfy the relation $AP - BP = 1$. Show that $12x^2 - 4y^2 = 3$.

Using distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

~~$AP = \sqrt{(x-1)^2+(y-0)^2} = \sqrt{x^2-2x+1+y^2}$~~

~~$BP = \sqrt{(x+1)^2+(y-0)^2} = \sqrt{x^2+2x+1+y^2}$~~

~~Given, $AP - BP = 1$~~

~~$\Rightarrow \sqrt{x^2-2x+1+y^2} - \sqrt{x^2+2x+1+y^2} = 1$~~

~~Squaring on both sides, $x^2 - \cancel{2x} + 1 + y^2 + x^2 + \cancel{2x} + \cancel{1} + y^2 - 2\sqrt{x^2-2x+1+y^2}\sqrt{x^2+2x+1+y^2} = \cancel{1}$~~

~~$\Rightarrow 2x^2 + 2y^2 + 1 - 2\sqrt{x}$~~

$AP = \sqrt{(x-1)^2+(y-0)^2} = \sqrt{(x-1)^2+y^2}$

$BP = \sqrt{(x+1)^2+(y-0)^2} = \sqrt{(x+1)^2+y^2}$

Given $AP - BP = 1 \Rightarrow \sqrt{(x-1)^2+y^2} - \sqrt{(x+1)^2+y^2} = 1$

$\Rightarrow \sqrt{(x-1)^2+y^2} = 1 + \sqrt{(x+1)^2+y^2}$

Squaring on both sides, $(x-1)^2 + \cancel{y^2} = 1 + (x+1)^2 + \cancel{y^2} + 2\sqrt{(x+1)^2+y^2}$

$\cancel{x^2} + \cancel{1} - 2x - \cancel{1} - \cancel{x^2} - 2x - 1 = 2\sqrt{(x+1)^2+y^2}$

$-4x - 1 = 2\sqrt{(x+1)^2+y^2}$

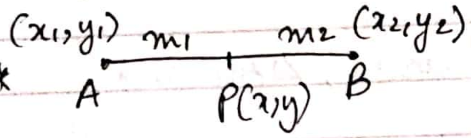
Again Squaring on both sides, $(-4x-1)^2 = 2^2\left(\sqrt{(x+1)^2+y^2}\right)^2$

$\Rightarrow 16x^2+1+8x = 4(x^2+2x+1+y^2)$

$\Rightarrow 16x^2+1+\cancel{8x} = 4x^2+\cancel{8x}+4+4y^2$

$\Rightarrow 12x^2-4y^2=3$

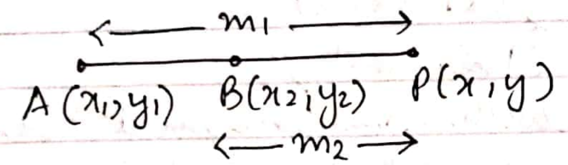
* Section formula (internally)



* P(x,y) divides AB in the ratio $m_1:m_2$

then $P(x,y) = \left(\dfrac{m_1x_2+m_2x_1}{m_1+m_2}, \dfrac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

* Section formula (externally)

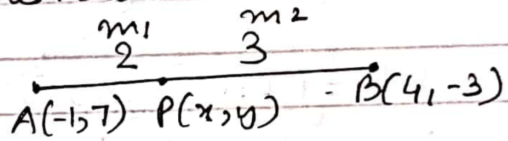


$P(x,y) = \left(\dfrac{m_1x_2-m_2x_1}{m_1-m_2}, \dfrac{m_1y_2-m_2y_1}{m_1-m_2}\right)$

Exercise 7.2

1) Find the coordinates of the point which divides the line Segment joining of (-1, 7) and (4, -3) in the ratio 2:3

Let P(x,y) divides AB in the ratio 2:3.



Using Section formula,

$P(x,y) = \left(\dfrac{m_1x_2+m_2x_1}{m_1+m_2}, \dfrac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$= \left(\dfrac{2\times4+3\times-1}{2+3}, \dfrac{2\times-3+3\times7}{2+3}\right) = \left(\dfrac{8-3}{5}, \dfrac{-6+21}{5}\right)$

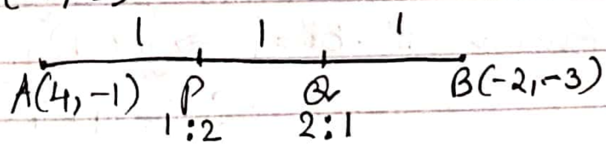
$= \left(\dfrac{5}{5}, \dfrac{15}{5}\right) = \underline{\underline{(1,3)}}$

Hence the coordinates of the required point are (1, 3)

2) Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3)

Let P and Q trisect AB.

Thus P divides AB in the ratio 1:2



Using Section formula, $P(x,y) = P\left(\dfrac{m_1x_2+m_2x_1}{m_1+m_2}, \dfrac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

X

COORDINATE GEOMETRY (TOPIC-2)

$$P(x,y) = \left(\frac{1\times -2+2\times 4}{1+2}, \frac{1\times -3+2\times -1}{1+2}\right) = \left(\frac{-2+8}{3}, \frac{-3-2}{3}\right)$$
$$= \left(\frac{6}{3}, \frac{-5}{3}\right) = \left(2, \frac{-5}{3}\right)$$

Similarly Q divides AB in the ratio 2:1

$$Q(x,y) = \left(\frac{2\times -2+1\times 4}{2+1}, \frac{2\times -3+1\times -1}{2+1}\right) = \left(\frac{-4+4}{3}, \frac{-6-1}{3}\right)$$
$$= \left(0, \frac{-7}{3}\right)$$

Hence the coordinates of the points of trisection are $\left(2, \frac{-5}{3}\right)$ and $\left(0, -\frac{7}{3}\right)$.

3) See the question and figure from your ncert textbook

Coordinates of green flag are $E(2,25)$

Coordinates of red flag are $F(8,20)$

Distance between the flags,

$$EF = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(8-2)^2+(20-25)^2}$$
$$= \sqrt{36+25} = \sqrt{61}\ m$$

$\frac{1}{4}$th of AD $= \frac{1}{4}\times 100m$

$= 25m$

$\frac{1}{5}$th of AD $= \frac{1}{5}\times 100m$

$= 20m$

mid-point of EF $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{8+2}{2}, \frac{25+20}{2}\right)$

$= (5, 22.5)$

Hence the blue flag is in the 5^{th} line at a distance of 22.5m above it.

4) Find the ratio in which the line segment joining the points $(-3,10)$ and $(6,-8)$ is divided by $(-1,6)$.

Let the ratio be $k:1$

$A(-3,10) \xrightarrow{k} P(-1,6) \xrightarrow{1} B(6,-8)$

Using Section formula,

$$P(x,y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$P(-1,6) = \left(\frac{6\times k+1\times -3}{k+1}, \frac{-8\times k+1\times 10}{k+1}\right) = \left(\frac{6k-3}{k+1}, \frac{-8k+10}{k+1}\right)$$

$\therefore \frac{6k-3}{k+1} = -1 \Rightarrow 6k-3 = -k-1$

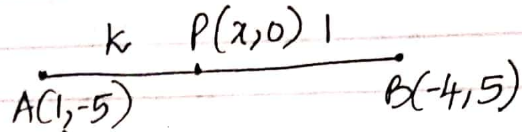
$\Rightarrow 7k = 2$

$k = 2/7$

Hence the required ratio is 2:7

5) Find the ratio in which the line segment joining $A(1,-5)$ and $B(-4,5)$ is divided by the x-axis. Also, find the coordinates of the point of division.

Let the ratio be $k:1$.



Using Section formula,

$$P(x,y) = P\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$P(x,0) = P\left(\frac{-4\times k+1\times 1}{1+k}, \frac{5\times k+1\times -5}{1+k}\right) = P\left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$$

$$\therefore \frac{5k-5}{k+1} = 0 \Rightarrow 5k-5=0$$
$$k=1$$

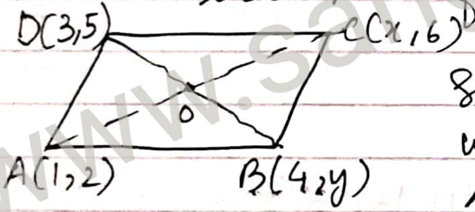
Hence the required ratio is $1:1$

Thus $\frac{-4k+1}{k+1} = x$

$$\frac{-4+1}{1+1} = x \Rightarrow -\frac{3}{2} = x.$$

$\therefore$ The point of division is $\left(-\frac{3}{2}, 0\right)$//

6) If $(1,2)$, $(4,y)$, $(x,6)$ and $(3,5)$ are the vertices of a ||gm taken in order, then find x and y.



Since diagonals bisect each other in ||gm ABCD, O is the mid point of AC and BD.

Using mid-section formula, $O(x,y) = O\left(\frac{1+x}{2}, \frac{6+2}{2}\right)$.

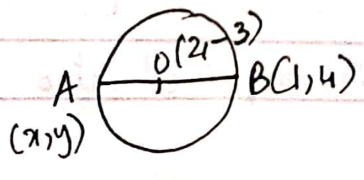
and also $O(x,y) = O\left(\frac{3+4}{2}, \frac{5+y}{2}\right)$

$$\therefore \left(\frac{1+x}{2}, \frac{8}{2}\right) = \left(\frac{7}{2}, \frac{5+y}{2}\right)$$

Then $\frac{1+x}{2} = \frac{7}{2} \Rightarrow x = 6$//

$$\frac{5+y}{2} = 4 \Rightarrow y = 3$$//

7) Find the coordinates of a point A, where AB is the diameter of a circle whose centre is $(2,-3)$ and B is $(1,4)$



Since O is the mid point of AB, using mid section formula.

$$O(x,y) = O\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

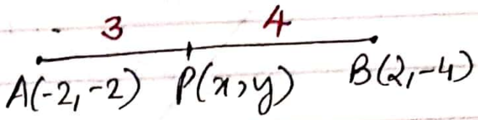
$O(2,-3) = C\left(\frac{x+1}{2}, \frac{4+y}{2}\right)$

$\therefore \frac{x+1}{2} = 2 \Rightarrow x = 3$

$\frac{4+y}{2} = -3 \Rightarrow 4+y = -6 \Rightarrow y = -10$

Hence the coordinates of A are $(3,-10)$

8) If A and B are $(-2,-2)$ and $(2,-4)$ respt. then find the coordinates of P such that $AP = \frac{3}{7}AB$ and P lies on the line segment AB.



Given $AP = \frac{3}{7}AB \Rightarrow \frac{AP}{AB} = \frac{3}{7}$

$\Rightarrow \frac{AB}{AP} = \frac{7}{3}$

$\Rightarrow \frac{PB+AP}{AP} = \frac{7}{3} \Rightarrow \frac{PB}{AP} + 1 = \frac{7}{3}$

$\Rightarrow \frac{PB}{AP} = \frac{7}{3} - 1 = \frac{4}{3}$

Hence $AP : PB = 3:4$

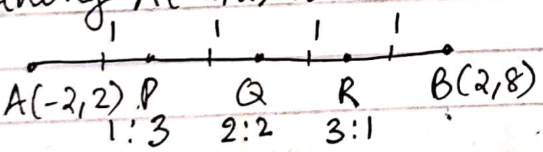
Using Section formula $P(x,y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$= \left(\frac{3\times 2+4\times -2}{3+4}, \frac{3\times -4+4\times -2}{3+4}\right)$

$= \left(\frac{6-8}{7}, \frac{-12-8}{7}\right) = \left(-\frac{2}{7}, \frac{-20}{7}\right)$

Hence the required coordinates are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$

9) Find the coordinates of the points which divide the line segment joining $A(-2,2)$ and $B(2,8)$ into four equal parts



Using section formula,

$P(x,y) = P\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

Let $P(x,y)$ divides AB in the ratio $1:3$

$P(x,y) = P\left(\frac{1\times 2+3\times -2}{1+3}, \frac{1\times 8+3\times 2}{1+3}\right) = P\left(\frac{2-6}{4}, \frac{8+6}{4}\right) = P\left(\frac{-4}{4}, \frac{14}{4}\right)$

$= P\left(-1, \frac{7}{2}\right)$

Let $Q(x,y)$ divides AB in the ratio $2:2$, i.e. Q is the mid-point of AB

$Q(x,y) = \left(\frac{-2+2}{2}, \frac{2+8}{2}\right) = (0,5)$

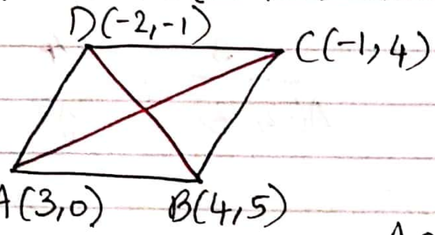
Let $R(x,y)$ divides AB in the ratio 3:1, then

$$R(x,y) = \left(\frac{3\times 2+1\times -2}{3+1}, \frac{3\times 8+1\times 2}{3+1}\right) = \left(\frac{6-2}{4}, \frac{24+2}{4}\right)$$

$$= \left(\frac{4}{4}, \frac{26}{4}\right) = \left(1, \frac{13}{2}\right)$$

$\therefore$ The required coordinates are $\left(-1, \frac{7}{2}\right), (0,5)$ and $\left(1, \frac{13}{2}\right)$

10) Find the area of a rhombus, if its vertices are $(3,0)$, $(4,5)$, $(-1,4)$ and $(-2,-1)$ taken in order.



Using distance formula,

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$AC = \sqrt{(-1-3)^2+(4-0)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}\text{ units}$$

$$BD = \sqrt{(-2-4)^2+(-1-5)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}\text{ units}$$

$$\text{area (rhombus ABCD)} = \frac{1}{2}\times AC\times BD = \frac{1}{\cancel{2}}\times 4\sqrt{2}\times 6\sqrt{2} = \underline{\underline{24\text{ sq. units}}}$$

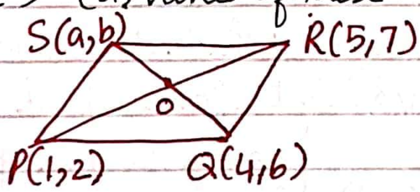
~~Topic-3~~

~~Area of~~

Additional Questions

1) If $P(1,2)$, $Q(4,6)$, $R(5,7)$ and $S(a,b)$ are the vertices of a parallelogram PQRS, then (a) $a=3, b=4$ (b) $a=2, b=3$ (c) $a=3, b=5$ (d) none of these.

Solution:-



Since diagonals bisect each other for a parallelogram, O is the mid-point of PR and SQ.

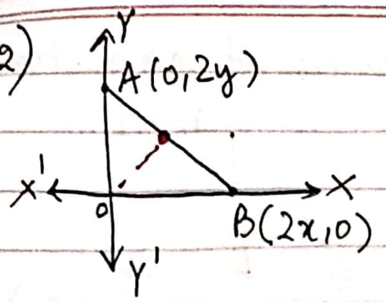
Thus using mid-section formula, $O(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$\Rightarrow \left(\frac{1+5}{2}, \frac{2+7}{2}\right) = \left(\frac{a+4}{2}, \frac{b+6}{2}\right)$$

$$\Rightarrow \left(3, \frac{9}{2}\right) = \left(\frac{a+4}{2}, \frac{b+6}{2}\right)$$

$\therefore \frac{a+4}{2} = 3 \Rightarrow a = 6-4$, $\underline{\underline{a=2}}$ | $\frac{b+6}{\cancel{2}} = \frac{9}{\cancel{2}} \Rightarrow b = 9-6$, $\underline{\underline{b=3}}$ (b)

2) The Coordinates of the point which is equidistant from the three vertices of the ΔAOB



(a) (x, y) (b) (y, x) (c) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (d) $\left(\frac{y}{2}, \frac{x}{2}\right)$

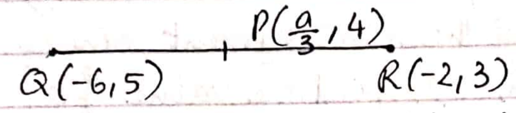
Solution:- Since ΔAOB is a right angled Δ, the mid-point of hypotenuse AB is equidistant from A, B and O.

$\therefore$ mid-point of AB $= \left(\frac{0+2x}{2}, \frac{2y+0}{2}\right) = \left(\frac{2x}{2}, \frac{2y}{2}\right) = \underline{(x, y)}$ (a)

3) If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points $Q(-6, 5)$ and $R(-2, 3)$, then the value of a is

(a) -4 (b) -12 (c) 12 (d) -6

Solution:-



Using mid-section formula, $P(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

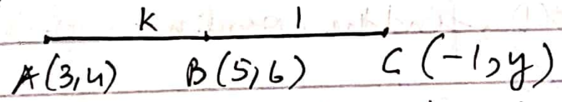
$\Rightarrow P\left(\frac{a}{3}, 4\right) = P\left(\frac{-6+-2}{2}, \frac{5+3}{2}\right)$

$\Rightarrow P\left(\frac{a}{3}, 4\right) = P(-4, 4)$

$\therefore \frac{a}{3} = -4 \Rightarrow \underline{a = -12}$ (b)

4) A straight line is drawn joining the points $(3,4)$ and $(5,6)$. If the line is extended, the ordinate of the point on the line, whose abscissa is -1 is ___

Solution:-



Let the ratio be $k:1$, Using Section formula,

$$B(x, y) = B\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$B(5, 6) = B\left(\frac{-1\times k+1\times 3}{k+1}, \frac{ky+1\times 4}{k+1}\right)$$

$$= B\left(\frac{-k+3}{k+1}, \frac{ky+4}{k+1}\right)$$

$\therefore \frac{-k+3}{k+1} = 5 \Rightarrow -k+3 = 5k+5$

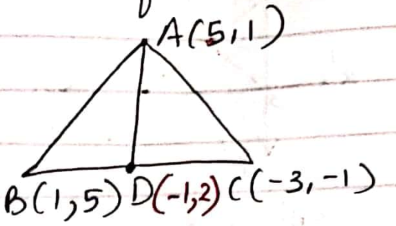
$\Rightarrow 6k = -2$

$k = -\frac{1}{3}$

Then $\frac{ky+4}{k+1} = 6 \Rightarrow \frac{-\frac{y}{3}+4}{-\frac{1}{3}+1} = 6 \Rightarrow -\frac{y}{3}+4 = \cancel{6}^{2}\times\frac{2}{\cancel{3}}$

$\Rightarrow -\frac{y}{3} = 0 \Rightarrow \underline{y = 0}$

5) A(5,1), B(1,5) and C(-3,-1) are the vertices of ΔABC. Find the length of median AD.



Solution:- Since D is the mid-point of BC, using mid-section formula

$$D(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{1-3}{2}, \frac{5-1}{2}\right) = \left(\frac{-2}{2}, \frac{4}{2}\right) = (-1,2)$$

Using distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$AD = \sqrt{(-1-5)^2+(2-1)^2} = \sqrt{(-6)^2+1^2} = \sqrt{36+1} = \sqrt{37} \text{ units}$$

Hence the length of median AD = $\sqrt{37}$ units

6) If the mid-point of the line segment joining the points P(6, b-2) and Q(-2,4) is (2,-3), find the value of b.

Solution:-

P(6, b-2) R(2,-3) Q(-2,4)

Using mid-section formula,

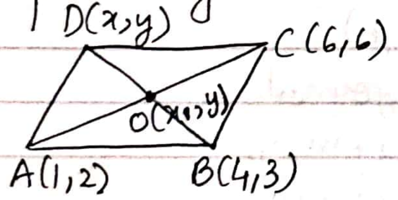
$$R(x,y) = R\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$R(2,-3) = R\left(\frac{6-2}{2}, \frac{b-2+4}{2}\right) = R\left(\frac{4}{2}, \frac{b+2}{2}\right)$$

$$= R\left(2, \frac{b+2}{2}\right)$$

$$\therefore \frac{b+2}{2} = -3 \Rightarrow b+2 = -6 \Rightarrow b = -8$$

7) If A(1,2), B(4,3) and C(6,6) are the three vertices of a parallelogram ABCD, find the coordinates of the fourth vertex D.



Since diagonals of a parallelogram bisect each other, O is the mid-point of AC and BD.

Using mid-section formula,

$$O(x,y) = O\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right), \text{ for diagonal AC}$$

$$= O\left(\frac{1+6}{2}, \frac{2+6}{2}\right) = O\left(\frac{7}{2}, 4\right)$$

Thus for diagonal BD, $O\left(\frac{7}{2}, 4\right) = O\left(\frac{x+4}{2}, \frac{y+3}{2}\right)$

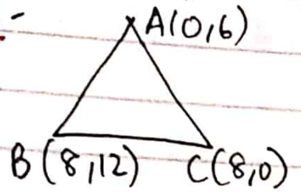
$$\therefore \frac{x+4}{\cancel{2}} = \frac{7}{\cancel{2}} \Rightarrow x = 3$$

$$\text{and } \frac{y+3}{2} = 4 \Rightarrow y = 5$$

Hence the coordinates of fourth vertex D is (3,5)

8) Find the coordinates of the centroid of a triangle whose vertices are (0,6), (8,12) and (8,0)

Soln:-

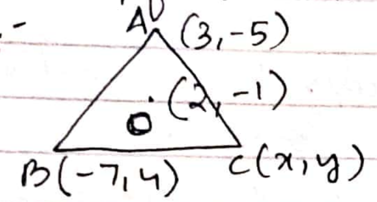


Centroid of a Δ whose vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3) is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$$= \left(\frac{0+8+8}{3}, \frac{6+12+0}{3}\right) = \left(\frac{16}{3}, \frac{18}{3}\right) = \underline{\underline{\left(\frac{16}{3}, 6\right)}}$$

9) Two vertices of a Δ are (3,−5) and (−7,4). If its centroid is (2,−1), find the third vertex.

Soln:-



Centroid of a Δ with vertices (x_1,y_1), (x_2,y_2) and (x_3,y_3) is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$$\Rightarrow (2,-1) = \left(\frac{3+-7+x}{3}, \frac{-5+4+y}{3}\right) = \left(\frac{-4+x}{3}, \frac{-1+y}{3}\right)$$

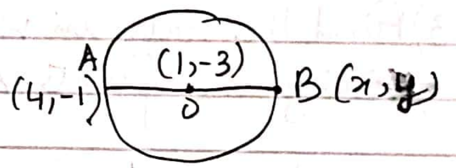
$$\therefore \frac{-4+x}{3} = 2 \Rightarrow -4+x = 6 \Rightarrow x = 10 //$$

$$\frac{-1+y}{3} = \cancel{-1} \Rightarrow -1+y = -3 \Rightarrow y = -2 //$$

Hence the third vertex is (10,−2)

10) The coordinates of one end point of a diameter of a circle are (4,−1) and the coordinates of the centre are (1,−3). Find the coordinates of the other end of the diameter.

Soln:- Since O is the Centre of AB, using mid-section formula



$$O(x,y) = O\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$O(1,-3) = O\left(\frac{4+x}{2}, \frac{-1+y}{2}\right)$$

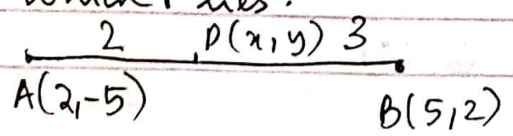
$$\therefore \frac{4+x}{2} = 1 \Rightarrow x = 2-4 = -2 //$$

$$\frac{-1+y}{2} = -3 \Rightarrow y = -6+1 = 5 //$$

Hence the coordinates of the other end of the diameter is (−2,−5) //

11) Point P divides the line segment joining the points A(2,−5) and B(5,2) in the ratio 2:3. Name the quadrant in which P lies.

Soln:-



Using Section formula,

$$P(x,y) = P\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$P(x,y) = \left(\frac{2\times5+3\times2}{2+3}, \frac{2\times2+3\times-5}{2+3}\right) = \left(\frac{10+6}{5}, \frac{4-15}{5}\right)$$

$$= \left(\frac{16}{5}, \frac{-11}{5}\right) = (3.2, -2.2)$$

Hence the $P(3.2, -2.2)$ lies in IV quadrant.

12) $P(5,-3)$ and $Q(3,y)$ are the points of trisection of the line segment joining $A(7,-2)$ and $B(1,-5)$. find y.

A(7,-2) P(5,-3) Q(3,y) B(1,-5)

Soln:-

A P Q B

1 1 1

1:2 2:1

Let $P(x,y)$ divides AB in the ratio 1:2 -

Using Section formula,

$$Q(x,y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$P(5,-3) = \left(\frac{1\times1+2\times7}{1+2}, \frac{1\times-5+2\times-2}{1+2}\right)$$

$$= \left(\frac{15}{3}, \frac{-9}{3}\right) = (5,-3)$$

$$Q(3,y) = \left(\frac{2\times1+1\times7}{2+1}, \frac{2\times-5+1\times-2}{2+1}\right) = \left(\frac{9}{3}, \frac{-12}{3}\right) = (3,-4)$$

$$\therefore \underline{y=-4}$$

13) Find the ratio in which $P(4,m)$ divides the line segment joining the points $A(2,3)$ and $B(6,-3)$. Hence find m.

k 1

A(2,3) P(4,m) B(6,-3)

Let the ratio be $k:1$

Using Section formula, $P(x,y) = P\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$$P(4,m) = P\left(\frac{k\times6+1\times2}{k+1}, \frac{k\times-3+1\times3}{k+1}\right)$$

$$= P\left(\frac{6k+2}{k+1}, \frac{-3k+3}{k+1}\right)$$

$$\therefore \frac{6k+2}{k+1} = 4 \Rightarrow 6k+2 = 4k+4 \Rightarrow 2k=2$$

$$\therefore k=1$$

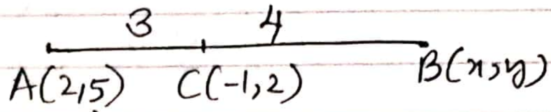
Hence the required ratio is 1:1

$\therefore m = \dfrac{-3k+3}{k+1}$

When $k = 1$, $m = \dfrac{-3+3}{2} = \underline{\underline{0}}$

14) If the point $C(-1,2)$ divides the line segment AB in the ratio $3:4$, where the coordinates of A are $(2,5)$, find the coordinates of B.

Soln:- Using Section formula,



$C(x,y) = C\left(\dfrac{m_1x_2+m_2x_1}{m_1+m_2}, \dfrac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$\Rightarrow C(-1,2) = \left(\dfrac{3x+8}{7}, \dfrac{3y+20}{7}\right)$

$\therefore \dfrac{3x+8}{7} = -1 \Rightarrow 3x = -7-8 = -15$

$x = \underline{\underline{-5}}$

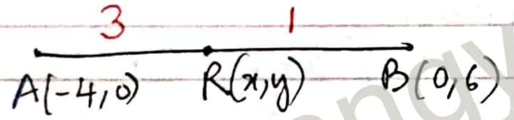
Also, $\dfrac{3y+20}{7} = 2$

$3y = 14-20 = -6$

$\underline{\underline{y = -2}}$

15) The point R divides the line segment AB where $A(-4,0)$, $B(0,6)$ are such that $AR = \frac{3}{4}AB$. Find the coordinates of R.

Soln:-



$AR = \dfrac{3}{4}AB$

$\dfrac{AR}{AB} = \dfrac{3}{4} \Rightarrow \dfrac{AB}{AR} = \dfrac{4}{3}$

$\Rightarrow \dfrac{AB}{AR} - 1 = \dfrac{4}{3} - 1$

$\Rightarrow \dfrac{AB-AR}{AR} = \dfrac{4-3}{3}$

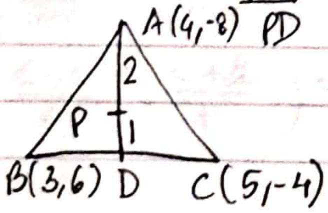
$\Rightarrow \dfrac{BR}{AR} = \dfrac{1}{3}$

Using Section Formula, $R(x,y) = R\left(\dfrac{3\times0+1\times-4}{3+1}, \dfrac{3\times6+1\times0}{3+1}\right)$

$= R\left(\dfrac{-4}{4}, \dfrac{18}{4}\right) = \underline{\underline{R\left(-1, \dfrac{9}{2}\right)}}$

16) If $A(4,-8)$, $B(3,6)$ and $C(5,-4)$ are the vertices of $\triangle ABC$, D is the mid point of BC and P is a point on AD joined such that $\dfrac{AP}{PD} = 2$, find the coordinates of P.

Soln:-

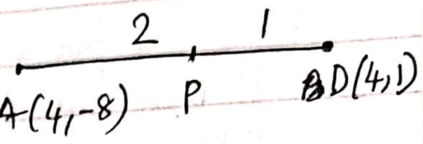


Since D is the mid-point of BC, using mid-section formula

$D(x,y) = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$$D(x,y) = D\left(\frac{3+5}{2}, \frac{6-4}{2}\right) = D(4,1)$$

Since P divides AD in the ratio 2:1, using section formula



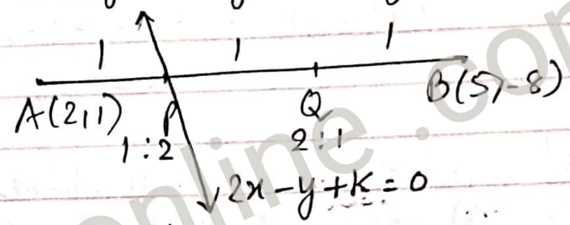
$$P(x,y) = P\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$= P\left(\frac{2\times4+1\times4}{2+1}, \frac{2\times1+1\times-8}{2+1}\right)$$

$$= P\left(\frac{12}{3}, \frac{-6}{3}\right) = P(4,-2)$$

17) The line segment joining the points A(2,1) and B(5,-8) is trisected at the points P and Q such that P is near to A. If P also lies on the line given by $2x-y+k=0$, find the value of k.

Soln:-



Let P divides AB in the ratio 1:2

Using Section Formula,

$$P(x,y) = P\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$= P\left(\frac{1\times5+2\times2}{1+2}, \frac{1\times-8+2\times1}{1+2}\right) = P\left(\frac{5+4}{3}, \frac{-8+2}{3}\right)$$

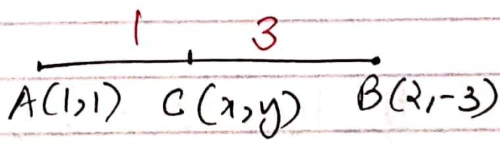
$$= P\left(\frac{9}{3}, \frac{-6}{3}\right) = P(3,-2)$$

Since P lies on $2x-y+k=0$

$$\Rightarrow 2\times3+2+k=0$$

$$\Rightarrow k=-8$$

18) If C is a point lying on the line segment AB joining A(1,1) and B(2,-3) such that $3AC = CB$, then find the coordinates of C.



Soln:- $3AC = CB$

$$\Rightarrow \frac{AC}{CB} = \frac{1}{3}$$

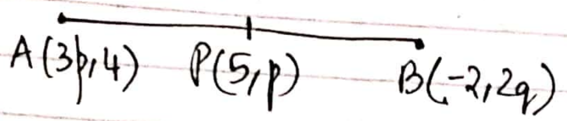
Using Section Formula, $C(x,y) = C\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$$C(x,y) = \left(\frac{1\times2+3\times1}{1+3}, \frac{1\times-3+3\times1}{1+3}\right) = \left(\frac{2+3}{4}, \frac{-3+3}{4}\right)$$

$$= \left(\frac{5}{4}, 0\right)$$

19) The coordinates of the mid-point of the line joining the points $(3p, 4)$ and $(-2, 2q)$ are $(5, p)$. Find the values of p and q.

Soln:-



Using mid-section formula,

$$P(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$P(5,p) = P\left(\frac{3p-2}{2}, \frac{4+2q}{2}\right)$$

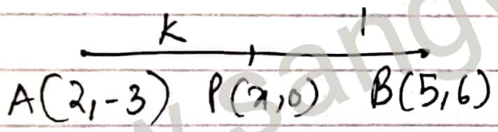
$$P(5,p) = P\left(\frac{3p-2}{2}, 2+q\right)$$

$$\therefore \frac{3p-2}{2} = 5 \Rightarrow 3p-2 = 10$$
$$\Rightarrow 3p = 12$$
$$p = 4$$

$$2+q = p \Rightarrow 2+q = 4$$
$$\Rightarrow q = 2$$

20) Find the ratio in which the line segment joining $(2,-3)$ and $(5,6)$ is divided by x-axis.

Soln:-



Let the ratio be $k : 1$

Using Section formula,

$$P(x,y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$P(x,0) = P\left(\frac{5k+2}{k+1}, \frac{6k-3}{k+1}\right)$$

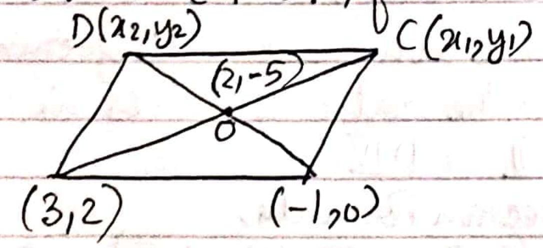
$$\therefore \frac{6k-3}{k+1} = 0$$

$$6k-3 = 0 \Rightarrow k = \frac{1}{2}$$

$\therefore$ The required ratio is $1:2$

21) If two vertices of a ||gm are $(3,2)$, $(-1,0)$ and diagonals cut at $(2,-5)$, find the other vertices of the ||gm.

Soln:-



Using mid-section formula

$$O(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

[$\because$ diagonals bisect each other if (x_1,y_1) and (x_2,y_2) are end points of a diagonal.

Thus for diagonal AC, $\frac{3+x_1}{2} = 2 \Rightarrow x_1 = 1$

$\frac{2+y_1}{2} = -5 \Rightarrow y_1 = -12$

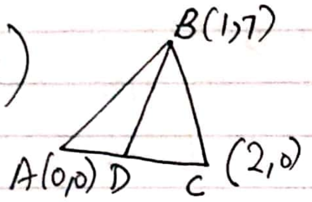
$\therefore$ The co-ordinates of C are $(1,-12)$

Also, for diagonal BD, $\frac{-1+x_2}{2} = 2 \Rightarrow x_2 = 5$

$\frac{y_2+0}{2} = -5 \Rightarrow y_2 = -10$

$\therefore$ The coordinates of D are $(5,-10)$

22)



BD bisects $\angle B$. Find the length of BD.

Soln:- Using angle bisector theorem, $\frac{AB}{BC} = \frac{AD}{DC} \rightarrow (1)$

Using distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB = \sqrt{(1-0)^2+(7-0)^2} = \sqrt{1+49} = \sqrt{50}$ units

$BC = \sqrt{(2-1)^2+(0-7)^2} = \sqrt{1+49} = \sqrt{50}$ units

From eq: (1), $\cancel{\frac{DC}{1}}$ $\cancel{\frac{AB}{BC}}$ $\frac{AD}{DC} = \frac{AB}{BC} = \frac{\sqrt{50}}{\sqrt{50}} = \frac{1}{1}$

$\therefore \frac{AD}{DC} = \frac{1}{1} \Rightarrow AD : DC = 1:1$

$\Rightarrow$ D is the mid-point of AC.

Using mid-section formula, $D(x,y) = D\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$D(x,y) = D\left(\frac{0+2}{2}, \frac{0+0}{2}\right)$

$= D(1,0)$

$\therefore BD = \sqrt{(1-1)^2+(0-7)^2} = \sqrt{49} = 7$ units

23) The point P which divides the line segment joining the points A(2,-5) and B(5,2) in the ratio 2:3 lies in the quadrant (a) I (b) II (c) III (d) IV

Soln:- A(2,-5) —2— P(x,y) —3— B(5,2)

Using Section Formula,

$P(x,y) = \left(\frac{2\times5+3\times\cancel{2}}{2+3}, \frac{2\times2+3\times-5}{2+3}\right) = \left(\frac{\cancel{16}}{5}, \frac{-11}{5}\right)$

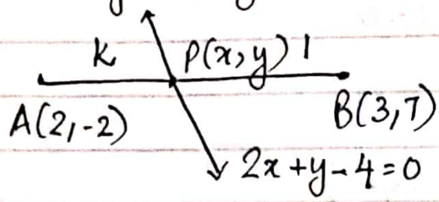
$= \left(\cancel{16}, -\frac{11}{5}\right)$

in IV quadrant.

Coordinate Geometry

* OPTIONAL EXERCISE 7.4

1) Determine the ratio in which the line $2x+y-4=0$ divides the line segment joining the points $A(2,-2)$ and $B(3,7)$.



Let the ratio be $k:1$.

Using Section Formula,

$$P(x,y) = P\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$P(x,y) = P\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$$

Since P lies on $2x+y-4=0$, $\frac{2(3k+2)}{k+1} + \frac{7k-2}{k+1} - 4 = 0$

$$6k+\cancel{4}+7k-2-4k-\cancel{4}=0$$

$$2k+7k-2=0$$

$$9k=2 \Rightarrow k=\frac{2}{9}$$

Hence the required ratio is $2:9$.

2) Find a relation between x and y, if the points (x,y), $(1,2)$ and $(7,0)$ are collinear.

Since the points are collinear, area formed $=0$.

area of a $\Delta = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right| = 0$

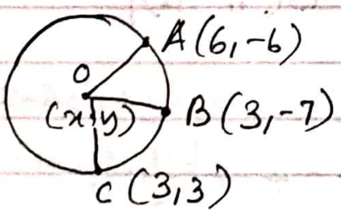
$$\Rightarrow \frac{1}{2}\left|x(2-0)+1(0-y)+7(y-2)\right|=0$$

$$\Rightarrow 2x-y+7y-2=0$$

$$\Rightarrow 2x+6y-14=0$$

$\Rightarrow x+3y-7=0$ is the required relation between x and y.

3) Find the centre of a circle passing through the points $(6,-6)$, $(3,7)$ and $(3,3)$



Using distance formula, $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Since OA, OB and OC are radii of the same circle, $OA=OB=OC$

$$\Rightarrow OA^2=OB^2=OC^2$$

$$\Rightarrow \underset{\mathrm{I}}{(6-x)^2+(-6-y)^2} = \underset{\mathrm{II}}{(3-x)^2+(-7-y)^2} = \underset{\mathrm{III}}{(3-x)^2+(3-y)^2}$$

$\Rightarrow$ From I and II, $36+\cancel{x^2}-12x+36+\cancel{y^2}+12y = 9+\cancel{x^2}-6x+49+\cancel{y^2}+14y$

$\Rightarrow -6x-2y+14=0 \Rightarrow 3x+y-7=0 \Longrightarrow (1)$

From II and III, $\cancel{(3-x)^2} + (-7-y)^2 = \cancel{(3-x)^2} + (3-y)^2$

$\Rightarrow 49 + \cancel{y^2} + 14y = 9 + \cancel{y^2} - 6y$

$\Rightarrow 20y = -40$

$\underline{y = -2}$

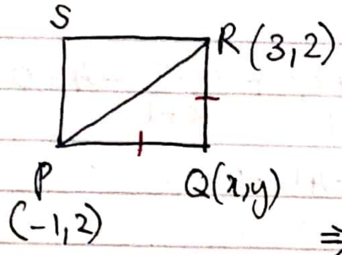
On Substituting in eq: (1), $3x - 2 - 7 = 0$

$3x = 9$

$\underline{x = 3}$

Hence the Centre of the Circle is $(3,-2)$

4) The two opposite vertices of a square are $(-1,2)$ and $(3,2)$. Find the coordinates of the other two vertices.



Using distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Since PQRS is a square,

$PQ = QR \Rightarrow PQ^2 = QR^2$

$\Rightarrow (x+1)^2 + \cancel{(y-2)^2} = (3-x)^2 + \cancel{(2-y)^2}$

$\Rightarrow \cancel{x^2} + 1 + 2x = 9 + \cancel{x^2} - 6x$

$\Rightarrow 8x = 8 \Rightarrow \underline{x = 1}$

Since $\angle Q = 90°$, using Pythagoras Theorem, $PQ^2 + QR^2 = PR^2$

$\Rightarrow (x+1)^2 + (y-2)^2 + (3-x)^2 + (2-y)^2 = (3+1)^2 + (2-2)^2$

$\Rightarrow \cancel{4} + y^2 + \cancel{4} - 4y + \cancel{4} + \cancel{4} + y^2 - 4y = \cancel{16}$

$2y^2 - 8y = 0$

$2y(y-4) = 0$

$\therefore \underline{y = 0, 4}$

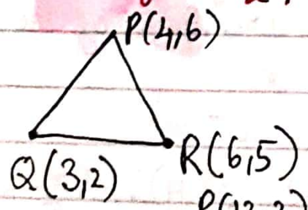
Hence the Coordinates of other two vertices are $(1,0)$ and $(1,4)$

5) (See the question from your text book)

(i) When A is taken as origin, AD is the x-axis and AB is the y-axis. The Coordinates are $P(4,6)$, $Q(3,2)$ and $R(6,5)$

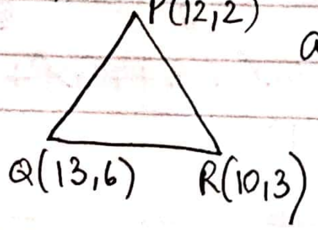
(ii) When C is taken as origin, then CB is the x-axis and CD is the y-axis. The coordinates are $P(12,2)$, $Q(13,6)$ and $R(10,3)$.

area of a $\Delta = \frac{1}{2} \left| x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) \right|$



Area$(\Delta PQR) = \frac{1}{2} \left| 4(2-5) + 3(5-6) + 6(6-2) \right|$

$= \frac{1}{2} \left| -12 - 3 + 24 \right| = \frac{9}{2}$ sq. units

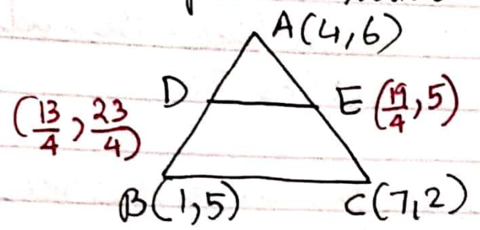


area $(\Delta PQR) = \frac{1}{2} \left| 12(6-3) + 13(3-2) + 10(2-6) \right|$

$= \frac{1}{2} \left| 36 + 13 - 40 \right| = \frac{9}{2}$ sq. units

Hence we observe that the area of Δ in both cases are same.

6) The vertices of a ΔABC are $A(4,6)$, $B(1,5)$ and $C(7,2)$. A line is drawn to intersect sides AB and AC at D and E respt, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the ΔADE and compare it with the area of ΔABC.



Given, $\frac{AD}{AB} = \frac{1}{4} \Rightarrow \frac{AB}{AD} = \frac{4}{1}$

$\Rightarrow \frac{AB}{AD} - 1 = \frac{4}{1} - 1$

$\Rightarrow \frac{AB - AD}{AD} = \frac{3}{1} \Rightarrow \frac{DB}{AD} = \frac{3}{1}$

$\Rightarrow \frac{AD}{DB} = \frac{1}{3}$

Thus D divides AB in the ratio 1:3

$A(4,6) \overset{1}{—} D(x,y) \overset{3}{—} B(1,5)$

Using section formula,

$$D(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$D(x,y) = D\left(\frac{1\times1 + 3\times4}{1+3}, \frac{1\times5 + 3\times6}{1+3}\right) = D\left(\frac{13}{4}, \frac{23}{4}\right)$$

Similarly, E also divides AC in the ratio 1:3

$A(4,6) \overset{1}{—} E(x,y) \overset{3}{—} C(7,2)$

$$E(x,y) = E\left(\frac{1\times7 + 3\times4}{1+3}, \frac{1\times2 + 3\times6}{1+3}\right) = E\left(\frac{19}{4}, 5\right)$$

area of a $\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$

$$\text{area}(\Delta ADE) = \frac{1}{2}\left|4\left(\frac{23}{4} - 5\right) + \frac{13}{4}(5-6) + \frac{19}{4}\left(6 - \frac{23}{4}\right)\right|$$

$$= \frac{1}{2}\left|3 - \frac{13}{4} + \frac{19}{16}\right| = \frac{1}{2}\left|\frac{48 - 52 + 19}{16}\right|$$

$$= \frac{1}{2} \times \frac{15}{16} = \frac{15}{32} \text{ sq. units}$$

$$\text{area}(\Delta ABC) = \frac{1}{2}\left|4(5-2) + 1(2-6) + 7(6-5)\right|$$

$$= \frac{1}{2}\left|12 - 4 + 7\right| = \frac{15}{2} \text{ sq. units}$$

$$\therefore \frac{\text{area}(\Delta ADE)}{\text{area}(\Delta ABC)} = \frac{15}{32} \times \frac{2}{15} = 1:16$$

7) Let $A(4,2)$, $B(6,5)$ and $C(1,4)$ be the vertices of $\triangle ABC$.

(i) The median from A meets BC at D. Find the coordinates of point D.

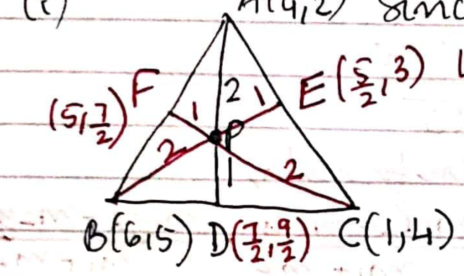
(ii) Find the coordinates of the point P on AD such that $AP:PD = 2:1$

(iii) Find the coordinates of points Q and R on medians BE and CF resp., such that $BQ:QE = 2:1$ and $CR:RF = 2:1$

(iv) What do you observe?

(v) If $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are the vertices of $\triangle ABC$, then find the coordinates of the centroid of the $\triangle$.

(i)



Since AD is the median of $\triangle ABC$, D is the mid-pt of BC

Using mid-section formula,

$$D(x,y) = D\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$$

(ii) Using section formula, $P(x,y) = P\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$$= \left(\frac{1\times4+2\times\frac{7}{2}}{2+1}, \frac{2\times\frac{9}{2}+1\times2}{2+1}\right) = P\left(\frac{11}{3}, \frac{11}{3}\right)$$

(iii) Since BE is the median of $\triangle ABC$, E is the mid-point of AC.

$$E(x,y) = E\left(\frac{4+1}{2}, \frac{2+4}{2}\right) = E\left(\frac{5}{2}, 3\right)$$

Since Q divides BE in the ratio 2:1,

$$Q(x,y) = Q\left(\frac{2\times\frac{5}{2}+1\times6}{2+1}, \frac{2\times3+1\times5}{2+1}\right) = Q\left(\frac{11}{3}, \frac{11}{3}\right)$$

Since CF is the median of $\triangle ABC$, F is the mid-point of AB.

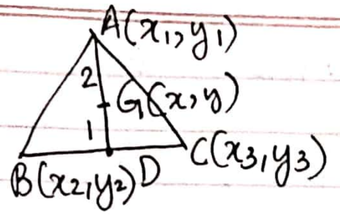
$$F(x,y) = F\left(\frac{4+6}{2}, \frac{2+5}{2}\right) = F\left(5, \frac{7}{2}\right)$$

Since R divides CF in the ratio 2:1,

$$R(x,y) = \left(\frac{1\times1+2\times5}{2+1}, \frac{1\times4+2\times\frac{7}{2}}{2+1}\right) = R\left(\frac{11}{3}, \frac{11}{3}\right)$$

(iv) We observe that the points P, Q and R coincide at $\left(\frac{11}{3}, \frac{11}{3}\right)$

(v) Three medians of a $\triangle$ intersect at a point called centroid. Centroid divides median in the ratio 2:1



Since AD is the median of $\triangle ABC$,
D is the mid-point of BC
$D(x, y) = D\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

Since $G(x,y)$ divides AD in the ratio 2 : 1,

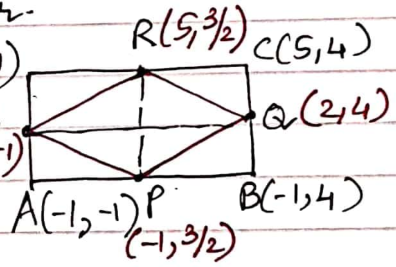
$$G(x,y) = G\left(\cancel{\frac{2x_3+x_1}{2+1}}, \frac{\cancel{2}\frac{(x_2+x_3)}{\cancel{2}}+x_1}{2+1}, \frac{\cancel{2}\times\frac{(y_2+y_3)}{\cancel{2}}+y_1}{2+1}\right)$$

$$= G\left(\frac{x_2+x_3+x_1}{3}, \frac{y_2+y_3+y_1}{3}\right)$$

Hence the coordinates of the Centroid of $\triangle ABC$

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

8) ABCD is a rectangle formed by the points A(-1,-1), B(-1,4), C(5,4) and D(5,-1). P, Q, R and S are the mid-points of AB, BC, CD and DA respt. Is the quad. PQRS a square, ? a rectangle or a rhombus? Justify your answer.



Since P, Q, R and S are the mid-points of AB, BC, DC and AD, using mid-section formula, Coordinates of mid-points

$$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$P(x,y) = P\left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = P\left(-1, \frac{3}{2}\right)$$

$$Q(x,y) = Q\left(\frac{5-1}{2}, \frac{4+4}{2}\right) = Q(2, 4)$$

$$R(x,y) = R\left(\frac{5+5}{2}, \frac{-1+4}{2}\right) = R\left(5, \frac{3}{2}\right)$$

$$S(x,y) = S\left(\frac{5-1}{2}, \frac{-1-1}{2}\right) = (2, -1)$$

Using distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$$PQ = \sqrt{(2+1)^2+\left(4-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}} = \frac{\sqrt{61}}{2} \text{ sq. units}$$

$$QR = \sqrt{(5-2)^2+\left(\frac{3}{2}-4\right)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2} \text{ sq. units}$$

$$RS = \sqrt{(2-5)^2+\left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2} \text{ sq. units}$$

$$SP = \sqrt{(-1-2)^2+\left(\frac{3}{2}+1\right)^2} = \sqrt{9+\frac{25}{4}} = \frac{\sqrt{61}}{2} \text{ sq. units}$$

diagonal PR $= \sqrt{(5+1)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} = \sqrt{36} = 6$ sq. units

diagonal SQ $= \sqrt{(2-2)^2 + (4+1)^2} = \sqrt{25} = 5$ sq. units.

Thus $PQ = QR = RS = PS$ and $PR \neq SQ$.
Hence quad. PQRS is a rhombus with all sides are equal and diagonals unequal.