XI

1. SETS

* Sets are used to define the concepts of relations and functions.
* The theory of sets was developed by German mathematician George Cantor (1845–1918)
* Some examples of sets used in Mathematics

  N → The set of Natural numbers
  Z → The set of all integers
  Q → The set of all rational numbers
  R → The set of real numbers (rational and irrational)
  Z^+ → The set of positive integers
  Q^+ → The set of positive rational numbers
  R^+ → The set of positive real numbers
* A set is a well-defined collection of objects
* There are two methods of representing a set:

  (i) Roster or tabular form

  eg:- The set of all vowels in the English alphabet
  $\{a, e, i, o, u\}$

  (ii) Set-builder form

  eg:- The set of all vowels in the English alphabet,
  $V = \{x : x \text{ is a vowel in English alphabet}\}$

Example 1:-

Write the solution set of the equation $x^2+x-2=0$ in roster form.

Solution:-

$x^2+x-2 = (x-1)(x+2) = 0$

S P
1 −2 < −1, 2

$\therefore x = 1, -2$

Hence the required solution set in roster form $= \{1, -2\}$

Example 2:-

Write the set $\{x : x \text{ is a positive integer and } x^2 < 40\}$ in the roster form.

Solution:-

$1^2 = 1 < 40$
$2^2 = 4 < 40$
$3^2 = 9 < 40$
$4^2 = 16 < 40$
$5^2 = 25 < 40$
$6^2 = 36 < 40$

Hence, the required set in roster form $= \{1, 2, 3, 4, 5, 6\}$

Example 3:-
Write the set $A = \{1, 4, 9, 16, 25 \ldots\}$ in set-builder form

Solution:-

$A = \{x : x$ is the square of a natural number$\}$

or $A = \{x : x = n^2$, where $n \in N\}$

Example 4:-
Write the set $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\}$ in the set-builder form

Solution:-

$$\{x : x = \frac{n}{n+1}, \text{ where } n \in N \text{ and } 1 \leq n \leq 6\}$$

Example 5:-
Match each of the set on the left described in the roster form with the same set on the right described in the set-builder form:-

(i) $\{P, R, I, N, C, A, L\}$

(ii) $\{0\}$

(iii) $\{1, 2, 3, 6, 9, 18\}$

(iv) $\{3, -3\}$

(a) $\{x : x$ is a positive integer and is a divisor of $18\}$

(b) $\{x : x$ is an integer and $x^2 - 9 = 0\}$

(c) $\{x : x$ is an integer and $x + 1 = 1\}$

(d) $\{x : x$ is a letter of the word PRINCIPAL$\}$

Solution:-

(i) $\rightarrow$ (d)

(ii) $\rightarrow$ (c)

(iii) $\rightarrow$ (a)

(iv) $\rightarrow$ (b)

EXERCISE 1.1

1) Which of the following are sets? Justify your answer.

(i) The collection of all the months of a year beginning with the letter J.

(ii) The collection of ten most talented writers of India.

(iii) A team of eleven best-cricket batsmen of the world.

(iv) The collection of all boys in your class.

(v) The collection of all natural numbers less than 100.

(vi) The collection of novels written by the writer Munshi Prem Chand.

(vii) The collection of all even integers

(viii) The Collection of questions in this Chapter.

(ix) A collection of most dangerous animals of the world.

Solution:-

(i) It is a well-defined collection of objects.
ie, January, June and July. Hence it is a set.

(ii) It is not a well-defined collection of persons since we cannot definitely decide which writer will be there in the collection. Hence it is not a set.

(iii) It is not a well-defined collection of players since we cannot definitely decide which player will be there in the collection. Hence it is not a set.

(iv) It is a well-defined collection of boys since we can definitely identify a boy who belongs to this collection. Hence it is a set.

(v) It is a well-defined collection of numbers since we can definitely identify a number which belongs to this collection. Hence it is a set.

(vi) It is a well-defined collection of books since we can definitely identify a book written by Munshi that belongs to this collection.

(vii) It is a well-defined collection of numbers since we can definitely identify an even number which belongs to this collection. Hence it is a set.

(viii) It is a well-defined collection of questions since we can definitely identify a question that belongs to this chapter. Hence it is a set.

(ix) It is not a well-defined collection of animals since we cannot definitely identify that which animal belongs to this collection. Hence it is not a set.

2) Let $A = \{1, 2, 3, 4, 5, 6\}$. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces.

(i) 5 _ A (ii) 8 _ A (iii) 0 _ A (iv) 4 _ A (v) 2 _ A (vi) 10 _ A

Solution:-

(i) $5 \in A$ (iv) $4 \in A$

(ii) $8 \notin A$ (v) $2 \in A$

(iii) $0 \notin A$ (vi) $10 \notin A$

3) Write the following sets in roster form:

(i) $A = \{x : x \text{ is an integer and } -3 \leq x < 7\}$

(ii) $B = \{x : x$ is a natural number less than $6\}$
(iii) $C = \{x : x$ is a two-digit natural number such that the sum of its digits is $8\}$
(iv) $D = \{x : x$ is a prime number which is divisor of $60\}$
(v) E = The set of all letters in the word TRIGONOMETRY
(vi) F = The set of all letters in the word BETTER.

Solution :-
(i) $A = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$
(ii) $B = \{1, 2, 3, 4, 5\}$
(iii) $C = \{17, 26, 35, 44, 53, 62, 71, 80\}$
(iv) $D = \{2, 3, 5\}$
(v) $E = \{T, R, I, G, O, N, M, E, Y\}$
(vi) $F = \{B, E, T, R\}$

4) Write the following sets in the set-builder form :
(i) $\{3, 6, 9, 12\}$
(ii) $\{2, 4, 8, 16, 32\}$
(iii) $\{5, 25, 125, 625\}$
(iv) $\{2, 4, 6, \ldots\}$
(v) $\{1, 4, 9, \ldots 100\}$

Solution :-
(i) $A = \{x : x = 3n$, where $n \in N$ and $n \leq 4\}$
(ii) $B = \{x : x = 2^n$, where $n \in N$ and $n \leq 5\}$
(iii) $C = \{x : x = 5^n$, where $n \in N$ and $n \leq 4\}$
(iv) $D = \{x : x = 2n$, where $n \in N\}$
(v) $E = \{x : x = n^2$, where $n \in N$ and $n \leq 10\}$

5) List all the elements of the following set :
(i) $A = \{x : x$ is an odd natural number$\}$
(ii) $B = \{x : x$ is an integer, $-\frac{1}{2} < x < \frac{9}{2}\}$
(iii) $C = \{x : x$ is an integer, $x^2 \leq 4\}$
(iv) $D = \{x : x$ is a letter in the word "LOYAL"$\}$
(v) $E = \{x : x$ is a month of a year not having 31 days$\}$
(vi) $F = \{x : x$ is a consonant in the English alphabet which precedes $k\}$

Solution :-
(i) $A = \{1, 3, 5, 7, 9, \ldots\}$
(ii) $B = \{0, 1, 2, 3, 4\}$

(iii) $C = \{-2, -1, 0, 1, 2\}$
(iv) $D = \{L, O, Y, A\}$
(v) $E = \{$February, April, June, September, November$\}$
(vi) $F = \{b, c, d, f, g, h, j\}$

6) Match each of the set on the left in the roster form with the same set on the right described in set-builder form.

(i) $\{1, 2, 3, 6\}$ (a) $\{x : x$ is a prime number and a divisor of $6\}$
(ii) $\{2, 3\}$ (b) $\{x : x$ is an odd natural number $< 10\}$
(iii) $\{M, A, T, H, E, I, C, S\}$ (c) $\{x : x$ is natural number and divisor of $6\}$
(iv) $\{1, 3, 5, 7, 9\}$ (d) $\{x : x$ is a letter of the word MATHEMATICS$\}$

Solution:-

(i) $\longrightarrow$ (c)
(ii) $\longrightarrow$ (a)
(iii) $\longrightarrow$ (d)
(iv) $\longrightarrow$ (b)

* A set which does not contain any element is called the empty set or the null set or the void set. It is denoted by ϕ or $\{\ \}$
* The number of distinct elements of a set S is denoted by $n(S)$
* A set which is empty or consists of a definite number of elements is called finite set; otherwise, the set is called infinite set.
* The set of real numbers cannot be described in the roster form.

Example 6:-

State which of the following sets are finite or infinite:

(i) $\{x : x \in N$ and $(x-1)(x-2) = 0\}$
(ii) $\{x : x \in N$ and $x^2 = 4\}$
(iii) $\{x : x \in N$ and $2x - 1 = 0\}$
(iv) $\{x : x \in N$ and x is prime$\}$
(v) $\{x : x \in N$ and x is odd$\}$

Solution:-

(i) The given set is $\{1, 2\}$. Hence it is finite.

(ii) The given set is $\{2\}$. Hence, it is finite.
(iii) The given set is $\{\ \}$. Hence, it is finite.
(iv) The given set is $\{2, 3, 5, 7, 11, \ldots\}$. Hence it is infinite.
(v) The given set is $\{1, 3, 5, 7, 9, \ldots\}$. Hence it is infinite.

* Two sets A and B are said to be equal, if they have exactly the same elements and denoted by $A = B$. Otherwise, the sets are said to be unequal and denoted by $A \neq B$.

Example 7:-

Find the pairs of equal sets, if any, give reasons:

$A = \{0\}$
$B = \{x : x > 15 \text{ and } x < 5\}$
$C = \{x : x - 5 = 0\}$
$D = \{x : x^2 = 25\}$
$E = \{x : x \text{ is an integral positive root of the equation } x^2 - 2x - 15 = 0\}$

Solution:- we have,

$A = \{0\}$
$B = \{\ \}$
$C = \{5\}$
$D = \{-5, 5\}$
$E = \{5\}$

$$x^2 - 2x - 15 = 0$$
$$(x + 3)(x - 5) = 0$$
$$x = 5, -3$$

Thus, the only pair of equal sets is C and E.

Example 8:-

Which of the following pairs of sets are equal? Justify your answer.

(i) X, the set of letters in "ALLOY" and B, the set of letters in "LOYAL".
(ii) $A = \{n : n \in Z \text{ and } n^2 \leq 4\}$ and $B = \{x : x \in R \text{ and } x^2 - 3x + 2 = 0\}$

Solution:-

(i) $X = \{A, L, O, Y\}$
$B = \{L, O, Y, A\}$
$\therefore X = B$

(ii) $A = \{-2, -1, 0, 1, 2\}$
$B = \{1, 2\}$

$\therefore A \neq B$

$x^2 - 3x + 2 = 0$
$(x-1)(x-2) = 0$
$\therefore x = 1, 2$

EXERCISE 1.2

1) Which of the following are examples of the null set.
(i) Set of odd natural numbers divisible by 2
(ii) Set of even prime numbers
(iii) $\{x : x$ is a natural number, $x < 5$ and $x > 7\}$
(iv) $\{y : y$ is a point common to any two parallel lines$\}$

Solution:-
(i) Since there is no odd number divisible by 2, it is a null set
(ii) Since 2 is the only even prime number, it is not a null set.
(iii) Since a number cannot be simultaneously less than 5 and more than 7, it is a null set.
(iv) Since parallel lines cannot have any point in common, it is a null set.

2) Which of the following sets are finite or infinite
(i) The set of months of a year
(ii) $\{1, 2, 3, \ldots\}$
(iii) $\{1, 2, 3, \ldots, 99, 100\}$
(iv) The set of positive integers greater than 100
(v) The set of prime numbers less than 99.

Solution:-
(i) It is a finite set because it has 12 elements.
(ii) Since the given set has infiniti number of natural numbers, it is an infinite set.
(iii) Since the give set has numbers from 1 to 100 only, it is a finite set.
(iv) Since there are infinitely many positive integers greater than 100, it is an infinite set.

(v) Since there are only 25 prime numbers less than 99, it is a finite set.

3) State whether each of the following set is finite or infinite.
(i) The set of lines which are parallel to the x-axis
(ii) The set of letters in the English alphabet
(iii) The set of numbers which are multiple of 5.
(iv) The set of animals living on the earth.
(v) The set of circles passing through the origin (0,0).

Solution:-
(i) Since we can draw infinitely many lines parallel to the x-axis, the given set is an infinite set.
(ii) Since there are only 26 letters in the English alphabet, the given set is a finite set.
(iii) Since there are infinitely many multiples of 5, the given set is an infinite set.
(iv) Since there are finite number of animals live on earth (eventhough it is a quite a big number), the given set is a finite set.
(v) Since infinite no. of circles can pass through the origin (0,0), the given set is an infinite set.

4) In the following, state whether A = B or not:
(i) $A = \{a, b, c, d\}$ $B = \{d, c, b, a\}$
(ii) $A = \{4, 8, 12, 16\}$ $B = \{4, 8, 16, 18\}$
(iii) $A = \{2, 4, 6, 8, 10\}$ $B = \{x : x \text{ is positive even integer and } x \leq 10\}$
(iv) $A = \{x : x \text{ is a multiple of } 10\}$ $B = \{10, 15, 20, 25, \ldots\}$.

Solution:-
(i) Since each element of set A is in set B and vice-versa
$\therefore A = B$
(ii) Since $12 \in A$ and $12 \notin B$, $A \neq B$
(iii) $B = \{2, 4, 6, 8, 10\}$
Thus each element of set A is in set B and vice-versa
$\therefore A = B$
(iv) $A = \{10, 20, 30, 40, \ldots\}$
Thus $15 \in B$ but $15 \notin A$. Thus $A \neq B$.

5) Are the following pair of sets equal? Give reasons.

(i) $A = \{2, 3\}$, $B = \{x : x \text{ is solution of } x^2+5x+6=0\}$

(ii) $A = \{x : x \text{ is a letter in the word FOLLOW}\}$

$B = \{y : y \text{ is a letter in the word WOLF}\}$

Solution:-

(i) $A = \{2, 3\}$

$B = \{-2, -3\}$

Thus $A \neq B$, the sets are not equal.

$x^2+5x+6=0$

$(x+3)(x+2)=0$

$\therefore x = -3, -2$

(ii) $A = \{F, O, L, W\}$

$B = \{W, O, L, F\}$

Since each element of set A is in set B, $A = B$.

The sets are equal.

6) From the sets given below, select equal sets.

$A = \{2, 4, 8, 12\}$, $B = \{1, 2, 3, 4\}$, $C = \{4, 8, 12, 14\}$,

$D = \{3, 1, 4, 2\}$, $E = \{-1, 1\}$, $F = \{0, a\}$, $G = \{1, -1\}$, $H = \{0, 1\}$

Solution :-

Since each element of set B is in set D and vice versa, $B = D$.

Since each element of set E is in set G and viceversa, $E = G$.

* A set A is said to be a subset of a set B, if every element of A is also an element of B. It is denoted by $A \subset B$, if $a \in A \Rightarrow a \in B$
* If $A = B$, then $A \subset B$ and $B \subset A$.
* Every set A is a subset of itself ie, $A \subset A$
* Null set is a subset of every set.
* $Q \subset R$; the set Q of rational numbers is a subset of the set R of real numbers.
* Let A and B be two sets. If $A \subset B$ and $A \neq B$, then A is called a proper subset of B and B is called superset of A.

  eg:- $A = \{1, 2\}$ is a proper subset of $B = \{1, 2, 3, 4\}$
* If a set A has only one element, then it is called a singleton set. eg :- $\{a\}$

Example 9:-

Consider the sets ϕ, $A = \{1, 3\}$, $B = \{1, 5, 9\}$, $C = \{1, 3, 5, 7, 9\}$.
Insert the symbol $\subset$ or $\not\subset$ between each of the following pair of sets:
(i) $\phi \ldots B$ (ii) $A \ldots B$ (iii) $A \ldots C$ (iv) $B \ldots C$

Solution:-

(i) Since null set is a subset of every set, $\phi \subseteq B$
(ii) Since $3 \notin B$, $A \not\subset B$
(iii) Since all elements of A are in C, then $A \subseteq C$
(iv) Since all elements of B are in C, then $B \subseteq C$

Example 10:-

Let $A = \{a, e, i, o, u\}$ and $B = \{a, b, c, d\}$. Is A a subset of B? No (why?). Is B a subset of A? No (why?)

Solution:-

Since all elements of A are not in B, A is not a subset of B. Similarly, since all elements of B are not in A, then B is also not a subset of A.

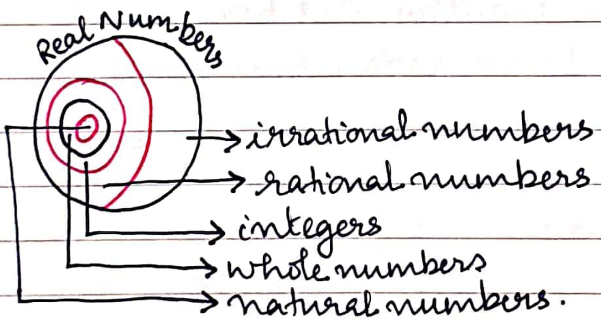
Example 11:-

Let A, B and C be three sets. If $A \in B$ and $B \subset C$, is it true that $A \subset C$? If not, give an example.

Solution:-

No. Let $A = \{3\}$; $B = \{\{3\}, 4, 5\}$; $C = \{\{3\}, 4, 5, 6\}$
Thus, $A \in B$ and $B \subset C$, but $A \not\subset C$ as $1 \in A$ and $1 \notin C$.
i.e., an element of a set can never be a subset of itself.

*



* The set of natural numbers, $N = \{1, 2, 3, 4, 5, \ldots\}$
* The set of integers, $Z = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$
* The set of rational numbers, $Q = \{x : x = \frac{p}{q}$ where $p, q \in Z$ and $q \neq 0\}$

* The set of irrational numbers, $T = \{x : x \in R \text{ and } x \notin Q\}$
eg:- $\sqrt{2}, \sqrt{5}, \pi$ etc.
* $N \subset Z \subset Q$
$Q \subset R$
$T \subset R$
$N \not\subset T$
* Let $a, b \in R$ and $a < b$, then
$\{y : a < y < b\}$ is called an open interval and is denoted by (a, b).
* The interval which contains the end points also is called closed interval and is denoted by $[a, b]$.
$\{x : a \leq x \leq b\}$
* $[a, b) = \{x : a \leq x < b\}$
* $(a, b] = \{x : a < x \leq b\}$
* $[0, \infty)$ defines the set of non-negative real numbers.
* $(-\infty, 0)$ defines the set of negative real numbers.
* $(-\infty, \infty)$ defines the set of real numbers from $-\infty$ to ∞.
* $(b - a)$ is the length of any of the intervals (a, b), $[a, b)$, $(a, b]$ or $[a, b]$.
* The collection of all subsets of a set A is called the power set of A. It is denoted by $P(A)$.
* In $P(A)$, every element is a set.
* If A is a set with $n(A) = m$, then $n[P(A)] = 2^m$.
* A universal set is a set which has elements of all the related sets, without any repetition of elements. It is denoted by U.

EXERCISE 1.3

1) Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces:
(i) $\{2, 3, 4\}$ ___ $\{1, 2, 3, 4, 5\}$
(ii) $\{a, b, c\}$ ___ $\{b, c, d\}$
(iii) $\{x : x$ is a student of class XI of your school$\}$ ___ $\{x : x$ is a student of your school$\}$
(iv) $\{x : x$ is a circle in the plane$\}$ ___ $\{x : x$ is a circle in the same plane with radius 1 unit$\}$

(v) $\{x : x$ is a triangle in a plane$\}$ — $\{x : x$ is a rectangle in the plane$\}$
(vi) $\{x : x$ is an equilateral Δ in a plane$\}$ — $\{x : x$ is a Δ in the same plane$\}$
(vii) $\{x : x$ is an even natural number$\}$ — $\{x : x$ is an integer$\}$

Solution:-

(i) $\{2,3,4\} \subseteq \{1,2,3,4,5\}$
(ii) $\{a,b,c\} \not\subseteq \{b,c,d\}$
(iii) $\{x : x$ is a student of class XI of your school$\} \subseteq \{x : x$ is a student of your school$\}$
(iv) $\{x : x$ is a circle in the plane$\} \not\subseteq \{x : x$ is a circle in the same plane with radius 1 unit$\}$
(v) $\{x : x$ is a Δ in a plane$\} \not\subseteq \{x : x$ is a rectangle in the plane$\}$
(vi) $\{x : x$ is an equilateral Δ in a plane$\} \subseteq \{x : x$ is a Δ in the same plane$\}$
(vii) $\{x : x$ is an even natural number$\} \subseteq \{x : x$ is an integer.

2) Examine whether the following statements are true or false:
(i) $\{a,b\} \not\subset \{b,c,a\}$
(ii) $\{a,e\} \subset \{x : x$ is a vowel in the English alphabet$\}$
(iii) $\{1,2,3\} \subset \{1,3,5\}$
(iv) $\{a\} \subset \{a,b,c\}$
(v) $\{a\} \in \{a,b,c\}$
(vi) $\{x : x$ is an even natural number less than $6\} \subset \{x : x$ is a natural number which divides $36\}$

Solution:-

(i) False; since all elements of $\{a,b\}$ are in set $\{b,c,a\}$
(ii) True; since all elements of $\{a,e\}$ are in set $\{a,e,i,o,u\}$
(iii) False; since 2 is an element of set $\{1,2,3\}$ but not an element of set $\{1,3,5\}$
(iv) True, since element a of set $\{a\}$ is in set $\{a,b,c\}$
(v) False, the elements of $\{a,b,c\}$ are a, b and c. Thus $\{a\} \subset \{a,b,c\}$
(vi) True, $\{2,4\} \subset \{1,2,3,4,6,9,12,18,36\}$

3) Let $A = \{1, 2, \{3,4\}, 5\}$. Which of the following statements are incorrect and why?

(i) $\{3,4\} \subset A$ (ii) $\{3,4\} \in A$ (iii) $\{\{3,4\}\} \subset A$
(iv) $1 \in A$ (v) $1 \subset A$ (vi) $\{1,2,5\} \subset A$
(vii) $\{1,2,5\} \in A$ (viii) $\{1,2,3\} \subset A$ (ix) $\phi \in A$
(x) $\phi \subset A$ (xi) $\{\phi\} \subset A$

Solution:-

(i) False; since $\{3,4\}$ is an element of A and $\{3,4\} \not\subset A$
(ii) True; since $\{3,4\}$ is an element of A and $\{3,4\} \in A$
(iii) True; since $\{3,4\}$ is an element of A.
(iv) True; since 1 is an element of A.
(v) False; since an element of a set can never be a subset of itself.
(vi) True; since elements of set $\{1,2,5\}$ are in Set A.
(vii) False; since $\{1,2,5\}$ are a subset of A but not elements.
(viii) False; since $3 \notin A$.
(ix) False; since null set is not an element of A but it is a subset of A.
(x) True; null set is a subset of every set.
(xi) False; since $\phi \subset A$ and $\{\phi\} \not\subset A$.

4) Write down all the subsets of the following sets:-
(i) $\{a\}$ (ii) $\{a, b\}$ (iii) $\{1,2,3\}$ (iv) ϕ

Solution:-

(i) $\phi, \{a\}$
(ii) $\{a\}, \{b\}, \{a,b\}, \phi$
(iii) $\{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}, \phi$
(iv) ϕ.

5) How many elements has P(A), if $A = \phi$?

Solution:- $A = \phi = \{\ \}$

$n(A) = 0$

$\therefore n[P(A)] = 2^0 = \underline{\underline{1}}$

6) Write the following as intervals:
(i) $\{x : x \in R, -4 < x \leq 6\}$ (ii) $\{x : x \in R, -12 < x < -10\}$
(iii) $\{x : x \in R, 0 \leq x < 7\}$ (iv) $\{x : x \in R, 3 \leq x \leq 4\}$

Solution:-

(i) $(-4, 6]$
(ii) $(-12, -10)$
(iii) $[0, 7)$
(iv) $[3, 4]$

7) Write the following intervals in set-builder form:
(i) $(-3, 0)$ (ii) $[6, 12]$ (iii) $(6, 12]$ (iv) $[-23, 5)$

Solution:-

(i) $\{x : x \in R, -3 < x < 0\}$
(ii) $\{x : x \in R, 6 \le x \le 12\}$
(iii) $\{x : x \in R, 6 < x \le 12\}$
(iv) $\{x : x \in R, -23 \le x < 5\}$

8) What universal set(s) would you propose for each of the following:
(i) The set of right triangles (ii) The set of isosceles triangles.

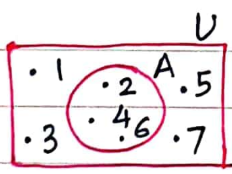
Solution:-

(i) The universal set can be the set of triangles
(ii) The universal set can be the set of triangles or the set of two dimensional figures.

9) Give the sets $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$ which of the following may be considered as universal set(s) for all the three sets A, B and C.
(i) $\{0, 1, 2, 3, 4, 5, 6\}$
(ii) ϕ
(iii) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
(iv) $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Solution:-

(iii) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$



* Venn diagrams are named after John Venn (1834 – 1883)
* The universal set is represented by a rectangular region and its subsets by circles.
* Also, an element of a set is represented by a point.

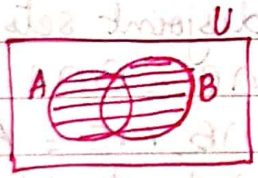
* **Union of Sets** :- Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and all the elements of B, the common elements being taken only once. It is denoted by the symbol '∪'.

Example 12:-
Let $A = \{2, 4, 6, 8\}$ and $B = \{6, 8, 10, 12\}$. Find $A \cup B$.
Solution:-
$A \cup B = \{2, 4, 6, 8, 10, 12\}$

Example 13:-
Let $A = \{a, e, i, o, u\}$ and $B = \{a, i, u\}$. Show that $A \cup B = A$.
Solution:-
$A \cup B = \{a, e, i, o, u\} = A$

* If $B \subset A$, then $A \cup B = A$.

Example 14:-
Let $X = \{$Ram, Geeta, Akbar$\}$ be the set of students of class XI, who are in school hockey team.
Let $Y = \{$Geeta, David, Ashok$\}$ be the set of students from class XI who are in school football team.
Find $X \cup Y$ and interpret the set.
Solution:-
$X \cup Y = \{$Ram, Geeta, Akbar, David, Ashok$\}$.
This set contains students who are in hockey team or the football team or both.

* 

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

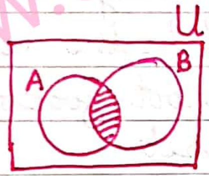
* Commutative law : $A \cup B = B \cup A$
* Associative law : $(A \cup B) \cup C = A \cup (B \cup C)$
* $A \cup \phi = A \rightarrow \phi$ is the identity of $\cup$
* Idempotent law : $A \cup A = A$
* Universal law : $U \cup A = U$

* $A \cap B = \{x : x \in A \text{ and } x \in B\}$
(Intersection of sets)

Example 15:-
Consider the sets A and B. Find $A \cap B$
$A = \{2, 4, 6, 8\}$ and $B = \{6, 8, 10, 12\}$
Solution:-
$A \cap B = \{6, 8\}$

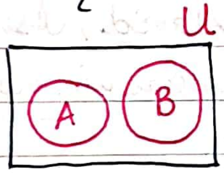
Example 16:-
Consider the sets X and Y. Find $X \cap Y$
$X = \{$Ram, Geeta, Akbar$\}$ and $Y = \{$Geeta, David, Ashok$\}$
Solution:-
$X \cap Y = \{$Geeta$\}$

Example 17:-
Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $B = \{2, 3, 5, 7\}$.
Find $A \cap B$ and hence show that $A \cap B = B$.
Solution:-
$A \cap B = \{2, 3, 5, 7\} = B$

* 

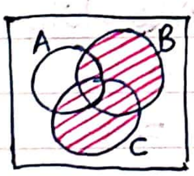
The intersection of two sets A and B is the set of all those elements which belong to both A and B.

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

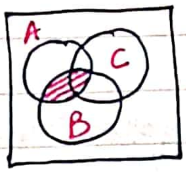
* 

If A and B are two sets such that $A \cap B = \phi$, then A and B are called disjoint sets

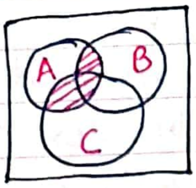
* Commutative law : $A \cap B = B \cap A$
* Associative law : $(A \cap B) \cap C = A \cap (B \cap C)$
* Law of ϕ and U : $\phi \cap A = \phi$, $U \cap A = A$
* Idempotent law : $A \cap A = A$
* Distributive law : $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

* 

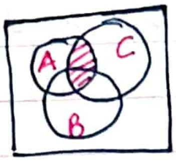
$B \cup C$

* 

$A \cap B$

* 

$A \cap (B \cup C)$ or $(A \cap B) \cup (A \cap C)$

* 

$A \cap C$

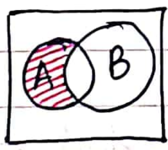
* Difference of sets :- $(A - B) \rightarrow$ the set of elements which belong to A but not to B.

Example 18:-
Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8\}$. find $A - B$ and $B - A$.
Solution:-
$A - B = \{1, 3, 5\}$
$B - A = \{8\}$

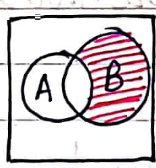
* $A - B \neq B - A$.

Example 19:-
Let $V = \{a, e, i, o, u\}$ and $B = \{a, i, k, u\}$.
Find $V - B$ and $B - V$.
Solution :-
$V - B = \{e, o\}$
$B - V = \{k\}$

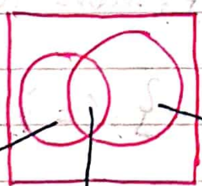
* $A - B = \{x : x \in A \text{ and } x \notin B\}$

* 

$A - B$



$B - A$

* 

$B - A$
$A - B$
$A \cap B$

$(A - B)$, $A \cap B$, $(B - A)$ are mutually disjoint sets.
$(A - B) \cap (A \cap B) \cap (B - A) = \phi$

EXERCISE 1.4

1) Find the union of each of the following pairs of sets:

(i) $X = \{1, 3, 5\}$ $\quad Y = \{1, 2, 3\}$

(ii) $A = \{a, e, i, o, u\}$ $\quad B = \{a, b, c\}$

(iii) $A = \{x : x$ is a natural number and multiple of $3\}$

$B = \{x : x$ is a natural number less than $6\}$

(iv) $A = \{x : x$ is a natural number, $1 < x \leq 6\}$

$B = \{x : x$ is a natural number and $6 < x < 10\}$

(v) $A = \{1, 2, 3\}$, $B = \phi$

Solution:-

(i) $X \cup Y = \{1, 2, 3, 5\}$

(ii) $A \cup B = \{a, b, c, e, i, o, u\}$

(iii) $A = \{3, 6, 9, 12, \ldots\}$

$B = \{1, 2, 3, 4, 5\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 9, 12, \ldots\}$

$= \{x : x = 1, 2, 4, 5$ or multiple of $3\}$

(iv) $A = \{2, 3, 4, 5, 6\}$

$B = \{7, 8, 9\}$

$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$

$= \{x : x \in N$ and $1 < x < 10\}$

(v) $A \cup B = \{1, 2, 3\} = A$.

2) Let $A = \{a, b\}$, $B = \{a, b, c\}$. Is $A \subset B$? What is $A \cup B$?

Solution:-

Yes $A \subset B$, since every element of A is in B.

$A \cup B = \{a, b, c\} = B$.

3) If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Solution:-

If A and B are two sets such that $A \subset B$, then $A \cup B = B$.

eg:- $A = \{a, b\}$, $B = \{a, b, c\}$, then $A \subset B$ and $A \cup B = \{a, b, c\}$

$= \underline{\underline{B}}$

4) If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$ find

(i) $A \cup B$ (ii) $A \cup C$ (iii) $B \cup C$ (iv) $B \cup D$ (v) $A \cup B \cup C$
(vi) $A \cup B \cup D$ (vii) $B \cup C \cup D$.

Solution:-

(i) $A \cup B = \{1, 2, 3, 4, 5, 6\}$
(ii) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$
(iii) $B \cup C = \{3, 4, 5, 6, 7, 8\}$
(iv) $B \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$
(v) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$
(vi) $A \cup B \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
(vii) $B \cup C \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

5) Find the intersection of each pair of sets
(i) $X = \{1, 3, 5\}$; $Y = \{1, 2, 3\}$
(ii) $A = \{a, e, i, o, u\}$; $B = \{a, b, c\}$
(iii) $A = \{x : x$ is a natural number and multiple of $3\}$
$B = \{x : x$ is a natural number less than $6\}$
(iv) $A = \{x : x$ is a natural number and $1 < x \leq 6\}$
$B = \{x : x$ is a natural number and $6 < x < 10\}$
(v) $A = \{1, 2, 3\}$, $B = \phi$

Solution:-

(i) $X \cap Y = \{1, 3\}$
(ii) $A \cap B = \{a\}$
(iii) $A = \{3, 6, 9, 12, \ldots\}$; $B = \{1, 2, 3, 4, 5\}$
$A \cap B = \{3\}$
(iv) $A = \{2, 3, 4, 5, 6\}$
$B = \{7, 8, 9\}$
$A \cap B = \phi$
(v) $A \cap B = \phi$

6) If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$; find
(i) $A \cap B$ (ii) $B \cap C$ (iii) $A \cap C \cap D$ (iv) $A \cap C$ (v) $B \cap D$
(vi) $A \cap (B \cup C)$ (vii) $A \cap D$ (viii) $A \cap (B \cup D)$
(ix) $(A \cap B) \cap (B \cup C)$ (x) $(A \cup D) \cap (B \cup C)$

Solution:-

(i) $A \cap B = \{7, 9, 11\}$

(ii) $B \cap C = \{11, 13\}$

(iii) $A \cap C \cap D = (A \cap C) \cap D = \{11\} \cap \{15, 17\} = \phi$

(iv) $A \cap C = \{11\}$

(v) $B \cap D = \phi$

(vi) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C) = \{7, 9, 11\} \cup \{11\}$
$= \{7, 9, 11\}$

(vii) $A \cap D = \{\ \}$

(viii) $A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$
$= \{7, 9, 11\} \cup \{\ \} = \{7, 9, 11\}$

(ix) $(A \cap B) \cap (B \cup C) = \{7, 9, 11\} \cap \{7, 9, 11, 13, 15\}$
$= \{7, 9, 11\}$

(x) $(A \cup D) \cap (B \cup C) = \{3, 5, 7, 9, 11, 15, 17) \cap \{7, 9, 11, 13, 15\}$
$= \{7, 9, 11, 15\}$

7) If $A = \{x : x$ is a natural number$\}$, $B = \{x : x$ is an even natural number$\}$, $C = \{x : x$ is an odd natural number$\}$ and $D = \{x : x$ is a prime number$\}$, find

(i) $A \cap B$ (ii) $A \cap C$ (iii) $A \cap D$ (iv) $B \cap C$ (v) $B \cap D$ (vi) $C \cap D$

Solution:-

$A = \{1, 2, 3, 4, 5 \ldots\}$

$B = \{2, 4, 6, 8, \ldots\}$

$C = \{1, 3, 5, 7, 9, \ldots\}$

$D = \{2, 3, 5, 7, \ldots\}$

(i) $A \cap B = \{2, 4, 6, 8, 10, 12, \ldots\} = B$

(ii) $A \cap C = \{1, 3, 5, 7, 9, \ldots\} = C$

(iii) $A \cap D = \{2, 3, 5, 7, \ldots\} = D$

(iv) $B \cap C = \phi$

(v) $B \cap D = \{2\}$

(vi) $C \cap D = \{3, 5, 7, 9, 11, \ldots\} = \{x : x$ is odd prime number$\}$

8) Which of the following pairs of sets are disjoint

(i) $\{1, 2, 3, 4\}$ and $\{x : x$ is a natural number and $4 \leq x \leq 6\}$

(ii) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$

(iii) $\{x : x$ is an even integer$\}$ and $\{x : x$ is an odd integer$\}$

Solution:-

(i) $A = \{1, 2, 3, 4\}$, $B = \{4, 5, 6\}$

$A \cap B = \{4\}$

Hence, not disjoint sets

(ii) $A = \{a, e, i, o, u\}$, $B = \{c, d, e, f\}$

$A \cap B = \{e\}$

Hence, not disjoint sets

(iii) $A = \{0, 2, 4, 6, 8, \dots\}$; $B = \{1, 3, 5, 7, \dots\}$

$A \cap B = \phi$

Hence, disjoint sets

9) If $A = \{3, 6, 9, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$, $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$, $D = \{5, 10, 15, 20\}$, find

(i) $A - B$ (ii) $A - C$ (iii) $A - D$ (iv) $B - A$ (v) $C - A$

(vi) $D - A$ (vii) $B - C$ (viii) $B - D$ (ix) $C - B$ (x) $D - B$

(xi) $C - D$ (xii) $D - C$

Solution:-

(i) $A - B = \{3, 6, 9, 15, 18, 21\}$

(ii) $A - C = \{3, 9, 15, 18, 21\}$

(iii) $A - D = \{3, 6, 9, 12, 18, 21\}$

(iv) $B - A = \{4, 8, 16, 20\}$

(v) $C - A = \{2, 4, 8, 10, 14, 16\}$

(vi) $D - A = \{5, 10, 20\}$

(vii) $B - C = \{20\}$

(viii) $B - D = \{4, 8, 12, 16\}$

(ix) $C - B = \{2, 6, 10, 14\}$

(x) $D - B = \{5, 10, 15\}$

(xi) $C - D = \{2, 4, 6, 8, 12, 14, 16\}$

(xii) $D - C = \{5, 15, 20\}$

10) If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, find

(i) $X - Y$ (ii) $Y - X$ (iii) $X \cap Y$

Solution:-

(i) $X - Y = \{a, c\}$

(ii) $Y - X = \{f, g\}$

(iii) $X \cap Y = \{b, d\}$

11) If R is the set of real numbers and Q is the set of rational numbers, then what is R−Q?

Solution:-

$R = \{x : x \in R\}$

$Q = \{x \mid x \in Q\}$

$R-Q = \{x : x \in T\}$ = set of irrational numbers.

12) State whether each of the following statement is true or false. Justify your answer.

(i) $\{2,3,4,5\}$ and $\{3,6\}$ are disjoint sets

(ii) $\{a,e,i,o,u\}$ and $\{a,b,c,d\}$ are disjoint sets.

(iii) $\{2,6,10,14\}$ and $\{3,7,11,15\}$ are disjoint sets

(iv) $\{2,6,10\}$ and $\{3,7,11\}$ are disjoint sets

Solution:-

(i) Let $A = \{2,3,4,5\}$, $B = \{3,6\}$

$A \cap B = \{3\}$

Hence the given sets are not disjoint sets. (False)

(ii) Let $A = \{a,e,i,o,u\}$ and $B = \{a,b,c,d\}$

$A \cap B = \{a\}$

Hence the given sets are not disjoint sets. (False)

(iii) Let $A = \{2,6,10,14\}$ and $B = \{3,7,11,15\}$

$A \cap B = \phi$

Hence the given sets are disjoint sets. (True)

(iv) Let $A = \{2,6,10\}$ and $B = \{3,7,11\}$

$A \cap B = \phi$

Hence the given sets are disjoint sets. (True)

* Let U be the Universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A. It is denoted by A'.

$A' = \{x : x \in U \text{ and } x \notin A\}$

$A' = U - A$

Example 20:-

Let $U = \{1,2,3,4,5,6,7,8,9,10\}$ and $A = \{1,3,5,7,9\}$

Find A'.

Solution:-

$A' = U - A = \{2, 4, 6, 8, 10\}$

* $(A')' = U - A'$
$= \{x : x \in U \text{ and } x \in A'\}$
$(A')' = A$
* $(A \cup B)' = A' \cap B'$

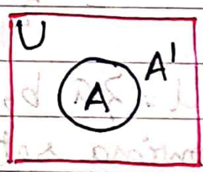
Example 21:-
Let U be universal set of all the students of class XI of a coeducational school and A be the set of all girls in class XI. find A'

Solution:-
$A' = U - A$ = Set of all boys in the class.

Example 22:-
Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3\}$ and $B = \{3, 4, 5\}$
find A', B', $A' \cap B'$, $A \cup B$ and hence show that $(A \cup B)' = A' \cap B'$

Solution:-
$A' = U - A = \{1, 4, 5, 6\}$
$B' = U - B = \{1, 2, 6\}$
$A' \cap B' = \{1, 6\}$
$A \cup B = \{2, 3, 4, 5\}$

LHS, $(A \cup B)' = U - (A \cup B) = \{1, 6\}$
RHS, $A' \cap B' = \{1, 4, 5, 6\} \cap \{1, 2, 6\} = \{1, 6\}$
$\therefore$ LHS = RHS
Hence $(A \cup B)' = A' \cap B'$



* $(A \cup B)' = A' \cap B'$
* $(A \cap B)' = A' \cup B'$

* De Morgan's law (named after the mathematician De Morgan):- The complement of the union of two sets is the intersection of their complements and the complement of the intersection of two sets is the union of their complements.

* Complement laws :
  (i) $A \cup A' = U$
  (ii) $A \cap A' = \phi$
* De Morgan's law :
  (i) $(A \cup B)' = A' \cap B'$
  (ii) $(A \cap B)' = A' \cup B'$
* Law of double complementation : $(A')' = A$
* Law of empty set and universal set
  $\phi' = U$
  $U' = \phi$

EXERCISE 1.5

1) Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. Find (i) A' (ii) B' (iii) $(A \cup C)'$ (iv) $(A \cup B)'$ (v) $(A')'$ (vi) $(B - C)'$

Solution :-

(i) $A' = U - A = \{5, 6, 7, 8, 9\}$

(ii) $B' = U - B = \{1, 3, 5, 7, 9\}$

(iii) $(A \cup C)' = U - (A \cup C) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4, 5, 6\}$
$= \{7, 8, 9\}$

(iv) $(A \cup B)' = U - (A \cup B) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 3, 4, 6, 8\}$
$= \{5, 7, 9\}$

(v) $(A')' = A = \{1, 2, 3, 4\}$

(vi) $(B - C)' = U - (B - C)$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 8\}$
$= \{1, 3, 4, 5, 6, 7, 9\}$

2) If $U = \{a, b, c, d, e, f, g, h\}$, find the complements of the following sets :
(i) $A = \{a, b, c\}$ (ii) $B = \{d, e, f, g\}$ (iii) $C = \{a, c, e, g\}$
(iv) $D = \{f, g, h, a\}$

Solution :-

(i) $A' = U - A = \{d, e, f, g, h\}$

(ii) $B' = U - B = \{a, b, c, h\}$

(iii) $C' = U - C = \{b, d, f, h\}$

(iv) $D' = U - D = \{b, c, d, e\}$

3) Taking the set of natural numbers as the Universal set, Write down the Complements of the following sets:

(i) $\{x : x$ is an even natural number$\}$

(ii) $\{x : x$ is an odd natural number$\}$

(iii) $\{x : x$ is a positive multiple of $3\}$

(iv) $\{x : x$ is a prime number$\}$

(v) $\{x : x$ is a natural number divisible by 3 and $5\}$

(vi) $\{x : x$ is a perfect square$\}$

(vii) $\{x : x$ is a perfect cube$\}$

(viii) $\{x : x + 5 = 8\}$

(ix) $\{x : 2x + 5 = 9\}$

(x) $\{x : x \geq 7\}$

(xi) $\{x : x \in N$ and $2x + 1 > 10\}$

Solution:-

(i) $\{1, 2, 3, 4, 5, \dots\} - \{2, 4, 6, 8, 10, \dots\}$
$= \{1, 3, 5, 7, 9, \dots\}$
$= \{x : x$ is an odd natural number$\}$

(ii) $\{1, 2, 3, 4, 5, \dots\} - \{1, 3, 5, 7, 9, \dots\}$
$= \{2, 4, 6, 8, 10, \dots\}$
$= \{x : x$ is an even natural number$\}$

(iii) $\{1, 2, 3, 4, 5, \dots\} - \{3, 6, 9, 12, 15, \dots\}$
$= \{1, 2, 4, 5, 7, 8, 10, \dots\}$
$= \{x : x \in N$ and x is not a multiple of $3\}$

(iv) $\{1, 2, 3, 4, 5, \dots\} - \{2, 3, 5, 7, 11, \dots\}$
$= \{1, 4, 6, 8, 9, 10, \dots\}$
$= \{x : x \in N$ where x is a composite number and $x = 1\}$

(v) $\{1, 2, 3, 4, 5, \dots\} - \{15, 30, 45, \dots\}$
$= \{1, 2, 3, 4, 5, \dots\}$
$= \{x : x \in N$ and x is not divisible by 3 and $5\}$

(vi) $\{1, 2, 3, 4, 5, \dots\} - \{1, 4, 9, 16, 25, \dots\}$
$= \{2, 3, 5, 6, 7, \dots\}$
$= \{x : x \in N$ and x is not a perfect square$\}$

(vii) $\{1, 2, 3, 4, 5, \dots\} - \{1, 8, 27, 64, 125, \dots\}$
$= \{x : x \in N$ and x is not a perfect cube$\}$

(viii) $\{1,2,3,4,5,\ldots\} - \{3\}$
$= \{x : x \in N \text{ and } x \neq 3\}$

(ix) $\{1,2,3,4,5,\ldots\} - \{2\}$
$= \{x : x \in N \text{ and } x \neq 2\}$

(x) $\{1,2,3,4,5,\ldots\} - \{7,8,9,10,11,\ldots\}$
$= \{x : x \in N \text{ and } x < 7\}$

(xi) $\{x : x \in N \text{ and } x < \frac{9}{2}\}$

4) If $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{2,4,6,8\}$ and $B = \{2,3,5,7\}$
Verify that (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Solution:-

(i) LHS, $(A \cup B)' = U - (A \cup B)$
$= \{1,2,3,4,5,6,7,8,9\} - \{2,3,4,5,6,7,8\}$
$= \{1,9\}$

RHS, $A' \cap B' = \{1,3,5,7,9\} \cap \{1,4,6,8,9\}$
$= \{1,9\}$

$\therefore$ LHS = RHS. Hence verified.

(ii) LHS, $(A \cap B) = \{2\}$
$(A \cap B)' = \{1,3,4,5,6,7,8,9\}$

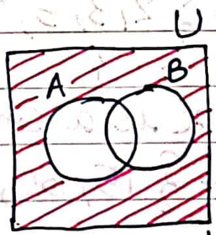
RHS, $A' \cup B' = \{1,3,5,7,9\} \cup \{1,4,6,8,9\}$
$= \{1,3,4,5,6,7,8,9\}$

$\therefore$ LHS = RHS. Hence verified.

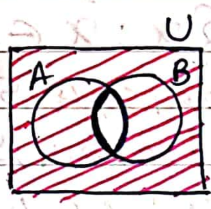
5) Draw appropriate Venn diagram for each of the following:-
(i) $(A \cup B)'$ (ii) $A' \cap B'$ (iii) $(A \cap B)'$ (iv) $A' \cup B'$

Solution:-

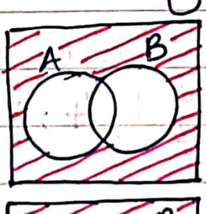
(i) $(A \cup B)'$



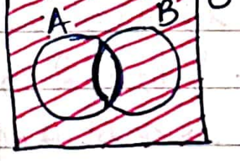
(iv) $A' \cup B'$



(ii) $A' \cap B'$



(iii) $(A \cap B)'$



6) Let U be the set of all triangles in a plane. If A is the set of all triangles with atleast one angle different from $60°$, what is A'?

Solution:-

A' is the set of all equilateral triangles.

7) Fill in the blanks to make each of the following a true statement:

(i) $A \cup A' = \_\_$ (ii) $\phi' \cap A = \_\_$ (iii) $A \cap A' = \_\_$ (iv) $U' \cap A = \_\_$

Solution:-

(i) $A \cup A' = \underline{U}$

(ii) $\phi' \cap A = U \cap A = \underline{A}$

(iii) $A \cap A' = \phi$

(iv) $U' \cap A = \phi$

* If $A \cap B = \phi$, then $n(A \cup B) = n(A) + n(B)$
* $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
* $(A-B) \cup (A \cap B) \cup (B-A) = A \cup B$
* $n(A-B) + n(A \cap B) + n(B-A) = n(A \cup B)$
* $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Example:- 23

If X and Y are two sets such that $X \cup Y$ has 50 elements, X has 28 elements and Y has 32 elements, how many elements does $X \cap Y$ have?

Solution:-

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$50 = 28 + 32 - n(X \cap Y)$

$\therefore n(X \cap Y) = 60 - 50 = 10$

Hence $X \cap Y$ has 10 elements

Example:- 24

In a school, there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics and 4 teach both physics and mathematics. How many teach physics?

Solution:-

Let M denotes set of teachers who teach mathematics and P denotes set of teachers who teach Physics.

Thus, $n(M \cup P) = 20$, $n(M) = 12$ and $n(M \cap P) = 4$

$\therefore n(M \cup P) = n(M) + n(P) - n(M \cap P)$

$20 = 12 + n(P) - 4$

$\therefore n(P) = 20 - 12 + 4 = 8 + 4 = \underline{12}$

Hence there are 12 teachers who teach physics.

Example 25:-

In a class of 35 students, 24 like to play cricket and 16 like to play football. Also, each student likes to play at least one of the two games. How many students like to play both cricket and football?

Solution:-

Let C denotes set of students who like to play cricket and F denotes set of students who like to play football.

Thus, $n(A \cup B) = 35$, $n(C) = 24$, $n(F) = 16$, $n(A \cap B) = ?$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$35 = 24 + 16 - n(A \cap B)$

$\therefore n(A \cap B) = 24 + 16 - 35 = 40 - 35 = 5$ //

Hence, no. of students who like to play both cricket and football = 5 students

Example 26:-

In a survey of 400 students in a school, 100 were listed as taking apple juice, 150 as taking orange juice and 75 were listed as taking both apple as well as orange juice. Find how many students were taking neither apple juice nor orange juice.

Solution:-

Let U denotes ~~the~~ set of all students in the school, A denotes set of students who take apple juice and O denotes set of students who take orange juice.

$$n(A \cup O)' = n(U) - n(A \cup O)$$
$$= n(U) - [n(A) + n(O) - n(A \cap O)]$$
$$= 400 - (100 + 150 - 75)$$
$$= 400 - 175 = 225 //$$

$\therefore$ No. of students who were taking neither apple juice nor orange juice = 225.

Example 27:-

There are 200 individuals with a skin disorder, 120 had been exposed to the chemical C_1, 50 to chemical C_2 and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to

(i) Chemical C_1 but not chemical C_2

(ii) Chemical C_2 but not chemical C_1

(iii) Chemical C_1 or Chemical C_2

Solution:-

Let U denotes total no. of individuals with skin disorder, A denotes set of individuals exposed to chemical C_1, B denotes set of individuals exposed to chemical C_2.

Thus $n(U) = 200$, $n(A) = 120$, $n(B) = 50$ and $n(A \cap B) = 30$

(i) $n(A - B) = n(A) - n(A \cap B) = 120 - 30 = 90$

$\therefore$ no. of individuals exposed to chemical C_1 but not C_2 $= 90$

(ii) $n(B - A) = n(B) - n(A \cap B) = 50 - 30 = 20$

$\therefore$ no. of individuals exposed to chemical C_2 but not C_1 $= 20$

(iii) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 120 + 50 - 30 = 120 + 20 = 140$

$\therefore$ no. of individuals exposed to chemicals C_1 or $C_2 = 140$

EXERCISE 1.6

1) If X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$ and $n(X \cup Y) = 38$, find $n(X \cap Y)$

Solution:-

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

$$38 = 17 + 23 - n(X \cap Y)$$

$$\therefore n(X \cap Y) = 17 + 23 - 38 = \underline{2}$$

2) If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements, how many elements does $X \cap Y$ have?

Solution:-

$n(X \cup Y) = 18$, $n(X) = 8$, $n(Y) = 15$

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$18 = 8 + 15 - n(X \cap Y)$

$\therefore n(X \cap Y) = 8 + 15 - 18 = 23 - 18 = \underline{\underline{5}}$

3) In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

Solution:-

Let U denotes total no. of people, A denotes the set of people who speak Hindi and B denotes the set of people who speak English.

$n(A \cup B) = 400$; $n(A) = 250$; $n(B) = 200$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$400 = 250 + 200 - n(A \cap B)$

$\therefore n(A \cap B) = 250 + 200 - 400 = 450 - 400 = 50$ //

$\therefore$ No. of people can speak both Hindi and English = 50

4) If S and T are two sets such that S has 21 elements, T has 32 elements and $S \cap T$ has 11 elements, how many elements does $S \cup T$ have?

Solution:-

$n(S) = 21$, $n(T) = 32$, $n(S \cap T) = 11$

$\therefore n(S \cup T) = n(S) + n(T) - n(S \cap T)$

$= 21 + 32 - 11 = 42$ //

Hence there are 42 elements in $S \cup T$.

5) If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have?

Solution:-

$n(X) = 40$, $n(X \cup Y) = 60$, $n(X \cap Y) = 10$

$\therefore n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$60 = 40 + n(Y) - 10$

Thus, $n(Y) = 60 - 40 + 10 = 30$ //

Hence there are 30 elements in set Y.

6) In a group of 70 people, 37 like coffee, 52 like tea and each person like atleast one of the two

drinks. How many people like both coffee and tea?

Solution:-

Let U denotes the set of total no. of people, A denotes the set of people who like coffee and B denotes the set of people who like tea.

Then, $n(U) = 70$, $n(A) = 37$, $n(B) = 52$.

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$70 = 37 + 52 - n(A \cap B)$

$n(A \cap B) = 37 + 52 - 70 = 19$

Hence, there are 19 people who like both coffee and tea.

7) In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

Solution:-

Let C denotes the set of people who like cricket and T denotes the set of people who like tennis.

$n(T \cup C) = 65$, $n(C) = 40$, $n(T) = ?$, $n(T \cap C) = 10$

Thus, $n(T \cup C) = n(T) + n(C) - n(T \cap C)$

$65 = n(T) + 40 - 10$

$\therefore n(T) = 65 - 40 + 10 = 35$

Hence there are 35 people who like tennis.

Also, $n(T - C) = n(T) - n(T \cap C)$

$= 35 - 10 = 25$

Hence there are 25 people who like tennis only.

8) In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak atleast one of these two languages?

Solution:-

Let F denotes the set of people who speak French and S denotes the set of people who speak Spanish.

Thus, $n(F) = 50$, $n(S) = 20$ and $n(S \cap F) = 10$

$\therefore n(F \cup S) = n(F) + n(S) - n(S \cap F)$

$= 50 + 20 - 10 = 60$

Hence, there are 60 people who speak atleast one of these two languages.

Example 28:-

Show that the set of letters needed to spell "CATARACT" and the set of letters needed to spell "TRACT" are equal.

Solution:-

Let X denotes the set of letters in "CATARACT" and Y denotes the set of letters in "TRACT".

Thus, $X = \{C, A, T, R\}$

and $Y = \{T, R, A, C\}$

$\therefore$ Every element in X is in Y and every element in Y is in X.

Thus, $X = Y$.

Example 29:-

List all the subsets of the set $\{-1, 0, 1\}$

Solution:-

Let $A = \{-1, 0, 1\}$.

Hence, all the subsets of A are $\phi, \{-1\}, \{0\}, \{1\}, \{-1, 0\}, \{0, 1\}, \{-1, 1\}$ and $\{-1, 0, 1\}$

Example 30:-

Show that $A \cup B = A \cap B$ implies $A = B$.

Solution:-

Let $a \in A$. Then, $a \in A \cup B$.

Since $A \cup B = A \cap B$, $a \in A \cap B$

Thus, $a \in B$. $\therefore A \subset B$

Similarly, if $b \in B$, then $b \in A \cup B$.

Since $A \cup B = A \cap B$, $b \in A \cap B$.

Thus $b \in A$ also

$\therefore B \subset A$

Therefore, $A \subset B$ and $B \subset A \Rightarrow A = B$.

Example 31:-

For any sets A and B, Show that $P(A \cap B) = P(A) \cap P(B)$

Solution:-

Let $X \in P(A \cap B)$.

Then $X \subset A \cap B \Rightarrow X \subset A$ and $X \subset B$.

Thus, $X \in P(A)$ and $X \in P(B) \Rightarrow X \in P(A) \cap P(B)$

$\therefore P(A\cap B) \subset P(A)\cap P(B)$

Let $Y \in P(A)\cap P(B) \Rightarrow Y \in P(A)$ and $Y \in P(B)$

$\Rightarrow Y \subset A$ and $Y \subset B$

$\Rightarrow Y \subset A\cap B$

$\Rightarrow Y \in P(A\cap B)$

$\therefore P(A)\cap P(B) \subset P(A\cap B)$

Hence, $P(A\cap B) = P(A)\cap P(B)$

Example 32:-

A market research group conducted a survey of 1000 consumers and reported that 720 consumers like product A and 450 consumers like product B, what is the least number that must have liked both products?

Solution:-

Let U denotes the set of all consumers, A denotes the set of consumers who like product A and B denotes the set of consumers who like product B.

Thus, $n(A\cup B) = 1000$, $n(A) = 720$, $n(B) = 450$.

$\therefore n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$\Rightarrow 1000 = 720 + 450 - n(A\cap B)$

$\Rightarrow n(A\cap B) = 720 + 450 - 1000 = 170$

Hence there are 170 consumers who liked both products.

Example 33:-

Out of 500 car owners investigated, 400 owned car A and 200 owned Car B, 50 owned both A and B cars. Is this data correct?

Solution:-

Let U denotes total set of car owners, A denotes set of owners who owned car A and B denotes set of owners who owned car B.

$$n(A\cup B) = n(A) + n(B) - n(A\cap B)$$

$$500 = 400 + 200 - n(A\cap B)$$

$$\therefore n(A\cap B) = 400+200-500 = 100 > 50$$, which is a contradiction.

Hence the given data is incorrect.

Example : 34

A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports?

Solution:-

Let F, B and C denote the set of men who received medals in football, basketball and cricket respectively.

Thus, $n(F) = 38$, $n(B) = 15$, $n(C) = 20$;

$$n(F\cup B\cup C) = 58, \quad n(F\cap B\cap C) = 3$$

$$\therefore n(F\cup B\cup C) = n(F)+n(B)+n(C) - n(F\cup B) - n(B\cup C) - n(F\cup C) + n(F\cap B\cap C)$$

$$58 = 38+15+20 - n(F\cup B) - n(B\cup C) - n(F\cup C) + 3$$

$$\therefore n(F\cup B) + n(B\cup C) + n(F\cup C) = 38+15+20+3-58 = 18$$

Hence no. of men who received medals in exactly two of the three sports = 18.