IX Revision - Heron's Formula

MCQs

1) If the side of a rhombus is 10cm and one diagonal is 12cm, the area of rhombus is
(a) $96cm^2$ (b) $48cm^2$ (c) $72cm^2$ (d) $80cm^2$

2) The area of an isosceles Δ having base 2cm and the length of one of the equal sides 4cm is
(a) $\sqrt{15}\ cm^2$ (b) $\sqrt{\frac{15}{2}}\ cm^2$ (c) $2\sqrt{15}\ cm^2$ (d) $4\sqrt{15}\ cm^2$

3) The sides of a triangle are 56cm, 60cm and 52cm long. Then the area of the triangle is
(a) $1322\ cm^2$ (b) $1311\ cm^2$ (c) $1344\ cm^2$ (d) $1392\ cm^2$

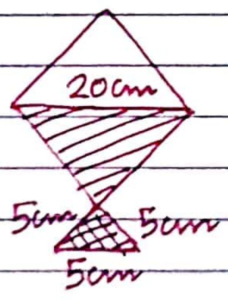
4) The area of an equilateral triangle with side $4\sqrt{3}$ cm is
(a) $20cm^2$ (b) $20\sqrt{3}\ cm^2$ (c) $18.784\ cm^2$ (d) $20.784\ cm^2$

5) The cost of levelling the ground in the form of triangle having sides 51m, 37m, 20m at the rate of ₹3 per m^2 is
(a) ₹306 (b) ₹918 (c) ₹725 (d) ₹900

6) The perimeter of an equilateral Δ is 60m. The area is
(a) $10\sqrt{3}\ m^2$ (b) $15\sqrt{3}\ m^2$ (c) $20\sqrt{3}\ m^2$ (d) $100\sqrt{3}\ m^2$

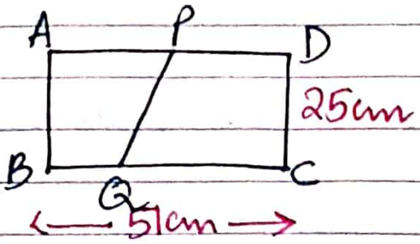
7) The edges of a triangular board are 6cm, 8cm and 10cm. The cost of painting it at ₹0.09 per cm^2 is
(a) ₹2.00 (b) ₹2.16 (c) ₹2.48 (d) ₹3.00

8) The length of each side of an equilateral Δ having an area of $9\sqrt{3}\ cm^2$ is
(a) 8cm (b) 36cm (c) 4cm (d) 6cm

9) 

A kite is in the shape of a square of diagonal 20cm and an equilateral Δ of base 5cm is made of three different shades as shown. How much paper for the shaded portion has been used?
(a) $100cm^2$ (b) $110.825cm^2$ (c) $215.588\ cm^2$ (d) $15.588\ cm^2$

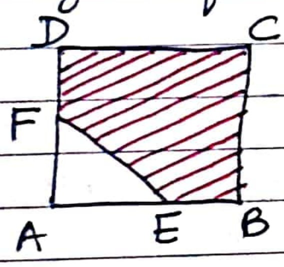
10) A rhombus shaped field has green grass for 36 cows to graze. If each side of the field is 30m and longer diagonal is 48m, then how much area of grass each cow will get, if $216m^2$ of area is not to be grazed.
(a) $6m^2$ (b) $12m^2$ (c) $18m^2$ (d) $29m^2$

11) 

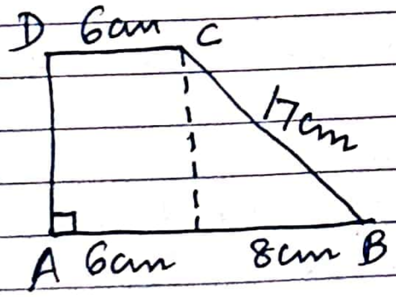
The dimensions of a rectangle ABCD are 51cm × 25cm. A trapezium PQCD with its parallel sides QC and PD in the ratio 9:8; is cut off from the rectangle. If the area of the trapezium PQCD is $\frac{5}{6}$th part of the area of the rectangle, find the lengths QC and PD.

12) Two parallel sides of a trapezium are 120cm and 154cm and other sides are 50cm and 52cm. Find the area of trapezium.

13) A forest reservoir is in the shape of quadrilateral whose sides taken in order are 9m, 40m, 15m and 28m. If the angle between first two sides is a right angle, find the area of a forest reservoir.

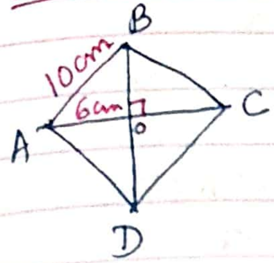
14) 

ABCD is a square of side 4cm. E and F are mid-points of AB and AD respectively. Find the area of the shaded region.

15) 

Calculate the area of trapezium.

IX Revision – Heron's Formula (Answers)

1) 

We know that diagonals of a rhombus bisect each other at 90°.

In rt. $\triangle OAB$, $OB^2 = AB^2 - OA^2 = 10^2 - 6^2$

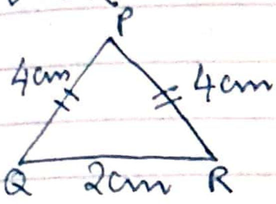
$= 100 - 36 = 64$

$OB = \sqrt{64} = 8\,cm$

$\therefore BD = 2OB = 2 \times 8 = 16\,cm$ //

Given, $AC = 12\,cm$

$$\text{area (rhombus } ABCD) = \frac{1}{2} \times AC \times BD = \frac{1}{\cancel{2}_1} \times \cancel{12}^{6} \times 16 = 96\,cm^2 \text{ (a)}$$

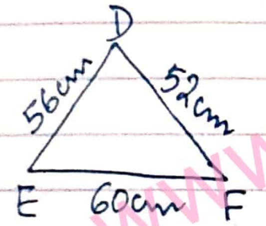
2) 

Let $a = 4\,cm$, $b = 4\,cm$, $c = 2\,cm$

$$S = \frac{a+b+c}{2} = \frac{4+4+2}{2} = \frac{10}{2} = 5\,cm$$

$$\text{area} (\triangle PQR) = \sqrt{S(S-a)(S-b)(S-c)} = \sqrt{5(5-4)(5-4)(5-2)}$$

$$= \sqrt{5 \times 1 \times 1 \times 3} = \sqrt{15}\,cm^2 \text{ (a)}$$

3) 

Let $a = 56\,cm$, $b = 52\,cm$, $c = 60\,cm$

$$S = \frac{a+b+c}{2} = \frac{56+52+60}{2} = \frac{168}{2} = 84\,cm$$

$$\text{area} (\triangle DEF) = \sqrt{S(S-a)(S-b)(S-c)} = \sqrt{84(84-56)(84-52)(84-60)}$$

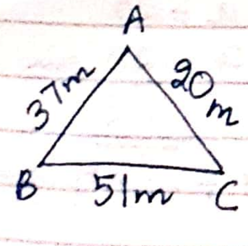
$$= \sqrt{\underbrace{84}_{12\times 7} \times \underbrace{28}_{7\times 4} \times \underbrace{32}_{16\times 2} \times \underbrace{24}_{12\times 2}} = 12 \times 7 \times 2 \times 4 \times 2$$

$$= 1344\,cm^2 \text{ (c)}$$

4) $a = 4\sqrt{3}\,cm$

$$\text{area of equilateral } \triangle = \frac{\sqrt{3}a^2}{4} = \frac{\sqrt{3} \times \cancel{4}^{1}\sqrt{3} \times 4\sqrt{3}}{\cancel{4}_1} = 12\sqrt{3}$$

$$= 12 \times 1.732 = 20.784\,cm^2 \text{ (d)}$$

5) 

Let $a = 51\,m$, $b = 37\,m$, $c = 20\,m$

$$S = \frac{a+b+c}{2} = \frac{51+37+20}{2} = \frac{108}{2} = 54\,m$$

$$\text{area} (\triangle ABC) = \sqrt{S(S-a)(S-b)(S-c)} = \sqrt{54(54-51)(54-37)(54-20)}$$

$$= \sqrt{\underbrace{54}_{9\times 3\times 2} \times \underbrace{3}_{3} \times \underbrace{17}_{17} \times \underbrace{34}_{17\times 2}} = 17 \times 2 \times 3 \times 3$$

$$= 306\,m^2$$ //

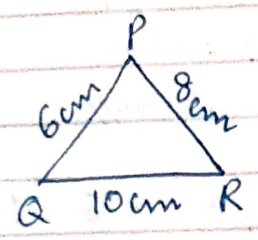
Cost of levelling = area × rate = 306 × 3 = ₹918 (b)

6) Perimeter of equilateral $\Delta = 3a = 60m$

$$a = \frac{60}{3} = 20m$$

area of equilateral $\Delta = \frac{\sqrt{3}a^2}{4} = \frac{\sqrt{3} \times 20 \times \cancel{20}^{5}}{\cancel{4}_{1}}$

$$= \underline{\underline{100\sqrt{3}\,m^2}} \text{ (d)}$$

7) 

Let $a = 6cm$, $b = 8cm$, $C = 10cm$

$$S = \frac{a+b+C}{2} = \frac{6+8+10}{2} = \frac{24}{2} = 12cm$$

area $(\Delta PQR) = \sqrt{S(S-a)(S-b)(S-c)}$

$$= \sqrt{12(12-6)(12-8)(12-10)}$$

$$= \sqrt{\underset{6\times2}{12} \times 6 \times 4 \times 2} = 6 \times 2 \times 2$$

$$= 24\,cm^2$$

$\therefore$ Cost of painting = area $\times$ rate

$$= 24 \times 0.09$$

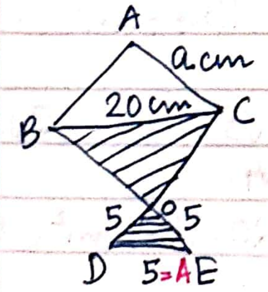
$$= \underline{\underline{₹\,2.16}} \text{ (b)}$$

8) area of an equilateral $\Delta = \frac{\sqrt{3}a^2}{4} = 9\sqrt{3}$

$$a^2 = \frac{9\cancel{\sqrt{3}}^{1} \times 4}{\cancel{\sqrt{3}}_{1}}$$

$$a^2 = 36$$

$$\therefore a = \sqrt{36} = 6cm \text{ (d)}$$

9) 

diagonal of a square $= \sqrt{2}a = 20$

$$a = \underline{\underline{\frac{20}{\sqrt{2}}}} \text{ cm}$$

$\therefore$ area of shaded portion $= \frac{\text{area of square}}{2} +$ area of equilateral Δ

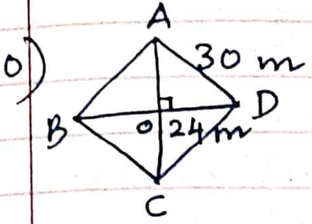
$$= \frac{a^2}{2} + \frac{\sqrt{3}A^2}{4}$$

$$= \frac{20}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \frac{1}{2} + \frac{\sqrt{3}}{4} \times 5 \times 5$$

$$= \frac{20\times20}{2\times2} + \frac{25\times1.732}{4} = \frac{400+43.3}{4}$$

$$= \frac{443.3}{4} = 110.825\,cm^2 \text{ (b)}$$

10)



Since diagonals of a rhombus bisect each other at right angle, $\angle AOD = 90°$.

In rt. ΔAOD, using Pythagoras Theorem

$$OA^2 = AD^2 - OD^2$$
$$= 30^2 - 24^2$$
$$= 900 - 576 = 324$$
$$OA = \sqrt{324} = 18\,m$$
$$\therefore AC = 2OA = 2\times18 = 36\,m$$

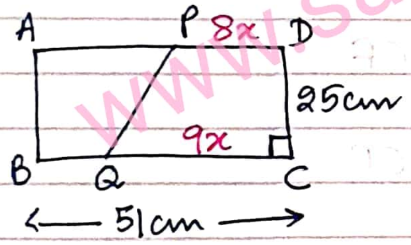
$$\therefore \text{area of field} = \frac{1}{2}\times d_1\times d_2 = \frac{1}{\cancel{2}}\times \cancel{48}^{24}\times36 = 864\,m^2$$

area of field not grazed by cows $= 216\,m^2$

$\therefore$ area of field grazed by cows $= 864 - 216 = 648\,m^2$

Then, area of field grazed by each cow $= \frac{648}{36} = 18\,m^2$ (C)

11)



Let QC and PD be $9x$ and $8x$ respectively.

Area of trapezium PDCQ $= \frac{1}{2}(PD+QC)\times DC$

$$= \frac{1}{2}(9x+8x)\times25 = \frac{17x\times25}{2}\,cm^2$$

area of rectangle ABCD $= BC\times DC = 51\times25\,cm^2$

ATQ, area (trap. PDCQ) $= \frac{5}{6}\times$ area (rect. ABCD)

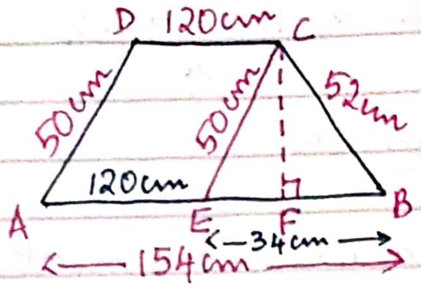
$$\Rightarrow \frac{17x\times\cancel{25}}{2} = \frac{5}{6}\times51\times\cancel{25}$$

$$\therefore x = \frac{5\times\cancel{51}^{3}\times\cancel{2}}{\cancel{6}_{3}\times\cancel{17}} = 5\,cm$$

Thus, $QC = 9x = 9\times5 = 45\,cm$ //

$PD = 8x = 8\times5 = 40\,cm$ //

12)



Construction : draw $CE \parallel AD$ to meet AB at E, to form AECD a parallelogram.

Then, $AD = CE = 50cm$

$DC = AE = 120cm$

Also, $EB = 154 - 120 = 34cm$

In $\triangle CEB$, let $a = 50cm$, $b = 34cm$ and $c = 52cm$.

$S = \frac{a+b+c}{2} = \frac{50+34+52}{2} = \frac{136}{2} = 68cm$

area of $\triangle CEB = \sqrt{S(S-a)(S-b)(S-c)}$ (using Heron's formula)

$= \sqrt{68(68-50)(68-34)(68-52)}$

$= \sqrt{\underset{34\times2}{68} \times \underset{9\times2}{18} \times \underset{34}{34} \times \underset{16}{16}}$

$= 34 \times 2 \times 3 \times 4$

$= 816\ cm^2$

Also, area($\triangle CEB$) $= \frac{1}{2} \times EB \times CF$

$\Rightarrow 816 = \frac{1}{2} \times 34 \times CF$

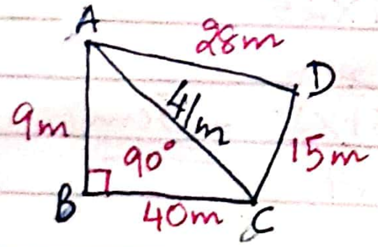
$\therefore CF = \frac{\cancel{2} \times \cancel{816}}{\cancel{34}\ \cancel{17}} = 48cm$ //

Thus, area(trap. ABCD) $= \frac{1}{2}(AB+DC) \times CF$

$= \frac{1}{\cancel{2}}(120+154) \times \cancel{48}^{24}$

$= 274 \times 24 = 6576\ cm^2$ //

13)



Using Pythagoras Theorem in rt. $\triangle ABC$,

$AC^2 = AB^2 + BC^2$

$= 9^2 + 40^2 = 81 + 1600 = 1681$

$\therefore AC = \sqrt{1681} = 41m$ //

area $(\Delta ABC) = \frac{1}{2} \times AB \times BC = \frac{1}{\cancel{2}_1} \times 9 \times \cancel{40}^{20} = 180 m^2$

Using Heron's formula in ΔADC, let $a = 41m, b = 15m, c = 28m$

$S = \frac{a+b+c}{2} = \frac{41+15+28}{2} = \frac{84}{2} = 42m$

area $(\Delta ADC) = \sqrt{S(S-a)(S-b)(S-c)}$

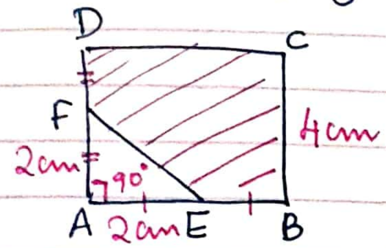
$= \sqrt{42(42-41)(42-15)(42-28)}$

$= \sqrt{\underbrace{42}_{3\ 14} \times 1 \times \underbrace{27}_{3\times 9} \times \underbrace{14}_{1\times 14}} = 14 \times 3 \times 3$

$= 126 m^2$

Thus, area of forest reservoir $=$ area$(\Delta ABC) +$ area(ΔADC)

$= 180 + 126 = \underline{\underline{306 m^2}}$

14) 

Area of square $ABCD =$ side $\times$ side

$= 4 \times 4 = \underline{\underline{16 cm^2}}$

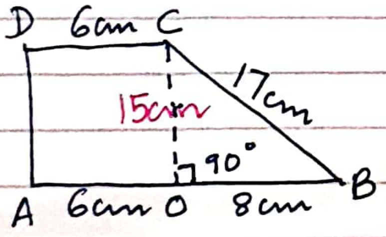
area $(\Delta FAE) = \frac{1}{2} \times AF \times AE$

$= \frac{1}{\cancel{2}_1} \times \cancel{2}^1 \times 2$ [$\because$ E and F are mid-pts]

$= \underline{\underline{2 cm^2}}$

$\therefore$ area of shaded region $=$ area(square ABCD) $-$ area(ΔFAE)

$= 16 - 2 = \underline{\underline{14 cm^2}}$

15) 

In rt. ΔCOB, using Pythagoras Theorem,

$OC^2 = BC^2 - OB^2 = 17^2 - 8^2 = 289 - 64$

$= 225$

$\therefore OC = \sqrt{225} = \underline{15 cm}$

Thus, area(trap. ABCD) $= \frac{1}{2}(AB + CD) \times OC$

$= \frac{1}{2}(14 + 6) \times 15$

$= \frac{1}{\cancel{2}_1} \times \cancel{20}^{10} \times 15 = \underline{\underline{150 cm^2}}$