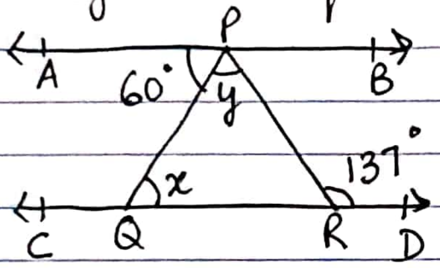
IX Homework – 10

1) Prove that $\dfrac{2^{30}+2^{29}+2^{28}}{2^{31}+2^{30}-2^{29}} = \dfrac{7}{10}$

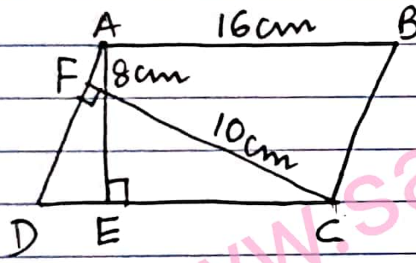
2) Factorise : $(a+b)^3 - 8$

3) If $3x+7y=14$, express y in terms of x. Also, check whether $(3,-2)$ is a point on the given line.

4) Prove that the sides opposite to equal angles of a triangle are equal.

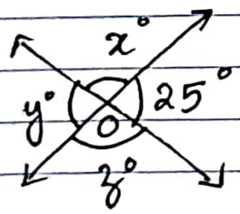
5) 

If $AB \parallel CD$, $\angle APQ = 60°$ and $\angle PRD = 137°$, then find the value of x and y.

6) 

ABCD is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16cm$, $AE = 8cm$ and $CF = 10cm$, find AD.

7) Simplify : $\dfrac{2}{\sqrt{5}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{2}} - \dfrac{3}{\sqrt{5}+\sqrt{2}}$

8) 

find the values of x, y and z

9) Find the remainder when $p(x) = 4x^4 - 3x^3 - 2x^2 + x - 7$ is divided by $(x+3)$

10) Express $1.4191919\ldots$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

IX Homework - 10 (Answers)

1)
$$\frac{2^{30}+2^{29}+2^{28}}{2^{31}+2^{30}-2^{29}} = \frac{\cancel{2^{28}}(2^2+2^1+1)}{\cancel{2^{28}}(2^3+2^2-2^1)}$$

$$= \frac{4+2+1}{8+4-2} = \frac{7}{10}$$

2)
$$(a+b)^3 - 8 = (\underset{x}{a+b})^3 - (\underset{y}{2})^3 \quad [x^3-y^3 = (x-y)(x^2+xy+y^2)]$$

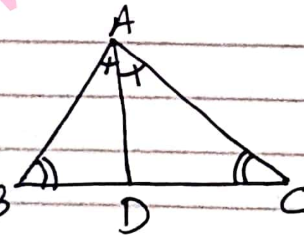
$$= (a+b-2)((a+b)^2 + 2(a+b) + 2^2)$$

$$= (a+b-2)[a^2+b^2+2ab+2a+2b+4]$$

3)
$$3x+7y = 14$$
$$7y = 14-3x$$
$$\therefore y = \frac{14-3x}{7}$$

When $x=3, y=-2$; $3x+7y = 3\times3+7\times(-2)$
$$= 9-14$$
$$= -5 \neq 14$$

Hence $(3,-2)$ is not a point on the given line.

4)


Given: In $\triangle ABC$, $\angle B = \angle C$

To prove: $AB = AC$

Construction: draw AD which bisects $\angle A$, to meet BC at D.

Proof: In $\triangle ADB$ and $\triangle ADC$, $\angle ABD = \angle ACD$ (given)

$\angle BAD = \angle CAD$ ($\because$ AD bisects $\angle A$)

$AD = AD$ (Common side)

$\therefore \triangle ADB \cong \triangle ADC$ (AAS Congruency)

Thus $AB = AC$ (by CPCT)

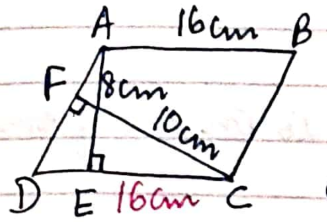
Hence sides opposite to equal angles of a $\triangle$ are equal.

5) $\angle PQR = \angle APQ = 60°$ (alternate interior angles, Since $AB \parallel CD$ and PQ is the transversal)

$\therefore x = 60°$

Using exterior angle property in $\triangle PQR$, $x+y=137^\circ$

$$y = 137^\circ - 60^\circ$$
$$= \underline{77^\circ}$$

6) 

$AB = DC = 16cm$ (opposite sides of a ||gm)

$$\text{area}(\|\text{gm } ABCD) = b \times h$$
$$= DC \times AE$$
$$= 16 \times 8 = 128\ cm^2$$

Also, $\text{area}(\|\text{gm } ABCD) = CF \times AD = 128$

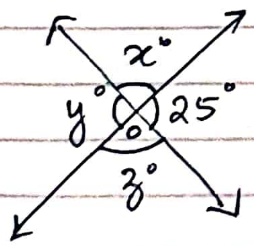
$$\therefore AD = \frac{128}{10} = \underline{\underline{12.8cm}}$$

7) $$\frac{2}{\sqrt{5}+\sqrt{3}} = \frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} = \frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{2(\sqrt{5}-\sqrt{3})}{5-3}$$
$$= \frac{\cancel{2}^{1}(\sqrt{5}-\sqrt{3})}{\cancel{2}_{1}} = \sqrt{5}-\sqrt{3}$$

$$\frac{1}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3}-\sqrt{2}$$

$$\frac{3}{\sqrt{5}+\sqrt{2}} = \frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} = \frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2} = \frac{3(\sqrt{5}-\sqrt{2})}{5-2} = \frac{\cancel{3}^{1}(\sqrt{5}-\sqrt{2})}{\cancel{3}_{1}}$$
$$= \sqrt{5}-\sqrt{2}$$

$$\therefore \frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}} = \cancel{\sqrt{5}} - \cancel{\sqrt{3}} + \cancel{\sqrt{3}} - \cancel{\sqrt{2}} - \cancel{\sqrt{5}} + \cancel{\sqrt{2}} = \underline{\underline{0}}$$

8) 

$x + 25^\circ = 180^\circ$ (linear pair)

$$x = 180^\circ - 25^\circ = 155^\circ$$
$x = z = 155^\circ$ (V.O.A)

$y = 25^\circ$ (V.O.A)

9) When $p(x)$ is divided by $(x+3)$, the remainder $= p(-3)$

$$= 4(-3)^4 - 3(-3)^3 - 2(-3)^2 + (-3) - 7$$
$$= 324 + 81 - 18 - 10$$
$$= \underline{377}$$

10) Let $x = 1.4191919\ldots$

$10x = 14.191919\ldots$

$1000x = 1419.191919\ldots$

$990x = 1405$

$x = \frac{\cancel{1405}^{281}}{\cancel{990}_{198}} = \frac{281}{198}$, which is in the form $\frac{p}{q}$.